SIMULATION OF DIGITAL FILTERS

A PROJECT REPORT

SUBMITTED BY

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UNDER THE GUIDANCE OF

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SYNOPSIS

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Digital Filters have the advantage of flexibility and sharp cut-off frequencies over analog filters particularly at a very low frequencies. Also since they are software oriented, by simple modification in the Program, the behaviour can be changed. Digital filters are used in image processing, coding, signal processing, Geo Physics, Bio-medical etc.

The Digital Filters have their own advantage and disadvantage. Digital Filters can be classified as one dimensional, two dimensional and n-dimensional and also depending on their orders.

In this work Elliptical Low Pass, elliptical High Pass, elliptical Band Pass and elliptical Band elimination filters are designed and analysed for frequency response.

An efficient BASIC Programming language has been choosen to implement these design and analysis.

INTRODUCTION

SOFTWARE SIMULATION

INTRODUCTION

Filtering is a process by which the frequency spectrum of a signal can be modified, reshaped or manipulated according to some desired specification and it may entail amplifying or alternating a range of frequency components, rejecting or isolating one specific frequency component etc. Filtering is done to eliminate contaminations (noise), signal distortion, to resolve signals into their components, to band limit signals and to convert discrete signals into continuous signals.

A digital filter is a digital system that can filter discrete time signals. It can be implemented by means of software or by dedicated hardware and in both cases can be used to filter real time or non-real time signals. A band limited continuous time signal can be transformed into discrete time signal by means of sampling. Conversely a discrete time signal so generated can be used to regenerate the original continuous time signal by virtue of sampling theorem. Hence hardware digital filters can be used to perform real time filtering tasks which in the not too distant past were performed almost exclusively by analog filters.

Why do we go in for digital filters :

As a result of the advances in digital electronics in recent years, it is becoming practical in the use of digital computers and special purpose hardware digital circutry to perform various signal processing functions that were traditionally achieved with analog equipment.

The primary advantage is the flexibility of digital filters. It is easier to change software than to solder joints and traces on a PCB; once it is in production. A digital filter can be tailored for many different tasks.

There is imperfection of standard filter components used in analog filtering. Resistors have some capacitance and inductance, inductors have resistance and capacitance and so on. Although digital filters have limitations, they approach the theoretical limits more closely. Digital filters also give better performance at very low frequencies where as to large resistors,. capacitors and inductors required in analog create size, weight and cost problems.

This project consists of software simulation as well as Hardware implementation. A resume about the two sections is given below.

A. SOFTWARE SIMULATION :

The digital filter consists of subroutines or programs that manipulates the frequency spectrum of a signal. Software for the Elliptic digital filter has been developed here. This particular filter has been developed here. This particular filter has been chosen because of its excellent performance at low frequencies and a very narrow transition bandwidth. The entire software has been developed in BASIC. The program designs the ELLIPTIC FILTER and gives the frequency response characteristics of it.

Chapter I

Discrete Signals and Sampling Procedure

DISCRETE SIGNALS AND SAMPLING PROCEDURE

CONTINUOUS & DISCRETE SIGNALS :

To get a better understanding of the fundamental steps involved in the design of digital filter let us start with an brief overview of discrete signals and sampling procedure.

A signal is a function of one or more independent variables. There are two types of signals.

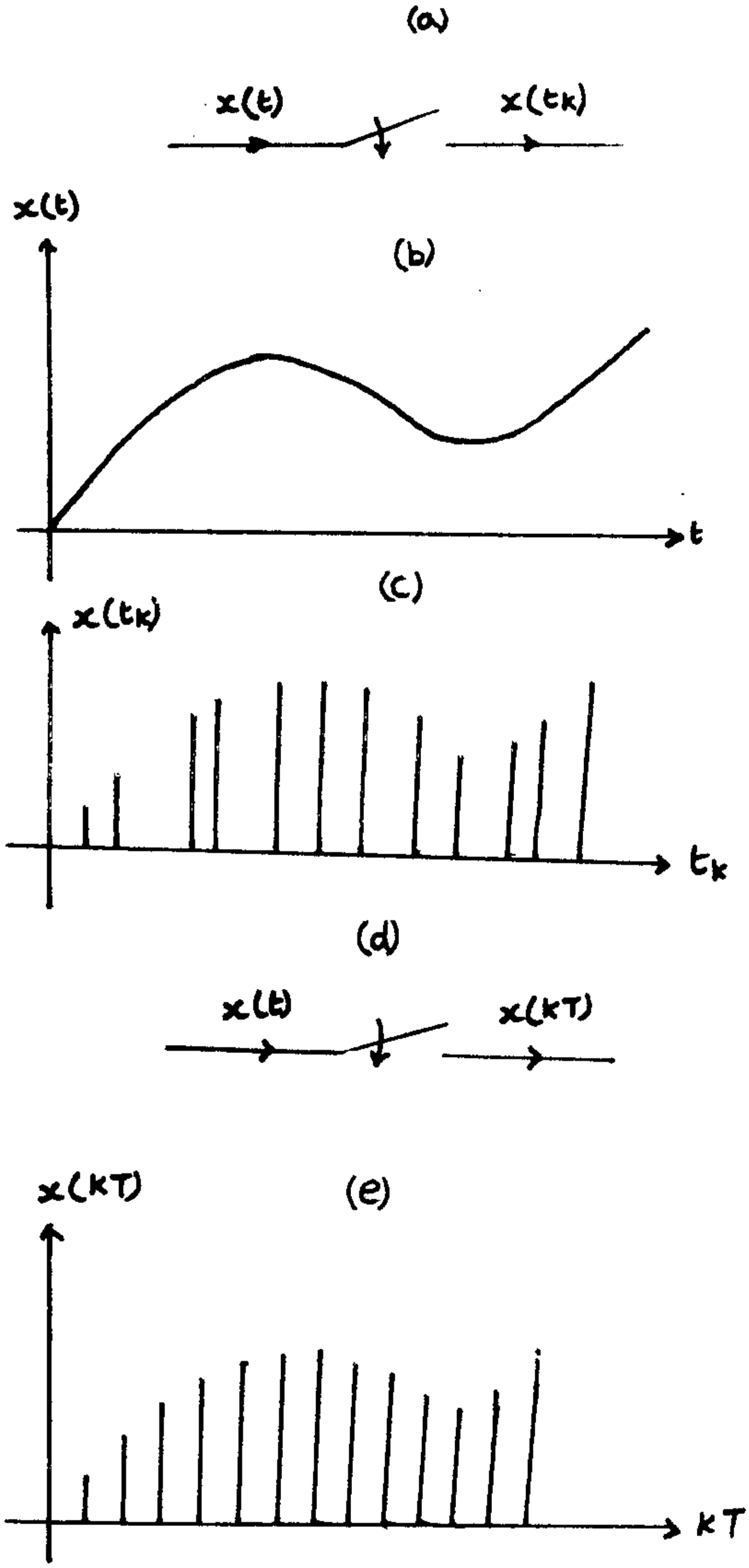
1. Continuous signals
2. Discrete signals

A continuous signal also called analog signal $x(t)$ varies continously with time 't' while a discrete signal called digital signal $x(k)$ is defined only as a sequence of discrete values of time 'K'.

a. Analog signal with continuous amplitude.
b. Discrete signal with continuous amplitude.

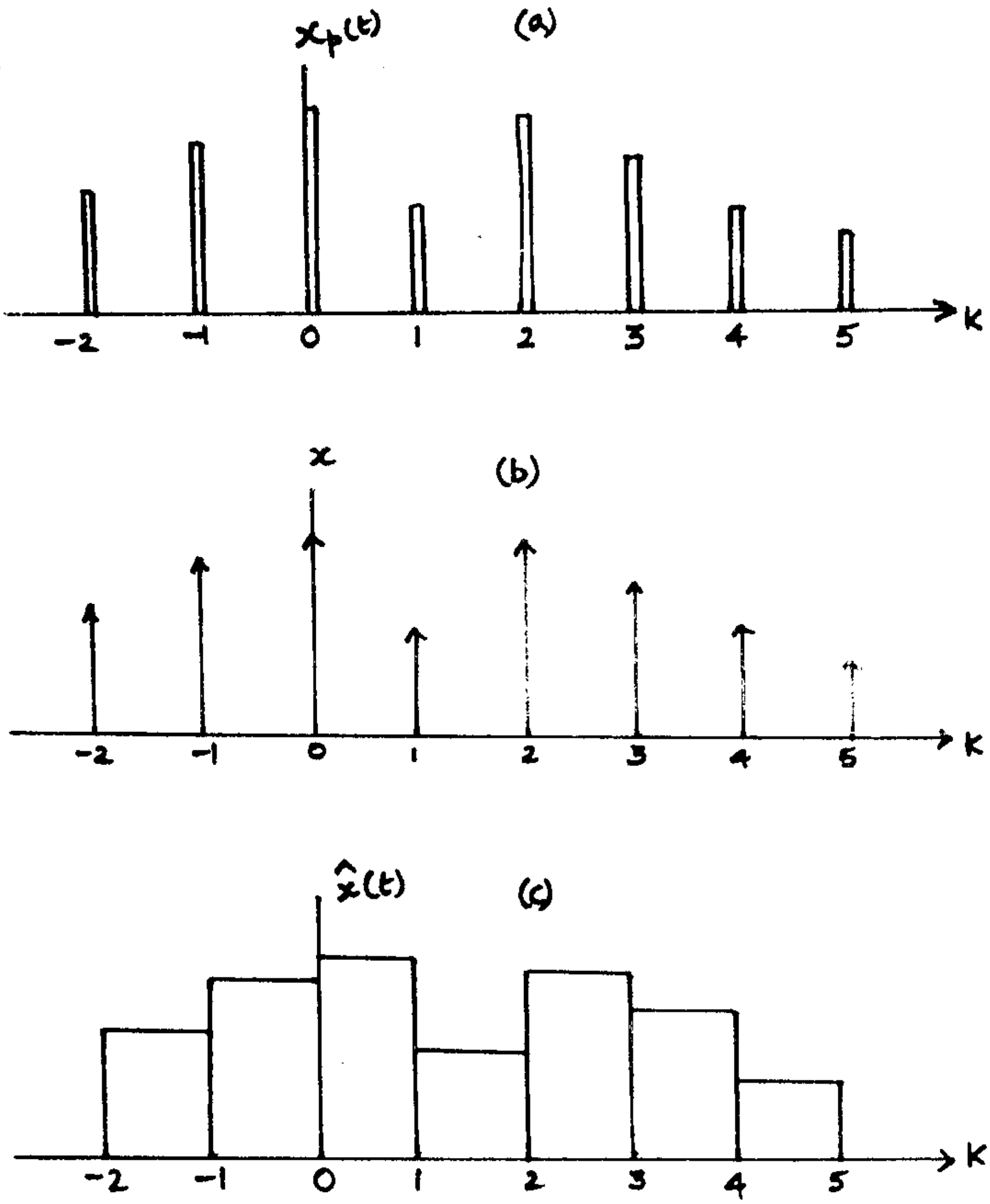
GENERATION OF A DISCRETE SIGNAL BY SAMPLING A CONTINUOUS SIGNAL.

In most cases we are concerned with an analog signal which is to be processed by a digital computer. This is accomplished by sampling the analog signal at discrete values of time.



(a) RANDOM SAMPLING (b) CONTINUOUS SIGNAL
(c) RANDOM SAMPLED SIGNAL (d) UNIFORM SAMPLING
(e) UNIFORMLY SAMPLED SIGNAL

Fig. 1



(a) SAMPLED SIGNAL AS A SERIES OF PULSES

(b) SAMPLED SIGNAL AS A SERIES OF IMPULSES

(c) A PIECEWISE – CONSTANT SIGNAL

Fig. 2

The sampling process is shown in fig.1. When the switch closes instantaneously at $t = tk$ and remains open during $t \neq tk$ it generates a sample of the signal $x\ (t_k)$. If the switch closes periodically at a uniform rate of T sec we get a discrete signal x (KT) indicating the value of the analog signal at time $t_k = KT$ and this signal is simply a sequence of numbers representing the sampled values of x(t). In a practical situation shown in Fig.2 the switch cannot open and close instantaneously and the sampled signal is a series of finite duration pulses $x_p\ (t)$, but it still caries information about $x^p\ (t)$.

The sampled signal is given by

$$x^*(t) = \sum_{k=-\infty}^{\infty} x\ (KT)\ \ \delta\ (t-KT)$$

where $\delta(t-KT)$ is a unit impulse occuring at time $t=KT$.

Sampling Theorem :

From sampling theorem, the continuous signal can be recovered from the sampled signal if the sampling rate is greater than at least twice the maximum frequency component of the analog signal and the pulse duration is much shorter than the sampling interval.

Chapter 2

Introduction to Digital Filters

DIGITAL FILTERS - INTRODUCTION

A digital filter is a computational process which manipulates on an input discrete signal to generate an output discrete signal according to some prescribed rules. The computational rule is either

A difference equation relating the input and output discrete signals.

or

The super position (convolution) summation expressing the output sequence in terms of the input sequence and the unit sample sequence of the filter.

Like analog filters, digital filters can be classified on the basis of magnitude - frequency response, as Low-pass, High - pass, Band - pass or Band - stop. The frequency response of digital filters is periodic. The period is equal to the sampling frequency of the filter.

The frequency response of a real filter (one with real coefficients) is symmetric about the folding frequency W_s /2, which is half of the sampling frequency.

The ideal magnitude - frequency relationship of the four types of digital filters is shown in fig.3. Since the sampling nature of the filter causes the folding of frequency response, only the lower of frequency

IDEAL AMPLITUDE RESPONSE

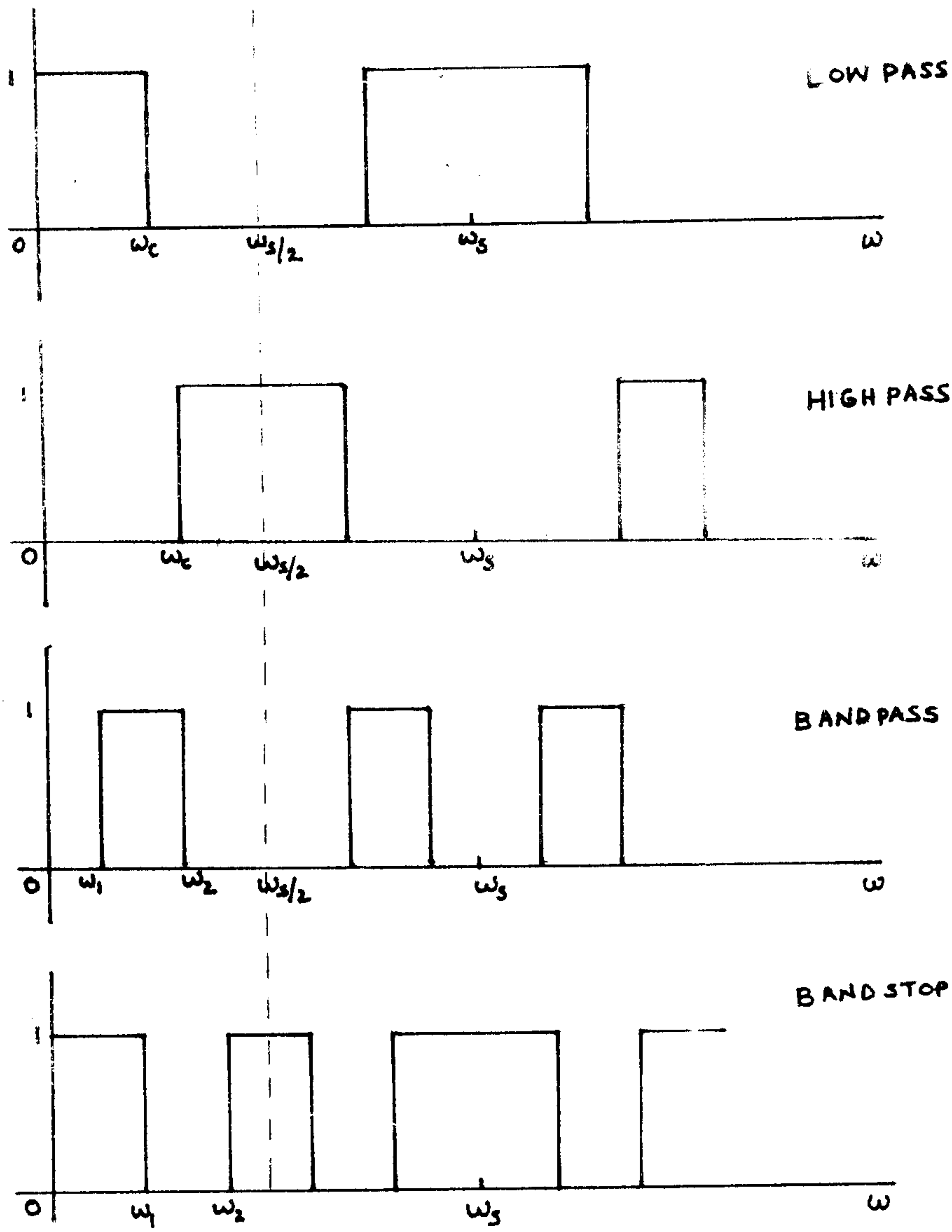


Fig. 3

response only the lower half of the response from dc to the folding frequency corresponds to the analog filter.

Therefore a high-pass digital filter actually implement an analog filter from the cut off frequency to the folding frequency. In this sense, it may be called a band pass filter in the analog domain in that it passes frequencies from the cut off point to the folding frequency. But the folding frequency is the highest frequency allowed to be present in order to avoid aliasing effect. That is this filter passes high frequency components from the cut off frequency to the highest frequency allowed to be present. Hence it is still a high pass filter.

A band-stop filter can be explained in a similar manner i.e. it passes frequencies from dc to the lower cut off-frequency and from the upper cut-off frequency to highest allowed but rejects the frequencies between the two cut-off points.

CLASSIFICATION

Based on their realisation procedures, digital filters can be classified as

a. Recursive

b. Non-recursive

Another method of classification is based on the response of digital filters to a unit sample input i.e.

i) Finite impulse response (F I R)

ii) Infinite inpulse response (IIR)

Recursive Filters :

As stated before, a digital filter is a computational algorithm which may be a difference or a convolution operation. The difference equation relates the present output value to the 'N' immediate past values of the output and the present and 'N' past values of the input. This type of input - output relationship is known as a recursive one and a digital filter realised by this method is called a recursive digital filter.

Non - Recursive Filters :

A digital filter can also be designed so that the input - output relationship is in the form of a super-position (convolution) summation. The computational algorithm in this form allows the present value of the output to be evaluated from the present and past values of the input. Since the past values of the output are not used directly to compute the present value of the output, a digital filter of this type is referred to as a non-recursive digital filter.

FIR & IIR Filters :

One characteristic of a digital filter is its response to a unit input. This unit impulse response sequence may be of finite or infinite duration. In the former case, the digital filter is called a Finite duration impulse response filter and the latter one is referred to as an Infinite duration impulse response filter.

An FIR system has a finite degree polynomial transfer function and an IIR system has a transfer function in the form of the ratio of two polynomials.

Next we shall have a brief review of elliptic approximation, based on which the digital filter has been designed.

Chapter 3

Review of Elliptic Approximation

ELLIPTIC APPROXIMATION

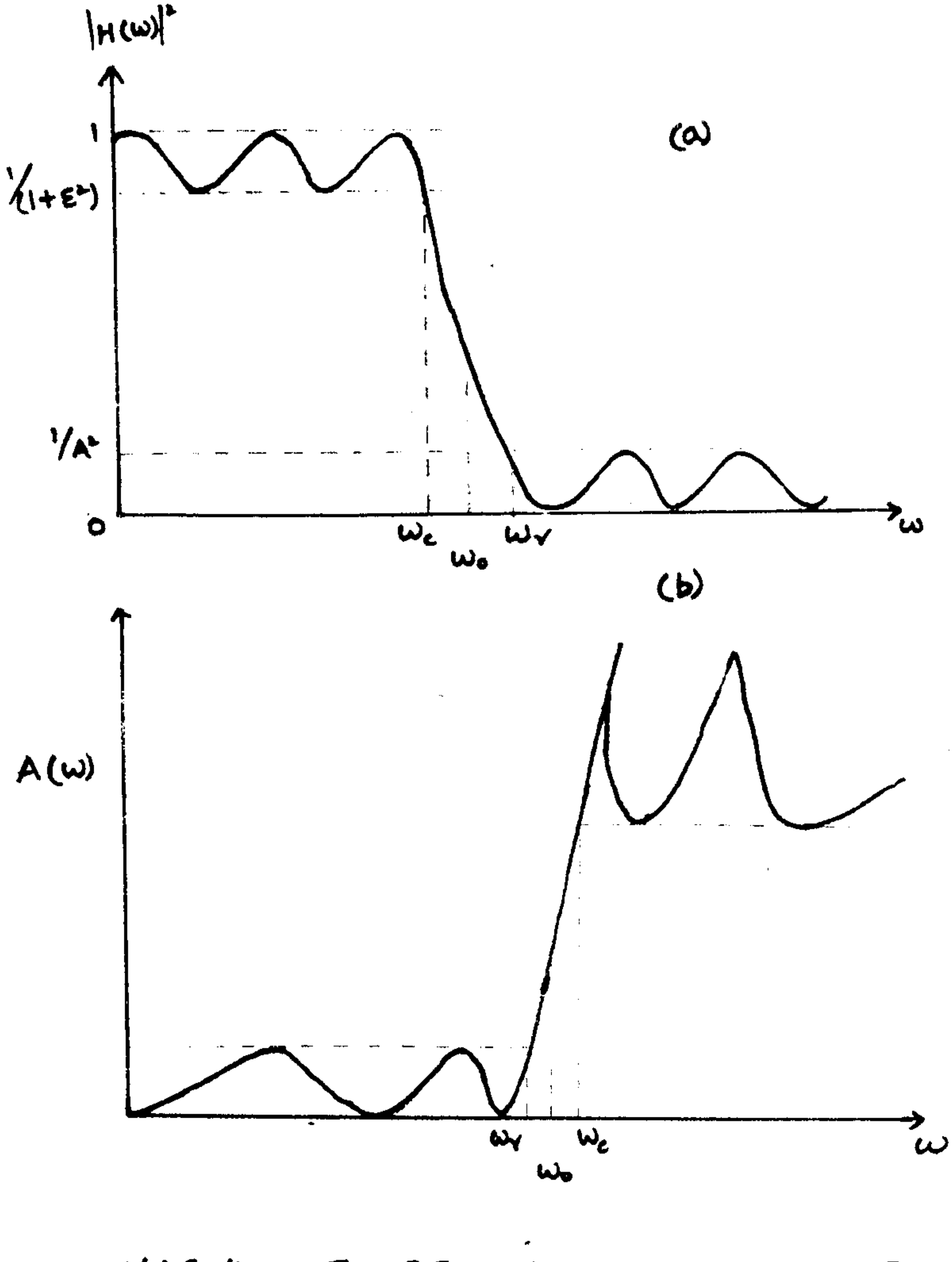
REASON FOR CHOOSING ELLIPTIC FILTER :

Though both Butterworth and Tchebyscheff filters are good at high frequencies, thay tend to deteriorate progressively as the frequency is decreased. An improved stop band characteristic can be achieved using elliptic approximation. In this the pass band loss oscillates between zero and a precribed maximum Ap and the stop band oscillates between infinity and a prescribed minimum Aa. The elliptic approximation is more efficient than the other two in that the transition between the pass band and stop band is steeper for a given order i.e. it has the narrowest transition band width.

Elliptic function filters have zeros and poles at finite frequencies which is responsible for the equiripple behaviour in the pass band and the finite transmission zeros in the stop band reduce the transition region so that extremely sharp roll off is obtained.

The following definitions apply to normalised elliptic function low pass filters and are illustrated in Fig.4.

r	-	pass band ripple
A	-	minimum stop band attenuation (dB)
Wr	-	lowest stop band frequency at which A occurs.



MAGNITUDE RESPONSE OF ELLIPTIC FILTERS
(a) LOW PASS (b) HIGH PASS

Fig. 4

The attenuation of elliptic filter can be expressed as

$$A_{dB} = 10 \log [1 + E^2 Zn^2(\omega)] \qquad 3\text{-}1$$

where E determines the ripple and $Zn(\omega)$ is an elliptic function of the n^{th} order.

In the elliptic filter response

W_c = pass band cut off freq.

W_r - stop band edge freq.

W_o - mid frequency in the transition band

$$W_o = \sqrt{W_c \quad W_r}$$

The transfer function of a normalised low pass elliptic filter of order N with $W_o = 1$ is given by

$$H(s) = \frac{H_o}{D_o(s)} \prod_{i=1}^{M} \frac{s^2 + a_i}{s^2 + b_i s + C_i} \qquad 3\text{-}2$$

where $M = \begin{cases} N/2 & \text{for even } N \\ \frac{N-1}{2} & \text{for odd } N \end{cases}$

and $D_o(s) = \begin{cases} 1 & \text{for N even} \\ s + \sigma_o & \text{for N odd} \end{cases}$

The transfer function can be found from the following relations for the given specifications of W_c, pass band ripple r and minimum attenuation is stop band (A).

$$w_r = 1/w_c$$

3-3

$$K = w_c^{\,2}$$

3-4

$$P = (1-K^2)^{\frac{1}{4}}$$

3-5

$$q_o = \tfrac{1}{2} \cdot \frac{1 - P}{1 + P}$$

3-6

$$q = q_o + 2q_o^{\,5} + 15q_o^{\,9} + 150q_o^{\,13}$$

3-7

$$b^2 = 10^{0.1\ A}$$

3-8

$$E^2 = 10^{0.1\ r}$$

3-9

$$D = \frac{b^2 - 1}{b^2 + 1}$$

3-10

$$N = \frac{\log 16\ D}{\log\ (1/q)}$$

3-11

$$\theta = \frac{1}{2N} \cdot \frac{E + 1}{E - 1}$$

3-12

$$\sigma_o = \left| \frac{2q^{\frac{1}{4}} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sinh\,(2m+1)\theta}{1 + 2 \sum_{m=1} (-1)^m q^{m^2} \cosh\,(2m)\theta} \right|$$

3-13

$$w_i = \sqrt{(1 + k\ \sigma_o)^2 \quad (1 + \sigma_o^2/k)}$$

3-14

$$W_i = \sqrt{(1 + k\sigma_o^2)(1 + \sigma_o^2/k)}$$

3-14

$$W_i = \frac{2q^{\frac{1}{4}} \sum_{m=0}^{\infty} (-1)^m q^{m(m+1)} \sinh((2m+1)\pi V/N)}{1 + 2\sum_{m=1}^{\infty} (-1)^m q^{m^2} \cos(2m\pi V/N)}$$

3-15

where $V = \begin{cases} i - \frac{1}{2} & \text{for even } N \\ i & \text{for odd } N \end{cases}$

and $i = 1, 2 \ldots\ldots M$

$$V_i = \sqrt{(1 - W_i^2 k)(1 - W_i^2/K)}$$

3-16

$$a_i = 1/W_i^2$$

$$b_i = 2\sigma_o V_i/(1 + \sigma_o^2 W_i^2)$$

$$C_i = ((\sigma_o V_i)^2 + (W_i W)^2)/(1 + \sigma_o^2 W_i^2)$$

$$HO = \begin{cases} 1/\epsilon \prod_{i=1}^{M} (C_i/a_i) & \text{for even } N \\ \sigma_o \prod_{i=1}^{M} (C_i/a_i) & \text{for odd } N \end{cases}$$

3-17

A program for evaluating the transfer function is presented at the end and it is used in designing the elliptic filter. Using the above background in elliptic approximation, next we shall deal with the design of the digital filter.

Chapter 4

Design of Digital Filter

DESIGN OF DIGITAL FILTER

Introduction:

Digital filter design is concerned with the selection of co-efficients of the difference equation on the unit sample response sequence used in the super position summation to meet some specified performance criteria, which may be given in the time or frequency domain.

Though filter design can be implemented in the time or frequency domain, we shall be dealing with the design of filters in the frequency domain because it is more precise and more suitable for electrical engineering practice.

Design Procedure:

Here, we are dealing with a detailed design procedure of elliptic filter in the recursive fashion.

Among the IIR digital filter design methods one important method is the bilinear transformation scheme, because of its case of implementation. In this, a digital filter transfer function is obtained from that of a proto-type analog filter through a simple substitution.

$$s \longrightarrow \frac{2}{T} \cdot \frac{z-1}{z+1}$$

where 'T' is the sampling interval.

The BASIC program which determines the transfer function is used as a subroutine. It is then utilised to obtain an analog prototype elliptic filter for the design of low pass, high pass, band pass and band stop elliptic digital filters. Since the prototype analog filter is the normalised low pass filter, the critical frequencies in the filter specifications must first be converted to the normalised low pass basis.

Warping Effect :

Let us digress a little and study about warping effect.

Let W and Ω represent the frequency variable in the analog filter and the derived digital filter respectively. Then according to the bilinear transformation method which states that,

$$H_D(z) = H_A(z) \Big|_{S = \frac{2Z-1}{TZ+1}}$$

where $H_D(z)$ & $H_A(z)$ are digital and analog filter transfer functions respectively, we have,

$$H_D(e^{j\Omega T}) = H_A(w)$$

provided that $w = 2/T \tan \Omega T/2$

For $\Omega < 0.3/T$ $W = \Omega$

As a result, the digital filter will have the same amplitude response as the analog filter at approximately the same frequency. For higher frequencies, however, the relation between W and Ω becomes nonlinear and distortion is introduced in the freq. of the digital filter relative to that of the analog filter. This is known as warping effect. To overcome warping the frequencies can be prewarped accordingly.

In order the encounter the problem of frequency warping introduced by the bilinear transformation, the prescribed critical frequencies are prewarped according to the following equation,

$$f_i = \frac{f_s}{\pi} \tan \frac{\pi F_i}{f_s} \qquad 4\text{-}2$$

where f_i denotes the prewarped frequency, based on which the analog prototype filter is to be designed. F_i is the corresponding prescribed frequency and f_s is the sampling frequency. This prewarping ensures that the digital filter designed by the bilinear transformation method will meet the specification at the prescribed frequencies i.e. the frequency warping effect will be cancelled. Through this prewarping scheme and the analog frequency transformation, the specified critical frequencies can be converted to the normalised low-pass prototype basis. There are four cases to consider in the frequency transformations.

1. LOW PASS FILTERS :

Let F_c and F_r be the specified passband and stopband edge frequencies in hertz, respectively, of the desired filter. Then because of the frequency prewarping scheme shown in 4-2, the passband and stopband edge frequencies in radians per second ω_c and ω_r respectively, of the normalized elliptic low pass analog filter must be such that,

$$\frac{\omega_c}{\omega_r} = \frac{(f_s/\pi)\ \tan(\pi F_c/f_s)}{(f_s/\pi)\ \tan(\pi F_r/f_s)}$$

$$= \frac{\tan(\pi F_c/f_s)}{\tan(\pi F_r/f_s)} = K_o \qquad 4\text{-}3$$

from which we find

$$\omega_c = \sqrt{K_o} \qquad 4\text{-}4$$

Since $\omega_r = 1/\omega_c$ for the normalised elliptic low-pass filter. Equation 4-4 gives a required parameter for the design of the normalised elliptic low pass prototype analog filter. The other two parameters are the passband ripple in decibels and the minimum stopband dB attenuation.

2. HIGH PASS FILTERS :

The same notations are used for the critical frequencies. The required passband edge frequency of of the normalised elliptic low-pass prototype analog filter is given by,

$$W_c = 1/\sqrt{K_o}$$

4-5

where K_o is given by

3. **BAND PASS FILTERS :**

Let F_{C1} and F_{C2} be the lower and upper passband edge frequencies, respectively and F_{r1} and F_{r2} be the lower and upper stopband edge of stop band filter frequencies. Then by using prewarped passband edge frequencies f_{C1} and f_{C2} given by

$$f_{C1} = \frac{f_s}{\pi} \tan \frac{F_{C1}}{f_s}$$

$$f_{C2} = \frac{f_s}{\pi} \tan \frac{F_{C2}}{f_s}$$

4-6

to design the analog prototype band-pass filter, the digital filter will achieve the exact passband characteristics. However, it is not, in general possible to achieve the prescribed stopband edges exactly. By properly selecting the design parameters of the analog prototype filter, it is possible to meet the stopband specifications with F_{r1} and F_{r2} representing, respectively, the lower and upper bounds of the stopband edge frequencies. To transform the critical frequencies to the normalized low pass basis, we define

$$K_B = \tan \frac{\pi F_{C1}}{f_s} \quad \tan \frac{\pi F_{C2}}{f_s} \qquad 4\text{-}7$$

and

$$K_C = \tan \frac{\pi F_{r1}}{f_s} \quad \tan \frac{\pi F_{r2}}{f_s} \qquad 4\text{-}8$$

Then, in order for the digital filter to meet the analog filter specifications at the two pasband edge frequencies, it can be shown that the passband edge frequency of the normalized low-pass elliptic analog prototype filter is given by

$$W_C = \sqrt{K_1} \quad \text{if } K_C \geq K_B \qquad 4\text{-}9$$

and

$$W_C = (\sqrt{K_2}\ ) \text{ if } K_C < K_B \qquad 4\text{-}10$$

where

$$K_1 = \frac{K_A \text{ TAN } (\pi F_{r_1}/f_s)}{K_B - \tan^2 (\pi F_{r1}/f_s)} \qquad 4\text{-}11$$

and

$$K_2 = \frac{K_A \tan (\pi F_{r2}/f_s)}{\tan^2 (\quad F_{r2}/f_s) - K_B} \qquad 4\text{-}12$$

with K_A given by

$$K_A = \tan \frac{\pi F_{C2}}{f_s} - \tan \frac{\pi F_{C1}}{f_s} \qquad 4\text{-}13$$

4. BAND-ELIMINATION FILTER :

Here, again, the same notations as used in the bandpass filter are used for the critical frequencies. The passband edge frequency required in the design of the normalized low-pass prototype filter is given by

$$W_C = 1/\sqrt{K_2} \quad \text{if } K_B \geq K_C \qquad 4\text{-}14$$

and

$$W_C = 1/\sqrt{K_1} \quad \text{if } K_C < K_B \qquad 4\text{-}15$$

where K_B, K_C and K_2, K_1 are as defined in the bandpass case.

Now, with the critical frequencies of the desired filter converted to the normalised low pass prototype basis, an analog low pass filter can be designed to meet the specified passband ripple and the stopband minimum attenuation requirements by using the formulas in 3-1 through 3-17 presented in chapter 3.0 and implemented in the BASIC subroutine. The form of the transfer function for this normalized elliptic low-pass analog filter, as given by Eq.3-2 is repeated here for easy reference.

$$H(s) = \frac{H_o}{D_o(s)} \prod_{i=1}^{M} \frac{s^2 + a_i}{s^2 + b_i s + C_i}$$

4-16

where

$$M = \begin{cases} N/2 & \text{for even } N \\ (N-1)/2 & \text{for odd } N \end{cases}$$

and

$$D_o(s) \begin{cases} 1 & \text{for even } N \\ S + \sigma_o & \text{for odd } N \end{cases}$$

with N being the order of the low-pass filter which is determined by the program to meet the specifications. In the program printout, the additional parameter for odd N is denoted by the symbol so. The low pass prototype transfer function is then transformed to the denormalized filter transfer function according to the analog frequency transformation schemes discussed earlier. We consider the four cases separately.

1. Low Pass to low pass transformation :

The low pass transfer function in 4-16 is denormalized to the desired frequency by replacing s by s/w_o, with w_o given by

$$W_o = 2f_s \tan(\pi F_c/f_s) / W_c$$

4-17

This results in the simple transformation for the factors in 4-16.

$$\frac{s^2 + a_i}{s^2+b_i s+C_i} \longrightarrow \frac{s^2 + a_i}{s^2+b_i' s+C_i'}$$

4-18

where

$$a_i^1 = a_i W_o^2, \quad b_i = b_i W_o, \quad C_i = C_i W_o^2$$

4-19

with Ho remaining unchanged. For odd N, the additional factor is changed as follows:

$$S + \sigma_o = S + \sigma_o' \quad \text{with } \sigma_o' = \sigma_o W_o$$

4-20

and

$$H_o \longrightarrow H_o' = W_o H_o$$

4-21

2. Low pass to high pass transformation :

To transform the normalized low pass transfer function in 4-16 to the desired high-pass transfer function, we apply the transformation in which is to replace S by Woh/S, where

$$W_o h = 2f_s \tan (\pi F_c/f_s)$$

4-22

The result is this simple substitution

$$\frac{s^2 + a_i}{s^2+b_i' s+C_i} \longrightarrow \frac{s^2 + a_i'}{s^2+b_i'+C_i'}$$

4-23

where

$$a_i' = W_o^2 h/a_i, \quad b_i' = b_i W_o h/C_i, \quad C_i' = W_o^2 h/C_i$$

with the corresponding change in H_o :

$$H_o \longrightarrow H_o a_i / C_i$$

4-25

the substitution for the additional factor for odd N is given by

$$1/(s+\sigma_o) \longrightarrow s/(s + \sigma_o') \text{ with } \sigma_o' = \frac{W_o h}{\sigma_o}$$

4-26

and

$$H_o = H_o' = H_o/\sigma_o$$

4-27

3. **Low-pass to band pass transformation :**

The analog frequency transformation is used to convert the low pass prototype transfer function in 4-16 to the desired bandpass transfer function. The results of this transformation are shown in the following factor substitutions :

$$\frac{s^2 + a_i}{s^2+bis+C_i} \longrightarrow \frac{s^4 + a_1 i + s^2 + a_2 i}{s^4+b_1 is^3+b_2 is^2+C_1 is+C_2 i}$$

4-28

where

$$a_1 i = 2W_{ob}^2 + a_i B^2 \qquad a_2 i = W_{ob}^4$$

$$b_1 i = b_i B \qquad b_2 i = 2W_o{}^2 b + C_i B^2$$

4-29

$$C_1 i = b_i B W_o^2 b \qquad C_2 i = W_o^4 b$$

and

$$W_o b = 2f_s \sqrt{K_B} \qquad B = 2Knf_s / W_o$$

4-30

with KA and KB defined in 4-7 and 4-13 the constant multiplier Ho remains unchanged. For odd N, the additional factor is changed as follows :

$$\frac{1}{s + \sigma o} \qquad \frac{B_s}{s^2 + \sigma o B_s + W_o^2 b}$$

4-31

4. Low-pass to bandstop transformation :

The transformation is used to obtain the desired bandstop transfer function for the normalized low-pass transfer function. It can be seen that the low-pass to band stop transformation can be effected by first replacing S by 1/S and then applying the low-pass to band pass transformation. Replacing S by 1/S is just the low-pass to high pass transformation with Woh = 1. Therefore, the factor substitution in 4-23 through 4-27 with Woh=1 are valid for this first step of the transformation. Then the factor substitution in 4-28 is used for the biquadratic factor to perform the additional low-pass to bandpass transformation. However, the bandwith parameter is different for this transformation and is given by

$$B' = 2 K_A f_s W_o$$ 4-32

The additional factor for odd N needs special attention, as shown below. The low-pass to bandpass transformation in applied to the result of the first step in 4-26 yields the substitution.

$$\frac{s}{s + \sigma_o} \longrightarrow \frac{s^2 + W_{ob}^2}{s^2 + \sigma_o' B' s + W_{ob}^2}$$ 4-33

with the analog transfer function of the desired filter found, we can now apply the bilinear transformation in 4-1 to obtain the digital filter transfer function. In all the formulas to be presented, we use K to denote the constant 2/T used in the bilinear transformation, (ie),

$$K = 2/T = 2f_s$$ 4-34

where f_s is the sampling frequency in hertz. The bilinear transformation applied to the biquadratic factor in 4-18 and 4-23 results in the transfer function of the low-pass and high-pass digital filter

$$\frac{s^2 + a_i}{s^2 + b_i' s + C_i} \longrightarrow \frac{z^2 + d_i z + 1}{z^2 + e_i z + f_i}$$

4-35

where

where

$$d_i = 2(a_i^1 - K^2) / \Delta_1$$
$$e_i = 2(C_i^1 - K^2) / \Delta_2$$
$$f_i = (K^2 - b_i^1 k + C_i^1) / \Delta_1 \qquad 4\text{-}36$$
$$\Delta_1 = a_i^1 + K^2$$
$$\Delta_2 = K^2 + b_i^1 k + c_i^1$$

and the constant multiplier is changed according to

$$H_o \longrightarrow H_o \Delta 1 / \Delta 2 \qquad 4\text{-}37$$

The additional factor for odd N in the low pass case in 4-20 becomes

$$\frac{1}{s + \sigma_o} \longrightarrow \frac{z_o + c_o}{z + z_o} \qquad 4\text{-}38$$

where

$$c_o = 1 \quad \text{and}$$
$$z = (\sigma_o^1 - K) / (\sigma_o^1 + K) \qquad 4\text{-}39$$

and the constant multiplier is changed to

$$H_o \Rightarrow H_o / (\sigma_o^1 + K) \qquad 4\text{-}40$$

The additional factor for odd N in the high-pass case in 4-26 is changed as

$$\frac{s}{s + \sigma_o^1} \longrightarrow \frac{z + C_o}{z + z_o} \qquad 4\text{-}41$$

where

$$C_o = -1 \quad \text{and}$$

$$Z_o = (\sigma_o{}^1 - K) / (\sigma_o{}^1 + K)$$

4-42

with the same change in the constant multiplier as given by 4-40. For the bandpass and bandstop filters, the bilinear transformation applied to the biquadratic factor in 4-28 yields the biquadratic factor for the digital transfer function

$$\frac{Z^4 + d_1i\ Z^3 + d_2i\ Z^2 + d_1i\ Z + 1}{Z^4 + e_1i\ Z^3 + e_2i\ Z^2 + e_3i\ Z + e_4i}$$

4-43

where

$$d_1i = 4\ (a_2i - K^4)\ /\ 3$$

$$d_2i = 2\ (3\ (a_2i + K^4) - a_1i\ K^2)\ /\ 3$$

$$e_1i = 2\ [2(C_2i - K^4) + C_1i\ K - b_1i\ K^3]/\ 4$$

$$e_2i = 2\ [3(C_2i + K^4) - b_2i\ K^2]/\ 4$$

$$e_3i = 2\ [2(C_2i - K^4) - (C_1iK - b_1iK^3)\ ]/\ 4$$

$$e_4i = [K^4 + b_2iK^2 + C_2i - b_1iK^3 - C_1iK]\ /\ 4$$

$$3 = K^4 + a_1iK^2 + a_2i$$

$$4 = K^4 + b_1i\ K^3 + b_2i\ K^2 + C_1iK + C_2i$$

with a corresponding change in the constant multiplier given by

$$H_o \longrightarrow H_o \cdot \Delta 3\ /\ \Delta 4$$

4-45

For odd N, the additional factor in 4-31 for the bandpass filter and the one in 4-33 for the bandstop filter are both transformed into the biquadratic form

$$\frac{Z^2 + a_1 Z + a_o}{Z^2 + b_1 Z + b_o}$$

4-46

where the parameters for the bandpass filter are given by

$$a_1 = 0 \qquad a_o = -1$$

$$b_1 = \frac{2\,(W_o{}^2 b - K^2)}{\Delta_5} \qquad b_o = \frac{K^2 - \sigma_o BK + W_o^2 b}{5}$$

4-47

$$\Delta_5 = K^2 + \sigma_o BK + W_o^2 b$$

with corresponding change in the constant multiplier given by

$$H_o \longrightarrow H_o\, BK/\Delta_5$$

and the parameters for the bandstop filter are given by

$$a_1 = \frac{2\,(W_o^2 b - K^2)}{\Delta_6} \qquad a_o = 1$$

$$b_1 = \frac{2(W_o^2\, b - K^2)}{\Delta_7} \qquad b_o = \frac{K^2 - \sigma_o'' B' K - W_o^2 b}{\Delta_7}$$

$$\Delta_6 = W_o^2 b + K^2$$

$$\Delta_7 = K^2 + \sigma_o'' B' K + W_o^2 b$$

4-49

with a corresponding change in Ho given by

$$H_o \longrightarrow H_o \cdot \quad \Delta_6 \;/\; \Delta_7$$ 4-50

The above design principles are used for implementing the digital filter in a computer program using BASIC. This is dealt in the following chapter.

Chapter 5

Basic Program Listing

BASIC Program to design a digital filter :

Reason for choosing BASIC:

Our primary consideration, in implementing the digital filter in BASIC, is the simiplicity of the language.

Moreover the graphic commands that are available in BASIC are easy to use.

Though a language like FORTRAN may be well suited for this application, conside ing the enormous amount of mathematics involved, the lack of graphic commands in FORTRAN has made us implement it in BASIC.

The transformation required to design a filter by using the analog frequency tranformation and the bilinear transformation as described in the previous chapter is implemented in the BASIC program as given below

The input to the program consists of the critical passband ripple in dB, stopband attennation (dB). After accepting the input data, the program then transforms the critical frequencies to the pass band edge frequency of the nomalised low-pass prototype and designs the low-pass prototype analog filter to meet the specifications. Analog frequency transformations are subsequently performed to obtain the analog version of the desired digital filter. The bilinear transformation applied to the analog filter yields the desired digital filter. The digital filter parameters are printed for the coefficients of the transfer function, together with the constant multiplier Ho.

In addition, the transfer function factors are multiplied in the form of two polynomials.

$$H(z) = Ho \frac{Z^2 + p_1 Z^{L-1} + p_2 Z^{L-2} + \ldots + p_L}{Z^2 + q_1 Z^{L-1} + Q_2 Z^{L-2} + \ldots + q_L} \quad 4\text{-}51$$

The coefficients p_i and q_i are then printed for i = 1,2,....L, where 'L' is equal to the order 'N' of the analog low-pass prototype filter for low-pass and high-pass filters and equal to 2N for bandpass and band-stop filters.

At the end of the program, the frequency response of the designed digital filter is plotted. The starting value of the normalised digital filter frequency step size of the frequency and number of points to be evaluated, are keyed in to get the desired response.

```
10 'PROGRAM TO SIMULATE DIGITAL FILTERS
20 CLS
30 LOCATE 9,16:PRINT "TYPE OF FILTER"
40 LOCATE 12,20: PRINT " L"
50 LOCATE 13,20: PRINT "H "
60 LOCATE 14,20: PRINT "BP "
70 LOCATE 15,20: PRINT "BS "
80 REM 1.LPF SPECIFICATIONS
90 A$=INKEY$
100 INPUT A$
110 IF A$="L" THEN 150
120 IF A$="H" THEN 210
130 IF A$="BP" THEN 260
140 IF A$="BS" THEN 310
150 CLS:PRINT "PLEASE ENTER THE FOLLOWING VALUES"
160 PRINT "LPF SPECIFICATIONS": PRINT
170 PRINT "F1=0"
180 PRINT "F2= PASS BAND EDGE FREQUENCY"
190 PRINT "F3= STOP EDGE FREQUENCY"
200 PRINT"F4=0":GOTO 360
210 CLS : PRINT  "HPF SPECIFICATIONS":PRINT
220 PRINT "F1=ANY NEGATIVE NUMBER"
230 PRINT  "F2=PASSBAND EDGE FREQ"
240 PRINT "F3= STOP BAND EDGE FREQ"
250 PRINT  "F4=0"  :GOTO 360
260 CLS: PRINT "BPF SPECIFICATIONS":PRINT
270 PRINT "F1=LOWER PASSBAND EDGE FREQ"
280 PRINT "F2=UPPER PASSBAND FREQ"
290 PRINT "LOWER STOPBAND EDGE FREQ. (F3<F1<F2<F4)"
300 PRINT  "F4=UPPER STOPBAND EDGE FREQ ":GOTO  360
310 CLS : PRINT  "BSF SPECIFICATIONS": PRINT
320 PRINT "F1=LOWERSTOP EDGE FREQUENCY"
330 PRINT  "F2=UPPER PASSBAND EDGE FREQUENCY
340 PRINT  "F3=LOWER STOPBAND EDGE FREQ
350 PRINT "F4=UPPER STOP EDGE FREQ (F1<F3<F4<F2)"
360 PRINT "F0 = SAMPLING FREQUENCY"
370 PRINT  "R0 = PASSBAND RIPPLE "
380 PRINT "B0 = ATTN. IN DB"
390 LOCATE 12,20 :PRINT "ENTER THE VALUES OF F1,F2,F3,F4,F0,R0,B0"
391 PRINT "F1=":INPUT F1
392 PRINT "F2=":INPUT F2
393 PRINT "F3=":INPUT F3
394 PRINT  "F4=0":INPUT F4
395 PRINT "R0=":INPUT R0
396 PRINT "B0=":INPUT B0
397 PRINT "F0=":INPUT F0
398  CLS
420 IF (F1=0) AND (F2<F3) GOTO 490
430 IF (F1<0) AND (F2>F3) GOTO 490
440 IF (F3<F1) AND (F2<F4) AND (F1<F2) GOTO 490
450 IF (F1<F3) AND (F4<F2)AND (F3<F4) GOTO  490
460 PRINT "THE DATA GIVEN IS WRONG "
470 PRINT "GIVE THE DATA IN THE CORRECT FORMAT"
480 GOTO 390
490 P=3.14159
500 P1=P/F0
510 IF (F1=0) GOTO 550
520  IF (F1<0) GOTO 570
530 IF (F3<F1) GOTO 640
```

```
540 IF (F1<F3) GOTO  660
550 PRINT "ELLIPTICAL LPF SPECIFICATIONS"
560 GOTO  580
570 PRINT "ELLIPTICAL HPF SPECIFICATIONS"
580 PRINT "THE PASS BAND EDGE FREQ =";F2;"HZ"
590 PRINT "STOP BAND EDGE FREQ =";F3;"HZ"
600 PRINT "THE SAMPLING FREQ =";F3;"HZ"
610 PRINT  "THE PASSBAND RIPPLE="R0;'DB"
620 PRINT "THE STOPBAND MINIMUM ATTENUATION=";B0;"DB"
630 GOTO 740
640 PRINT "ELLIPTICAL BPF SPECIFICATIONS "
650 GOTO 670
660 PRINT "ELLIPTICAL BPF SPECIFICATIONS"
670 PRINT "LPB EDGE FREQ=F1";"HZ"
680 PRINT "THE UPPER PASSBAND EDGE FREQ=";F2;"HZ"
690 PRINT "THE LOWER STOPBAND EDGE FREQ=";F3;"HZ"
700 PRINT "THE UPPER STOPBAND EDGE FREQ=";F4;"HZ"
710 PRINT "THE SAMPLING FREQ=";F0;"HZ"
720 PRINT "THE PASSBAND RIPPLE="RO";"DB"
730 PRINT "THE STOPBAND MIN ATTEN=";BO;"DB"
740 T5=2*F0
750 T6=T5^2
760 T7=T6*T5
770 T8=T7*T5
780 T2=TAN(P1*F2*P/180)
785 PRINT T2
790 T3=TAN(P1*F3*P/180)
795 PRINT T3
800 IF (F1<=0) GOTO 1000
810 T1= TAN(P1*F1*P/180)
820 T4= TAN(P1*F4*P/180)
830 A=T2-T1
840 B=T1*T2
850 C=T3*T4
860 A1=A*T3/(B-T3^2)
870 A2=A*T4/(T4^2-B)
880 IF (F1<F3) GOTO  920
890 W1=(A1)^.5
900 IF (B>C) THEN W1=(A2)^.5
910 GOTO 940
920 W1=1/(A2^.5)
930 IF (B>C) THEN W1=1/(A1^.5)
940 W2=T5*B^.5
950 W3=W2^2
960 W4=W3^2
970 IF (F1>F3) THEN B2=T5*A/W1
980 IF (F1<F3) THEN B2=T5*A*W1
990 GOTO 1080
1000 IF(F1<0) GOTO  1050
1010 W1=(T2/T3)^.5
1011 PRINT
1020 W2= T5*T2/W1
1030 W3=W2^2
1040 GOTO 1080
```

```
1050 W1=SQR(T3/T2)
1060 W2=T5*T2*W1
1070 W3=W2^2
1080 GOSUB 3800
1090 PRINT "THE PROTOTYPE ANALOG LPF"
1100 PRINT "THE ORDER OF THE FILTER =N=";N
1101 PRINT "I ","A(I) ","B(I)","C(I)"
1120 FOR I=1 TO N2
1130 PRINT I,A(I),B(I),C(I)
1140 NEXT I
1150 IF (F5=0) GOTO 1170
1160 PRINT "THE ADDL PARAMETER FOR ODD N=S0=";S0
1170 PRINT "THE CONST MULTIPLIER H0=";H0
1180 IF(F1=0) GOTO 1280
1190 IF (F1>0) GOTO 2050
1200 FOR I=1 TO N2
1210 T1=1/C(I)
1220 H0=H0*A(I)*T1
1230 A(I)=W3/A(I)
1240 B(I)=B(1)*W2*T1
1250 C(I)=W3*T1
1260 NEXT I
1270 GOTO 1330
1280 FOR I=1 TO N2
1290 A(I)=A(I)*W3
1300 B(I)=B(I)*W2
1310 C(I)=C(I)*W3
1320 NEXT I
1330 FOR I= 1 TO N2
1340 T1=A(I)+T6
1350 D(I)=2*(A(I)-T6)/T1
1360 T2=B(I)
1370 T3=C(I)
1380 T4=1/(T6+T2*T5+T3)
1390 E(I)=2*(T3-T6)*T4
1400 F(I)=(T6-T2*T5+T3)*T4
1410 H0=H0*T1*T4
1420 NEXT I
1425 PRINT H0
1430 PRINT "THE DIGITAL FILTER PARAMETERS ARE GIVEN BELOW"
1440 PRINT "I  "," D(I)  ", " E(I)"," F(I)"
1450 FOR I = 1 TO N2
1460 PRINT I,D(I),E(I),F(I)
1470 NEXT I
1480 IF (F5=0) GOTO 1620
1490 IF (F1<0) GOTO 1550
1500 T1=1/(S0*W2+T5)
1510 Z0=(S0*W2-T5)*T1
1520 H0 = H0*T1*W2
1530 C0=1
1540 GOTO 1600
1550 T1=S0*T5
1560 T2=1/(W2+T1)
1570 Z0=(W2-T1)*T2
1580 H0=H0*T5*T2
1590 C0=-1
1600 LINE(X,20)-(X,199)
```

```
1610 PRINT "CØ=";CØ,"ZØ=";ZØ
1620 PRINT "THE CONSTANT MULTIPLIER HØ=";HØ
1630 P(1)=1
1640 P(2)=D(1)
1650 P(3)=1
1660 Q(1)=F(1)
1670 Q(2)=E(1)
1680 Q(3)=1
1690 IF N2=1 THEN 1860
1700 FOR  I = 2 TO N2
1710 K= 2*I+1
1720 P(K)=1
1730 P(K-1)=P(K-3)+D(I)
1740 Q(K)=1
1750 Q(K-1)=Q(K-3)+E(I)
1760 L=2*I-3
1770 FOR M=1 TO L
1780 G= K-1-M
1790 P(G)=P(G)+D(I)*P(G-1)+P(G-2)
1800 Q(G)=F(I)*Q(G)+E(I)*Q(G-1)+Q(G-2)
1810 NEXT M
1820 P(2)=P(2)+D(I)
1830 Q(2)=F(I)*Q(2)+E(I)*Q(1)
1840 Q(I)=F(I)*Q(I)
1850 NEXT I
1860 N3=2*N2+1
1870 IF (F5=Ø) THEN 1990
1880 N4=N3-1
1890 P(N4+1) =1
1900 Q(N4+1)=1
1910 FOR I = 1 TO N4
1920 G=N3-I+1
1930 P(G)=P(G)*CØ+P(G-1)
1940 Q(G)=Q(G)*ZØ+Q(G-1)
1950 NEXT I
1960 P(1)=P(1)*CØ
1970 Q(1)=Q(1)*ZØ
1980 N3=N3+1
1990 N4=N3-1
2000 PRINT "I","P(I)","Q(I)"
2010 FOR I= 1 TO N4
2020 PRINT 1,P(N3-I),Q(N3-I)
2030 NEXT I
2040 GOTO  3160
2050 IF(F1>F3) GOTO  2120
2060 FOR  I = 1 TO N2
2070 HØ=HØ*A(I)/C(I)
2080 A(I)=1/A(I)
2090 B(I)=B(I)/C(I)
2100 C(I)=1/C(I)
2110 NEXT I
2120 FOR I =1 TO N2
2130 A3=2*W3+A(I)*B2^2
2140 B1=B(I)*B2
```

```
2150 B3=2*W3+C(I)*B2^2
2160 C1=B(I)*B2*W3
2170 T1=T8+A3*T6+W4
2180 T2=1/T1
2190 D1(I)=4*(W4-T8)*T2
2200 D2(I)=2*(3*(T8+W4)-A3*T6)*T2
2210 T2=1/(T8+B3*T6+W4+B1*T7+C1*T5)
2220 T3=C1*T5-B1*T7
2230 E1(I)= 2*(2*(W4-T8)+T3)*T2
2240 E2(I)= 2*(3*(T8+W4)-B3*T6)*T2
2250 E3(I)= 2*(2*(W4-T8)-T3)*T2
2260 E4(I)=(T8+B3*T6+W4-B1*T7-C1*T5)*T2
2270 H0=H0*T1*T2
2280 NEXT I
2290 PRINT "THE DIGITAL FILTER PARAMETERS ARE GIVEN BELOW"
2300 PRINT "I","D1(I)",'D2(I)"
2310 FOR I = 1 TO N2
2320 PRINT I,D1(I),D2(I)
2330 NEXT I
2340 PRINT TAB(3);"I";TAB(10);"E1(I)";TAB(25);'E2(I)"
2350 PRINT TAB(40);"E3(I)";TAB(55);"E4(I)"
2360 FOR I = 1 TO N2
2370 PRINT TAB(2); I;TAB(7);E1(I);TAB(22);E2(I)
2380 PRINT TAB(37);E3(I);TAB(52);E4(I)
2390 NEXT I
2400 IF (F5=0) THEN  2580
2410 IF (F1>F3) THEN  2500
2420 T1=W3+T6
2430 A1=2*(W3-T6)/T1
2440 A0=1
2450 T2=1/(T6+B2*T5/S0+W3)
2460 B1=A1*T1*T2
2470 B0=(T6-B2*T5/S0+W3)*T2
2480 H0=HO*T1*T2/S0
2490 GOTO 2560
2500 T1=1/(T6+S0*B2*T5+W3)
2510 A1=0
2520 A0=-1
2530 B1=2*(W3-T6)*T1
2540 B0=(T6-S0*B2*T5+W3)*T1
2550 H0 =H0*B2*T5*T1
2560 PRINT "ADDNL FACTORS FOR ODD N ARE"
2570 PRINT "A1=";A1;"A0=";A0;"B1=";B1;"B0=";B0
2580 PRINT "THE CONST MULTIPLIERS=H0=";H0
2590 P(1)=1
2600 P(2)=D1(1)
2610 P(3)=D2(1)
2620 P(4)=D1(1)
2630 P(5)=1
2640 Q(1)=E4(1)
2650 Q(2)=E3(1)
2660 Q(3)=E2(1)
2670 Q(4)=E1(1)
2680 Q(5)=1
2690 IF(N2<2) GOTO 2940
2700 FOR  I = 2 TO N2
```

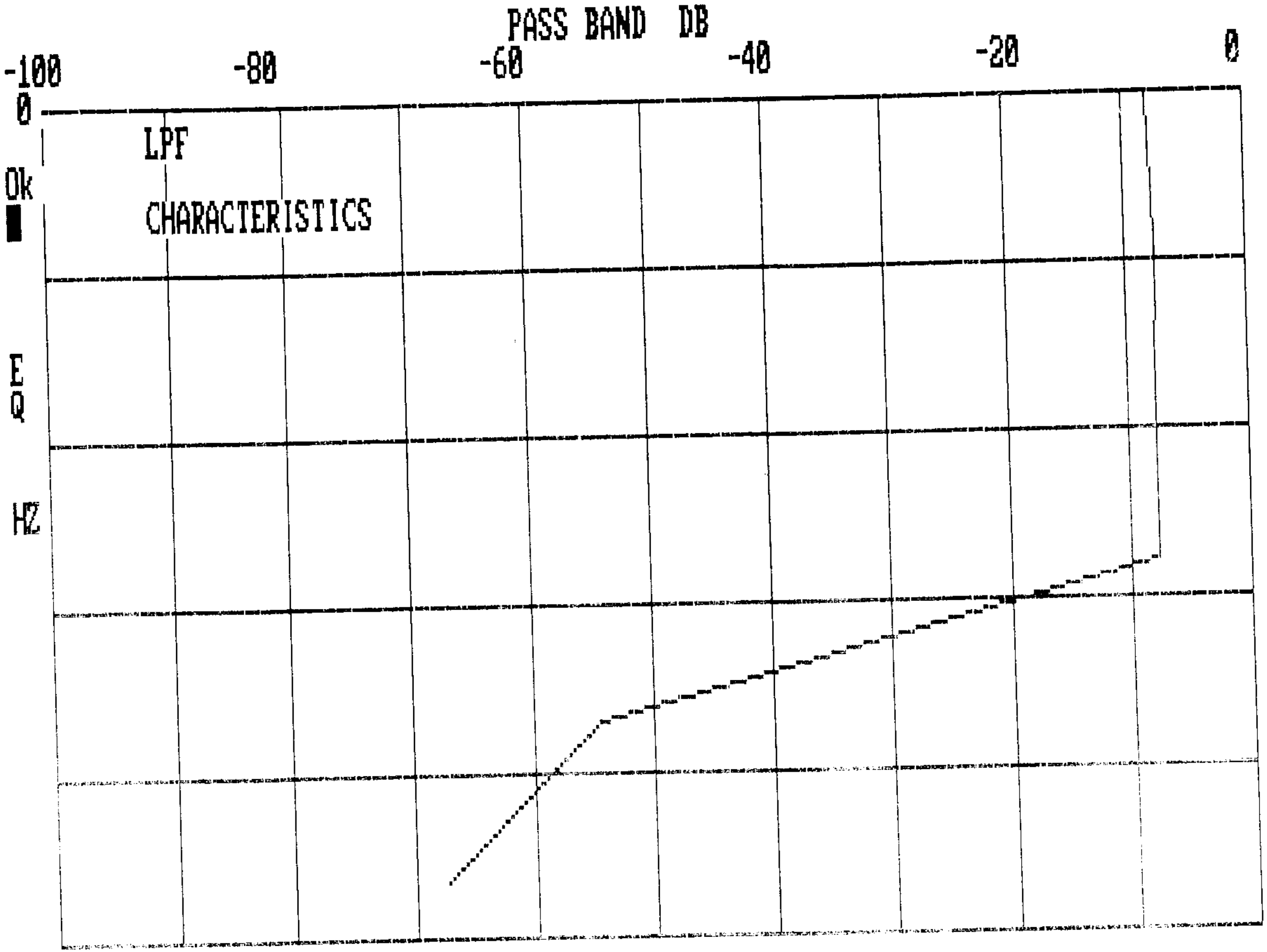
```
2710 X=4*I+1
2720 P(X)=1
2730 P(X-1)=P(X-5)+D1(I)
2740 P(X-2)=P(X-6)+D1(I)*P(X-5)+D2(I)
2750 P(X-3)=P(X-7)+D1(I)*P(X-6)+D2(I)*P(X-5)+D1(I)
2760 Q(X)=1
2770 Q(X-1)=Q(X-5)+E1(I)
2780 Q(X-2)=Q(X-6)+E1(I)*Q(X-5)+E2(I)
2790 Q(X-3)=Q(X-7)+E1(I)*Q(X-6)+E2(I)*Q(X-5)+E3(I)
2800 X3=4*I-7
2810 FOR  J = 1 TO X3
2820 Y= X-3-J
2830 P(Y)=P(Y)+D1(I)*P(Y-1)+D2(I)*P(Y-2)+D1(I)*P(Y-3)+P(Y-4)
2840 Q(Y)=Q(Y)+E1(I)*Q(Y-1)+E2(I)*Q(Y-2)+E3(I)*Q(Y-3)+E4(I)*Q(Y-4)
2850 NEXT J
2860 P(4)=P(4)+D1(I)*P(3)+D2(I)*P(2)+D1(I)
2870 P(3)=P(3)+D1(I)*P(2)+D2(I)
2880 P(2)=P(2)+D1(I)
2890 Q(4)=E4(I)*Q(4)+E3(I)*Q(3)+E2(I)*Q(2)+E1(I)*Q(1)
2900 Q(3)=E4(I)*Q(3)+E3(I)*Q(2)+E2(I)*Q(1)
2910 Q(2)=E4(I)*Q(2)+E3(I)*Q(1)
2920 Q(1)=E4(I)*Q(1)
2930 NEXT I
2940 NT=4*N2+1
2950 IF (F5=Ø) GOTO 311Ø
2955 P(NT+2)=1
2960 Q(NT+2)=1
2970 P(NT+1)=P(NT-1) +A1
2980 Q(NT+1)=Q(NT+1)+B1
3000 X4=NT-2
3010 FOR J= 1 TO X4
3020 IX=NT+1-J
3030 P(IX)=AØ*P(IX)+A1*P(IX-1)+P(IX-2)
3040 Q(IX)=BØ*Q(IX)+B1*Q(IX-1)+Q(IX-2)
3050 NEXT J
3060 P(2)=AØ*P(2)+A1
3070 P(1)=AØ*P(1)
3080 Q(2)=BØ*Q(2)+B1*Q(1)
3090 Q(1)=BØ*Q(1)
3100 NT=NT+2
3110 X=NT-1
3120 PRINT "I","P(I)","Q(I)"
3130 FOR Z= 1 TO X
3140 PRINT Z,P(NT-Z),Q(NT-Z)
3150 NEXT Z
3160 Z=FØ/F2
3170 IF(F1<=Ø) GOTO 32ØØ
3180 F9=(F1+F2)/2
3190 Z=FØ/F9
3200 PRINT "ENTER THE DATA :WØ,SØ,NØ"
3210 INPUT WØ,SØ,NØ
3220 PRINT "THE STARTING NORMALISED FREQ=:";WØ;"HZ"
3230 PRINT "THE STEP SIZE=:";SØ;"HZ"
3240 PRINT "THE NUMBER OF POINTS TO BE EVALUATED=";NØ
3250 PRINT "I","W(I)","AM","H(I)"
3260 IF(F1>Ø) GOTO  351Ø
3270 W=WØ
3280 FOR K= 1 TO NØ
```

```
3290 WP=W*P
3300 X1=COS(WP)
3310 X2=COS(2*WP)
3320 Y1=SIN(WP)
3330 Y2=SIN(2*WP)
3340 AM=H0^2
3350 FOR I = 1 TO N2
3360 AM=AM*((1+X2+D(I)*X1)^2+(Y2+D(I)*Y1)^2)
3370 AM=AM/((F(I)+X2+E(I)*X1)^2+(Y2+E(I)*Y1)^2)
3380 NEXT I
3390 IF (F5=0) GOTO 3410
3400 AM=AM*((C0+X1)^2+Y1^2)/((Z0+X1)^2+Y1^2)
3410 H(K) = 10 *ABS(LOG(AM)) / LOG(10)
3420 W(K)=W*Z
3430 PRINT K,W(K),AM,H(K)
3440 W= W+S0
3445 NEXT K
3450 PRINT "RUN AGAIN? YES/NO"
3460 INPUT N$
3470 IF (N$="YES") GOTO 3200
3490 GOSUB 4540
3500 GOTO 4790
3510 W=W0
3520 FOR K= 1 TO N0
3530 WP=W*P
3540 X1=COS(WP)
3550 X2=COS(2*WP)
3560 X3=COS(3*WP)
3570 X4=COS(4*WP)
3580 Y1=SIN(WP)
3590 Y2=SIN(2*WP)
3600 Y3=SIN(3*WP)
3610 Y4=SIN(4*WP)
3620 AM=H0^2
3630 FOR I =1 TO N2
3640 MR=(1+D1(I)*(X1+X3)+D2(I)*X2+X4)^2
3650 RM=(D1(I)*(Y1+Y3)+D2(I)*Y2+Y4)^2
3660 AM=AM*(RM+MR)
3670 MR=(E4(I)+E3(I)*X1+E2(I)*X2+E1(I)*X3+X4)^2
3680 RM=(E3(I)*Y1+E2(I)*Y2+E1(I)*Y3+Y4)^2
3690 AM=AM/(RM+MR)
3700 NEXT I
3710 IF(F5=0) GOTO 3740
3720 AM=AM*((A0+A1*X1+X2)^2+(A1*Y1+Y2)^2)
3730 AM=AM/((B0+B1*X1+X2)^2+(B1*Y1+Y2)^2)
3740 H(K)=10*LOG(AM)/LOG(10)
3750 W(K)=W*Z*1.739231
3760 PRINT K,W(K),AM,H(K)
3770 W=W+S0
3780 NEXT K
3790 GOTO 3460
3800 REM THE SUBROUTINE STARTS HERE
3805 CLS
3810 K=1
3820 S2=0
3830 E2=10^(.1*R0)
```

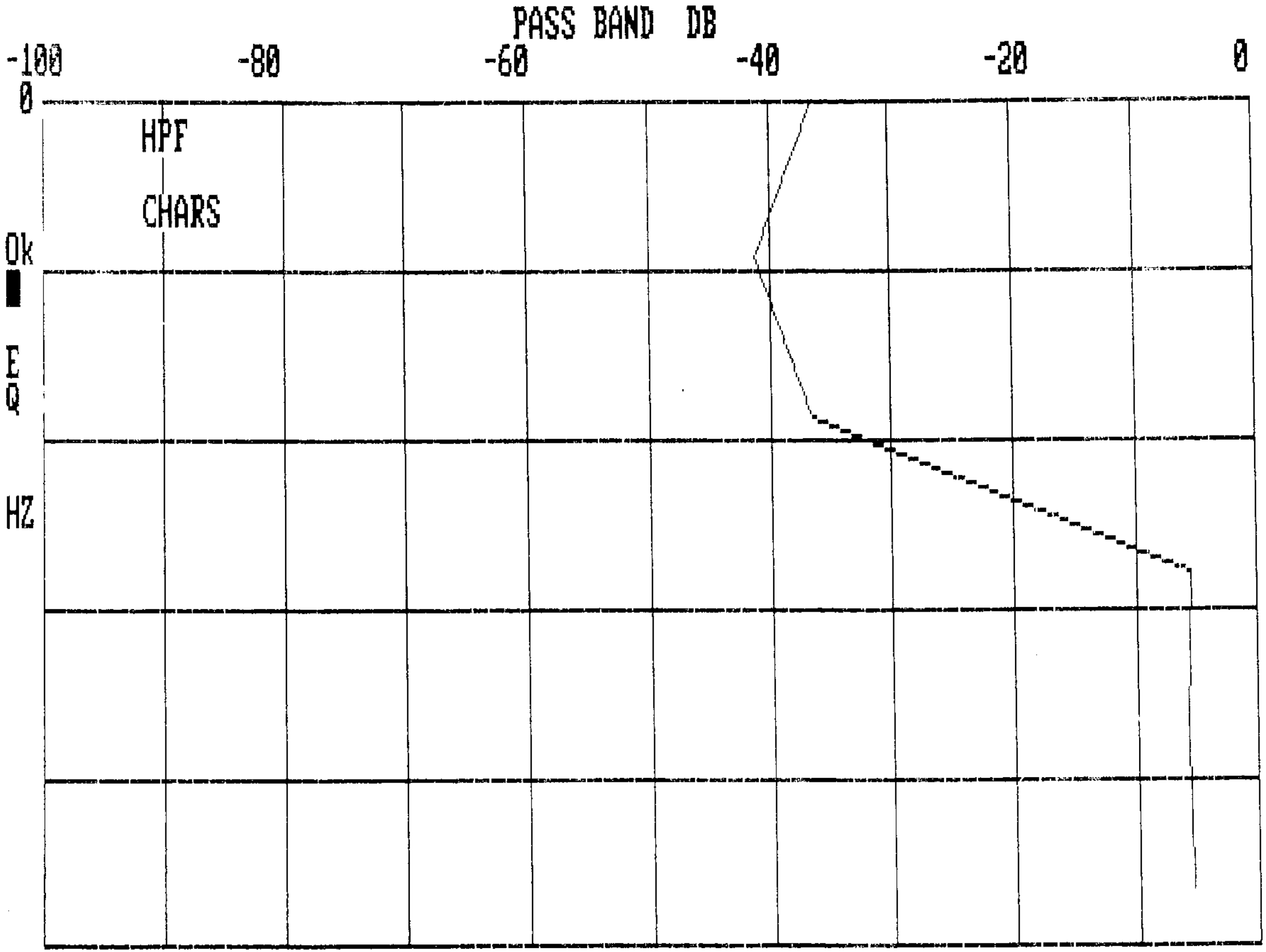
```
3840 E1=E2^.5
3850 E3=.05*R0
3860 E4=10^E3
3870 B5=10^(.1*B0)
3880 E5=-E3
3890 D1=(B5-1)/(E2-1)
3899 PRINT
3900 K1=W1^2
3901 OK=SQR(ABS(1-K1*K1))
3910 K2=SQR(OK)
3920 CLS:Q0=.5*(1-K2)/(1+K2)
3930 Q1=Q0^4
3940 Q2=Q1*Q0
3950 Q3=Q2*Q1
3960 Q4=Q3*Q1
3970 Q=Q0+2*Q2+15*Q3+150*Q4
3980 T1=LOG(16*D1)/LOG(1/Q)
3990 CLS:N=INT(T1)
4000 P1=P/N
4010 P2=2*P1:PRINT P2
4020 T2=(E4+1)/(E4-1):PRINT T2
4030 A=LOG(T2)/(2*N):PRINT A
4040 S1=(EXP(A)-EXP(-A))/2:PRINT S1
4050 FOR  M = 1 TO 4
4060 K=-K
4070 A2=2*M*A
4080 A1=A2+A
4090 S=(EXP(A1)-EXP(-A1))/2
4100 S1=S1+(K^M)*Q^(M*(M+1))*S
4110 C=(EXP(A2)+EXP(-A2))/2
4120 S2=S2+(K^M)*Q^(M*M)*C
4130 NEXT M
4140 Q5=2*Q^.25
4150 S0=Q5*S1/(1+2*S2)
4160 S0=ABS(S0)
4170 S4=S0*S0
4180 W=((1+S4*K1)*(1+S4/K1))^.5
4190 IF(INT(N/2)*2=N) GOTO 4230
4200 N2=(N-1)/2
4210 F5=1
4220 GOTO 4250
4230 N2=N/2
4240 F5=0
4250 FOR I = 1 TO N2
4260 A5=I
4270 IF(F5=0) THEN A5=A5-.5
4280 P5=P1*A5
4290 S1=SIN(P5)
4300 S2=0
4310 K=1
4320 FOR M= 1 TO 4
4330 K=-K
4340 A2=2*M*P5
4350 A1=A2+P5
4360 S1=S1+(K^M)*Q^(M*(M+1))*SIN(A1)
4370 S2=S2+(K^M)*Q^(M*M)*COS(A2)
4380 NEXT M
4390 X1=Q5*S1/(1+2*S2)
4400 X2=X1*X1
4410 V1=((1-X2*K1)*(1-X2/K1))^.5
```

```
4420 A(I) =1/X2
4430 S5=S0*V1
4440 D1=1+S4*X2
4450 B(I)=2*S5/D1
4460 C(I)=((S5^2+(X1*W)^2))/(D1^2)
4470 NEXT I
4471 PRINT
4480 H0=S0
4490 IF(F5=0) THEN H0=1/(10^E5)
4500 FOR G = 1 TO N2
4510 H0=H0*C(G)/A(G)
4520 NEXT G
4530 RETURN
4540 REM THE GRAPHICS SUBROUTINE STARTS HERE
4550 KEY OFF:CLS
4560 SCREEN 2:CLS
4570 LOCATE 1,34:PRINT "PASS BAND  DB":LOCATE 10,1:PRINT "F":PRINT "R":PRIN
4575 LOCATE 10,1:PRINT "E":PRINT "Q":PRINT :PRINT :PRINT "HZ"
4580 LINE (20,20)-(639,20):LINE-(639,199):LINE-(20,199):LINE-(20,20)
4590 FOR X=82 TO 640 STEP 62
4600 LINE(X,20)-(X,199)
4610 NEXT X
4620 FOR Y = 20 TO 199 STEP 36
4630 LINE (20,Y)-(639,Y)
4640 NEXT Y
4650 LOCATE 2,80:PRINT "0":LOCATE 2,1:PRINT "-100"
4660 FOR X=80 TO 1 STEP -16
4670 LOCATE 2,X
4680 READ M:DATA 0,-20,-40,-60,-80
4690 PRINT M
4700 NEXT X
4710 IF F1=0 THEN :LOCATE 4,10
4720 IF F1=0 THEN :PRINT "LPF":LOCATE 6,10:PRINT "CHARACTERISTICS"
4730 IF F4=0 THEN :LOCATE 4,10
4740 IF F4=0 AND F1<>0 THEN :PRINT "HPF":LOCATE 6,10:PRINT "CHARS"
4750 IF F3<F1 AND F2<F4 THEN :IF F1<F2 THEN :LOCATE 4,10
4760 IF F3<F1 AND F2<F4 THEN :PRINT "BPF":LOCATE 6,10:PRINT "CHARS"
4770 IF F1<F3 AND F4<F2 THEN :IF F3<F4 THEN :LOCATE 4,10
4780 IF F1<F3 AND F4<F2 THEN :IF F3<F4 THEN :PRINT "BSF":LOCATE 6,10:PRINT
4790 FOR I = 1 TO N0
4800 H(I)=ABS(H(I))
4810 X(I)=INT(5*H(I))
4820 IF(H(I)<1) THEN X(I)=INT(30*H(I))
4830 X(I)=639-X(I):X(I)=ABS(X(I))
4840 Y(I)=(I-1)*200/N0+20
4850 IF(I>1) THEN 4870
4860 LINE (20,20)-(X(I),Y(I))
4870 LINE -(X(I),Y(I))
4880 NEXT I
4890 END
```

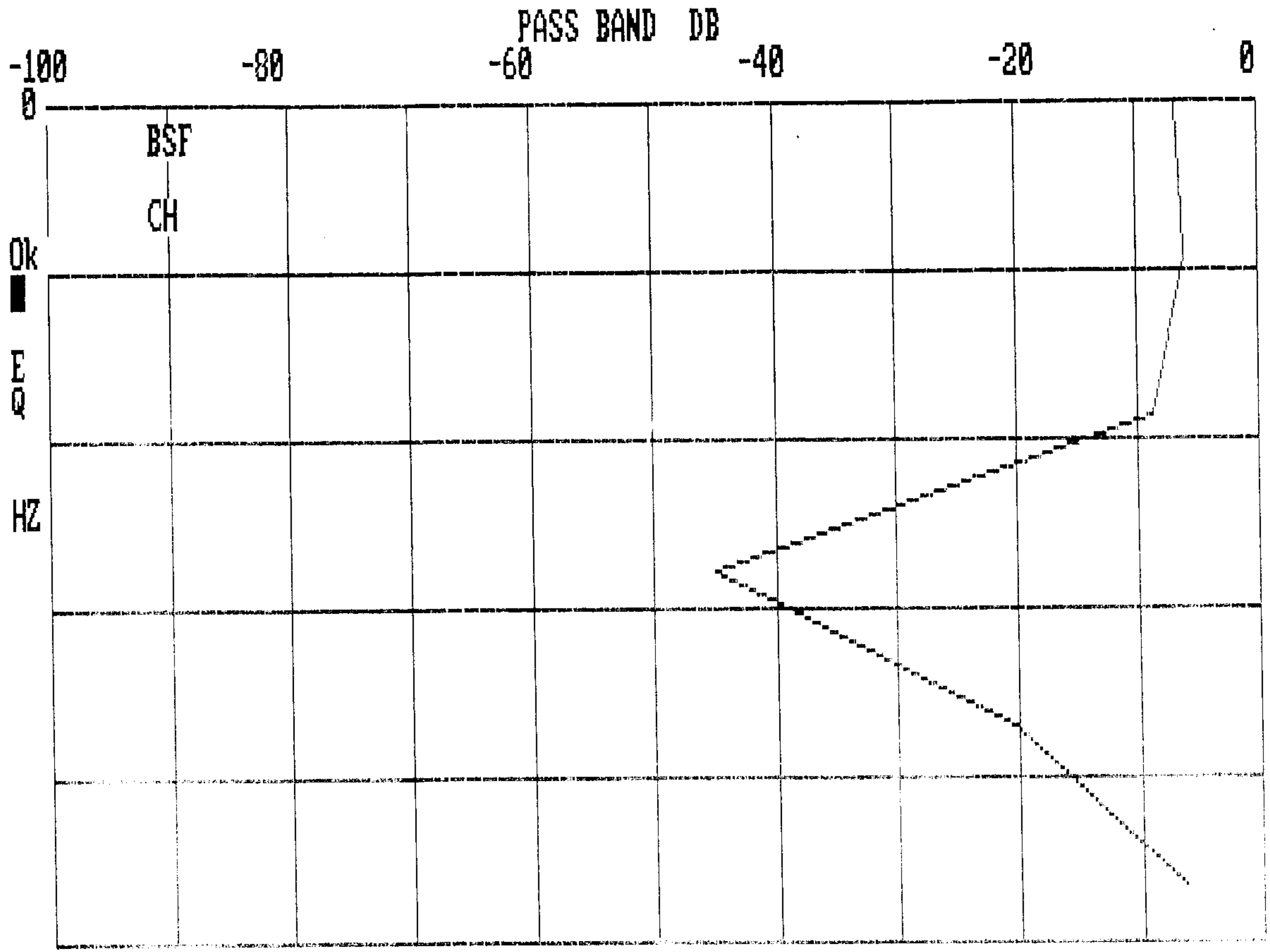
```
RUN
ELLIPTICAL LOW PASS FILTER SPECIFICATIONS
THE PASSBAND EDGE FREQUENCY= 2 HZ
THE STOPBAND EDGE FREQUENCY= 2.2 HZ
THE SAMPLING FREQUENCY= 2.2 HZ
THE PASSBAND RIPPLE= .5 DB
THE STOPBAND MINIMUM ATTENUATION= 60 DB
THE PROTOTYPE ANALOG LOW PASS FILTER
THE ORDER OF THE FILTER=N= 8
 I              A(I)           B(I)              C(I)
 1              15.78099        5568969          1519992
 2              2.389997        3318909          4895723
 3              1.375591        .1401918         .766581
 4              1.158892        3.613083E-02     8841126
THE CONSTANT MULTIPLIER H0= 8.885057E-04
THE DIGITAL FILTER PARAMETERS ARE GIVEN BELOW
  I              D(I)           E(I)             F(I)
 1              1.618507      -1.193143         .4330705
 2              .3583012      -.909721          .6683268
 3             -.1896278      -.6871865         .861496
 4             -.3577279      -.6010074         .9640744
THE CONSTANT MULTIPLIER H0= 1.216337E-02
I              P(I)           Q(I)
 1              1.429452      -3.391058
 2              3.565731       7.134297
 3              4.105036      -9.50097
 4              5.1708         9.269925
 5              4.105036      -6.198463
 6              3.565731       3.403019
 7              1.429452      -1.208496
 8              1               .4330705
ENTER THE DATA;W0,S0,N0
```


PASS BAND DB
-100
-80
-60
-40
-20
0
0
LPF
CHARACTERISTICS
0k
E
Q
HZ

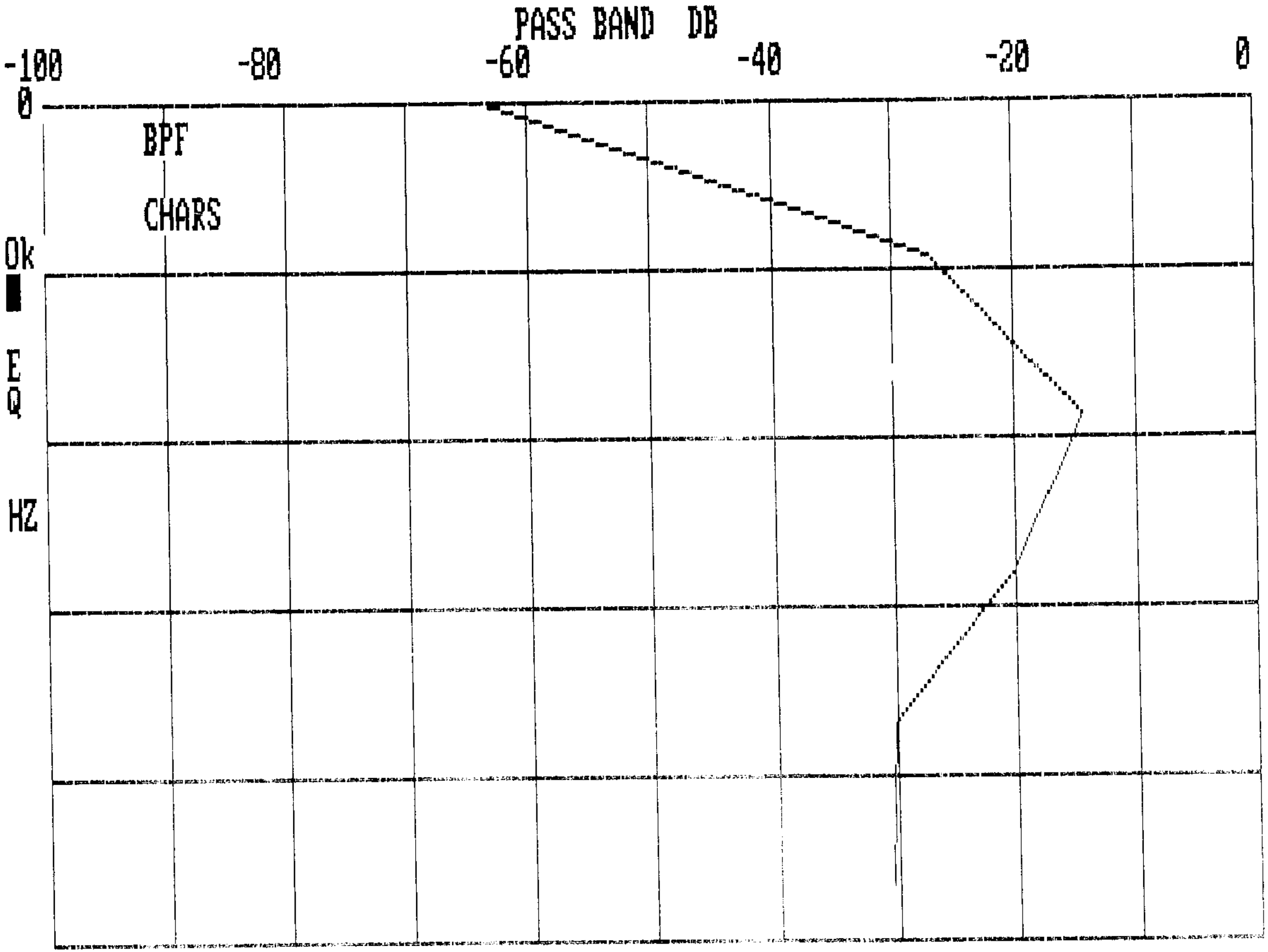
```
RUN
ELLIPTICAL HIGH PASS FILTER SPECIFICATIONS
THE PASSBAND EDGE FREQUENCY= 2 HZ
THE STOPBAND EDGE FREQUENCY= 1.725 HZ
THE SAMPLING FREQUENCY= 1.725 HZ
THE PASSBAND RIPPLE= .3 DB
THE STOPBAND MINIMUM ATTENUATION= 45 DB
THE PROTOTYPE ANALOG LOW PASS FILTER
THE ORDER OF THE FILTER=N= 6
 I            A(I)            B(I)            C(I)
 1            13.4516         .692067         .2157225
 2            2.169112        .3595419        .5671365
 3            1.375164        9.594129E-02    .7884865
THE CONSTANT MULTIPLIER H0= 2.488672E-03
THE DIGITAL FILTER PARAMETERS ARE GIVEN BELOW
 I            D(I)            E(I)            F(I)
 1            -1.575741       1.02775         .3489984
 2            -.3044204       .786194         .6529181
 3            .1486659        .6445543        .9032561
THE CONSTANT MULTIPLIER H0= .0237811
 I            P(I)            Q(I)
 1            -1.731496       2.458499
 2            3.200172        3.882369
 3            -3.391678       3.422332
 4            3.200172        2.151282
 5            -1.731496       .7825102
 6            1               .3489984
ENTER THE DATA;W0,S0,N0
```


PASS BAND DB
-100
-80
-60
-40
-20
0
0
HPF
CHARS
Ok
E
Q
HZ

```
RUN
ELLIPTICAL BAND STOP FILTER SPECIFICATIONS
THE LOWER PASSBAND EDGE FREQUENCY=F1HZ
THE UPPER PASSBAND EDGE FREQUENCY= 2 HZ
THE LOWER STOPBAND EDGE FREQUENCY= 1.1 HZ
THE UPPER STOPBAND EDGE FREQUENCY= 1.82 HZ
THE SAMPLING FREQUENCY= 10 HZ
THE PASSBAND RIPPLE= .2 DB
THE STOPBAND MINIMUM ATTENUATION= 40 DB
THE PROTOTYPE ANALOG LOW PASS FILTER
THE ORDER OF THE FILTER=N= 5
 I           A(I)           B(I)          C(I)
 1           3.109936       6054553       .527962
 2           1.45386        1533984       .799691
THE ADDITIONAL PARAMETER FOR ODD N=S0= .4986173
THE CONSTANT MULTIPLIER H0= 4.656055E-02
THE DIGITAL FILTER PARAMETERS ARE GIVEN BELOW
 I           D1(I)          D2(I)
 1           -2.411861      3.393082
 2           -2.346683      3.247341
  I      E1(I)          E2(I)
                                          E3(I)          E4(I)
  1    -1.954646      2.204223
                                         -1.415852       5641073
  2    -2.205879      2.896988
                                         -2.090984       9070486
ADDITIONAL FACTORS FOR ODD N ARE
A1=-1.236069  A0= 1         B1=-.7927951  B0= .2827681
THE CONSTANT MULTIPLIER=H0= .4102958
 I              P(I)         Q(I)
 1            -5.994613    -4.95332
 2             19.18218     12.99413
 3            -40.51573    -22.67065
 4             62.04367     28.95543
 5            -71.19007    -27.72204
 6             62.04367     20.18144
 7            -40.51573    -11.00784
 8             19.18218     4.329544
 9            -5.994613    -1.102332
 10            1            .1446847
ENTER THE DATA;W0,S0,N0
?
```


PASS BAND DB
-100
-80
-60
-40
-20
0
0
BSF
CH
Ok
E
Q
HZ

```
ELLIPTICAL BANDPASS FILTER SPECIFICATIONS
THE LOWER PASSBAND EDGE FREQUENCY=F1HZ
THE UPPER PASSBAND EDGE FREQUENCY= 4 HZ
THE LOWER STOPBAND EDGE FREQUENCY= 2.427 HZ
THE UPPER STOPBAND EDGE FREQUENCY= 4.3 HZ
THE SAMPLING FREQUENCY= 10 HZ
THE PASSBAND RIPPLE= .1 DB
THE STOPBAND MINIMUM ATTENUATION= 60 DB
THE PROTOTYPE ANALOG LOW PASS FILTER
THE ORDER OF THE FILTER=N= 5
 I              A(I)            B(I)            C(I)
 1              5.427973         600699          .3427077
 2              2.233397         1960799          573471
THE ADDITIONAL PARAMETER FOR ODD N=S0= .4112871
THE CONSTANT MULTIPLIER H0= 6.667712E-03
THE DIGITAL FILTER PARAMETERS ARE GIVEN BELOW
                D1(I)           D2(I)
 1              1 138131        .544941
 2              1.667798        1.729315
 I       E1(I)           E2(I)
                                             E3(I)           E4(I)
 1       2.081943        2.497781
                                             1.571969        .5874216
 2       2.127459        2.699857
                                             1.94206         .8500082
ADDITIONAL FACTORS FOR ODD N ARE
A1= 0           A0=-1           B1= 1.037733  B0= .6790909
THE CONSTANT MULTIPLIER=H0= 5.094355E-03
 I              P(I)            Q(I)
 1              2.805929        5.247134
 2              3.172429        14.67421
 3              2.877039        27.2976
 4              2.566291        37.2567
 5              0               38.24485
 6             -2.566291        30.01212
 7             -2.877039        17.72081
 8             -3.172429        7.661747
 9             -2.805929        2.200258
 10            -1               .339079
```


PASS BAND DB
-100
-80
-60
-40
-20
0
0
BPF
CHARS
0k
E
Q
HZ

CONCLUSION

CONCLUSION

Digital filters, in the form of software have been used extensively in the past and will undoubtedly be used in the future at a progressively increasing rate.

Typical Applications are

1. Data smoothing and prediction
2. Image enhancement
3. Pattern recognition
4. Speech processing
5. Processing of telemetry signals
6. Processing of bio medical signals
7. Simulation of analog systems

Programmable hardware digital filters have already made their appearance in the form of digital signal processors such as FFT processors, Frequency synthesizers and wave analysers. Many signal processing chips have been developed, which eliminate the endless hardware components and simplifies digital filtering.

Non-programmable digital filters are currently considered as a possible replacement of analog filters in many communication sub systems.

The main disadvantage of hardware digital filters at present in their relatively high cost. However, with the tremendous advancements in the domain of large Scale integration, the cost of hardware digital filters is likely to drop drastically in the not-too-distant future. At that time digital filters will become more attractive than analog filters in many more applications. It is not expected that digital filters will replace analog filters, on the other hand like crystal, mechanical monolithic and active filters, digital filters will become an invaluable addition to the bag of tricks available to the filter designer.

Future Developments

FUTURE DEVELOPMENTS

The PCB for the A/D convertor has been designed such that using analog multiplier chip and then selecting a single channel it will be possible to process more than one signal simultaneously. Thus using two 7506 chips 32 single ended or 16 differential inputs can be connected to the ADC. Then using the control yegister a single channel is selected and processed. If the software is written in order to perform multitasking then it will appear as if 8 signals were filtered simultaneously.

The DAC will have have only 8 channels and each channel signal can be filtered with different cut off frequencies. The D/A convertor is designed such that it will have 8 channels. So, 8 signals can be processed simultaneously. (Time slots and allotted to each signal).

Bibliography

BIBLIOGRAPHY

1) Digital Filters - Analysis and Design

- by Andreas [illegible]

2) Digital and Kalman Filtering

- by S.M. Bozic

3) Digital Signal Processing

- by Rabiner and Gold

4) "A simple Digital Filter" by R.D.Grappel

'BYTE', Vol.3, No.2, [illegible]

(pg 168-171)

5) "Digital Filters Explained" by

- H.J. Hutchings, B.A.

'Electronics & Wireless World' Dec.1985

(pg. 23-25)

6) "Digital Filter Design procedure - 3" by

- by J.T.R. Sylvester Bradley M.A

'Wireless world' , March - 1985 (pg.46-47)

7) 'Assembly Language Techniques for the IBM PC'

- Alan R.Miller