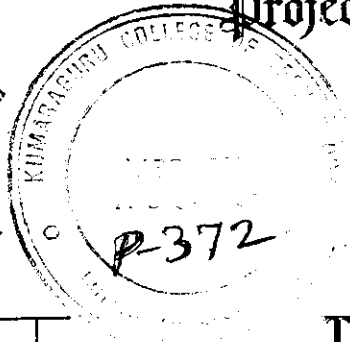
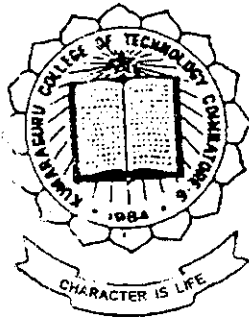


# Analysis of Flow Induced Vibration in Steel Chimneys by FEM Approach

Project Report 1998 - 99



Submitted by

**K. RAJESH**

**T. VIJAYA KANNAN**

**S. MURUGANANTHAM**

Under the Guidance of

**MR. MOHANDAS GANDHI M.E., M.B.A.,**

In partial fulfilment of the requirements  
for the award of the Degree of  
**BACHELOR OF ENGINEERING**  
in Mechanical Engineering Branch  
of Bharathiar University, Coimbatore

Department of Mechanical Engineering

**Kumaraguru College of Technology**


Coimbatore - 641 006

Department of Mechanical Engineering  
**Kumuraguru College of Technology**  
Coimbatore - 641 006

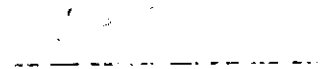
**PROJECT REPORT 1998 - 99**

**CERTIFICATE**

This is to certify that the report entitled "ANALYSIS OF FLOW INDUCED VIBRATION IN STEEL CHIMNEYS BY FEM APPROACH" has been submitted by Mr. K. RAJESH, Roll No. 95ME30,  
Mr. T. VIJAYA KANNAN, Roll No. 95ME53  
Mr. S. MURUGANANTHAM, Roll No. 95ME 24  
in partial fulfilment for the award of  
BACHELOR OF ENGINEERING in Mechanical Engineering branch of the  
Encrathiar University, Coimbatore - 641 006 during the academic year 1998-99.



Guide



Head of Department

The candidate was Examined by us in the Project Work  
Viva-voce Examination held on \_\_\_\_\_ and his University  
Register Number is \_\_\_\_\_



Internal Examiner



External Examiner

GCI Education  
ACI 2/ II Avenue,  
Devi Nilayam,  
Annanagar,  
Chennai

27<sup>th</sup> February 1999

**TO WHOMSOEVER IT MAY CONCERN**

This is to inform that **Mr. T. Vijay Kannan, Mr. S. Muruganatham, Mr. K. Rajesh**, Final Year Mechanical, Kumaraguru College of Technology, Coimbatore -6 has approached us for their project on "Analysis of Flow Induced Vibration steel Chimney By FEM Approach". Their model was developed and analyzed using **ANSYS 5.4 FEM** software and solution extracted on deflection, stress and natural frequency.

For Giri Computer Info Systems Pvt. Ltd.,

**M.K. Dhandapani**  
**Manager – Technical**

*M.K. Dhandapani*

For GIRI COMPUTER INFORMATION  
SYSTEMS PVT. LTD.

Managing Director

## ACKNOWLEDGEMENT

We wish to show our sincere and heartfelt thanks to **Dr.K.Padmanabhan**, Principal, Kumaraguru College of Technology for providing us the environment to do the project successfully

We wish to express our greatfulness to **Dr.T.L.Seetharamarao**, head of Department, Mechanical Engineering for his motivation and encouragement in carrying out this project.

We accord ourselves the privilege of thanking **Mr. N.Mohandas Gandhi M.E, M.B.A**, Senior Lecturer Mechanical Engineering for his valuable guidance and immense help at all stages of the project in completing this project work

We are very much thankful to **Mr.V.Velmurugan**, Senior Lecturer Mechanical Engineering and **Mr.V.Govindharaj**, Senior Lecturer Civil Engineering for their valuable guidance in our project specifically in the area of FEM and software applications

We are responsible for thanking **Mr.V.Gunaraj**, Assistant Professor Mechanical Engineering, **Ms Deveki**, and Assistant Professor Computer Science Engineering Department for their valuable guidance in MATLAB software

A special thanks is due for **Mr.R.James Ganna Xavier**, Senior Lecturer Mechanical Engineering, our faculty advisor. We will be failing in our duty if we do not thank all the other staff members who have always been a source of inspiration to us.

We wish to record our sincere thanks to **Mr.K.Dhandapani**. Technical manager of Giri Computer Information (GCI) System, Chennai for providing the ANSYS software for completing our project.

## **SYNOPSIS :**

This project titled “analysis of flow induced vibration in steel chimneys by FEM approach “ deals about the causes of vibration due to Von – Karman vortex shedding formed by the flow of wind over the cylindrical structure of the chimney.

In this project, the thickness of the steel plate for the chimney structure is designed based on the bending moment on each zone corresponding to the wind load value and only the required thickness value is provided in respective heights. For this whole chimney structure, the mass matrix and stiffness matrix are formed and these matrices are solved for eigen value problem to obtain the natural frequency for the chimney structure. In this regard, we have chosen the C++ software for design calculations and MATLAB for solving the eigen value problem.

The analysis part is done in the ANSYS (version 5.1) software .ie. for finding the frequency under loading condition, deflection and stress calculations etc. then after comparing the above two results the conclusion has been given.

# CONTENTS

0.	INTRODUCTION	
1.	CHIMNEY	
1.1.	What is Chimney	
1.2.	Function of Chimney	
1.3.	Types of steel chimneys	
1.4.	Forces acting on steel chimney	
2.	INTRODUCTION TO VIBRATION	2
2.1.	Terms used in Vibratory motion	
2.2.	Types of Vibratory motion	
2.3.	Types of Free Vibration	
2.4.	Vibration Theory	
2.5.	Vibration Analysis Procedure	
2.6.	Mathematical Model	
2.7.	Mass Stiffness and Damping Matrices	
2.8.	Solution of mathematical models	
2.9.	Multi degree of freedom systems	
2.10.	Influence coefficients.	
3.	ABOUT FEM	24
3.1.	Why FEA	
3.2.	Application of FEA	
3.3.	Parameters required for FEA	
3.4.	Steps in Finite Element method	
4.	METHODOLOGY	30
4.1.	To find natural frequency	
4.2.	To find the stiffness matrix	
4.3.	To find the influence coefficient.	
4.4.	To find the critical condition (or) Resonance condition	

5.	DESIGN	31
5.1.	Exit Velocity	
5.2.	Wake Bufetting	
5.3.	Vortex formation	
6.	DESIGN PROCEDURE	35
6.1.	Design of Chimney Dimensions	
6.2.	Design Calculations	
6.3.	Design in C++ Software	
7.	NATURAL FREQUENCY CALCULATION	63
7.1.	Formation of Stiffness matrix	
7.2.	Formation of mass matrix	
7.3.	About MATLAB Software	
7.4.	Solution phase in MATLAB.	
8.	ANALYSIS IN ANSYS SOFTWARE	71
8.1.	Pre - processing	
8.2.	Solution phase	
8.3.	Post processing	
9.	CONCLUSION	113
10.	BIBLIOGRPAHY	114

## **INTRODUCTION:**

When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a vibratory motion. This is due to the reason that, when a body is displaced, the internal forces in the form of elastic or strain energy are present in the body. At release these forces in the body to its original position. When the body reaches the equilibrium position, the whole of the elastic or strain energy is converted into kinetic energy due to which the body continues to move in the opposite direction. The whole of the kinetic energy is again converted into strain energy due to which the body again returns to the equilibrium position. In this way, the vibratory motion is repeated indefinitely.

Any motion that repeats itself after an interval of time is called vibration. Vibration that is caused by fluid flowing around or through objects is referred to as flow induced vibration. The subject of flow induced vibration is a broad one. Our discussion will be limited to some of the more important aspects of vibration caused by the vortex shedding by finite element analysis (FEM), since it is one of the most common sources of flow induced vibration.

Every vibrating system has its frequency known as natural frequency. Whenever the natural frequency of a vibrating system coincides with the frequency of the external excitation, there occurs a phenomenon known as resonance, which leads to excessive deflections and failure.

The project is full of accounts of design analysis based on resonance and excessive vibration of the systems. Because of the devastating effects that vibration can



have on the system, vibration testing has become a standard procedure in the design and development of most engineering systems.

## **OBJECTIVES:**

### **Importance of study:**

- In 1940 the suspension bridge across the Tacoma narrows in Washington, after in operation for months, was completely wrecked due to flow induced vibration at moderate wind speed.
- The oil storage tank in Venezuela was collapsed due to wind excited oscillation.
- Chimneys also some crack failure due to the lack of knowledge in flow induced vibration in designing their structure.
- After this, the study of flow induced vibration became very important in design and development of most the engineering system.

### **The main objectives of this project are:**

- \* The frequency vortex shedding does not coincide with the natural frequency of the system to avoid resonance.
- \* Stable and economical design by reducing the factor of safety.
- \* Innovative developed design method, which involves both static and dynamic design.
- \* Since the design analysis is based on FEM, it is easy to analyze complex problems and the results will be very accurate.
- \* FEM offers scope for developing general-purpose programs with the properties of various types of elements forming an “Element library” and the other procedures of analysis forming the common cone segments. The element library is a very important component in a package and allows the

user to idealize the system and arrive at the finite element model as realistically possible.

- \* This modular structure of the program organisation is well exploited in the large number of program packages such as "STAR DYNE" "PROMECHANICA" etc., now available for practical application to various disciplines of engineering.
- \* Since in this project vibration analysis is done in chimneys by taking it as a cantilever structure, we can extend this procedure in analyzing the vibrations for the submarine periscopes, telescoping car antenna, wind turbine blades, aircraft wings, propeller shaft of a ship, micro waves & TV transmission towers, TV antennas, electric transmission lines, Nuclear fuel rods, heat exchanger tubes, water and oil storage tanks, street lighting towers, cantilever bridge and wind tunnels etc depending upon the nature of loading and boundary conditions.

## **1.CHIMNEYS :**

### **1.1 WHAT IS CHIMNEY:**

Chimneys as we know them today are tall slender structures, which fulfil an important function. They had a humble beginning as household vents and over the years, as vents grew larger and taller, they came to be known as chimneys. A cluster of them is a stack.

During early days the term “stack” was used to describe the extension piece added to a flue duct to convey and discharge combustion gases away from the operating area of an industry. A stack, which was scientifically designed to take cognizance of gas temperature and velocity effects, corrosion aspects, etc., was called a chimney. By usage, the term “stack” has gained popularity and today it also signifies a chimney.

Steel chimneys are also known as steel stacks. The steel chimneys are made of steel plates and supported on foundation. The steel chimneys are used to escape and disperse the flue gases to such a height that the gases do not contaminate surrounding Atmosphere. The cross-sectional area of steel chimney is kept large enough to allow the passage of burnt gases.

#### **The cross-sectional area of steel chimney depends on**

- 1.Type and quantity of fuel to be used in a plant.
2. Available draft for carrying the burnt gases up the chimney
3. Losses due to friction within the chimney

The height of steel chimney is kept to provide the required draft. The draft is defined as the difference between absolute gas pressure at any point in the duct or steel chimney and the ambient atmospheric pressure.

**The draft depends on**

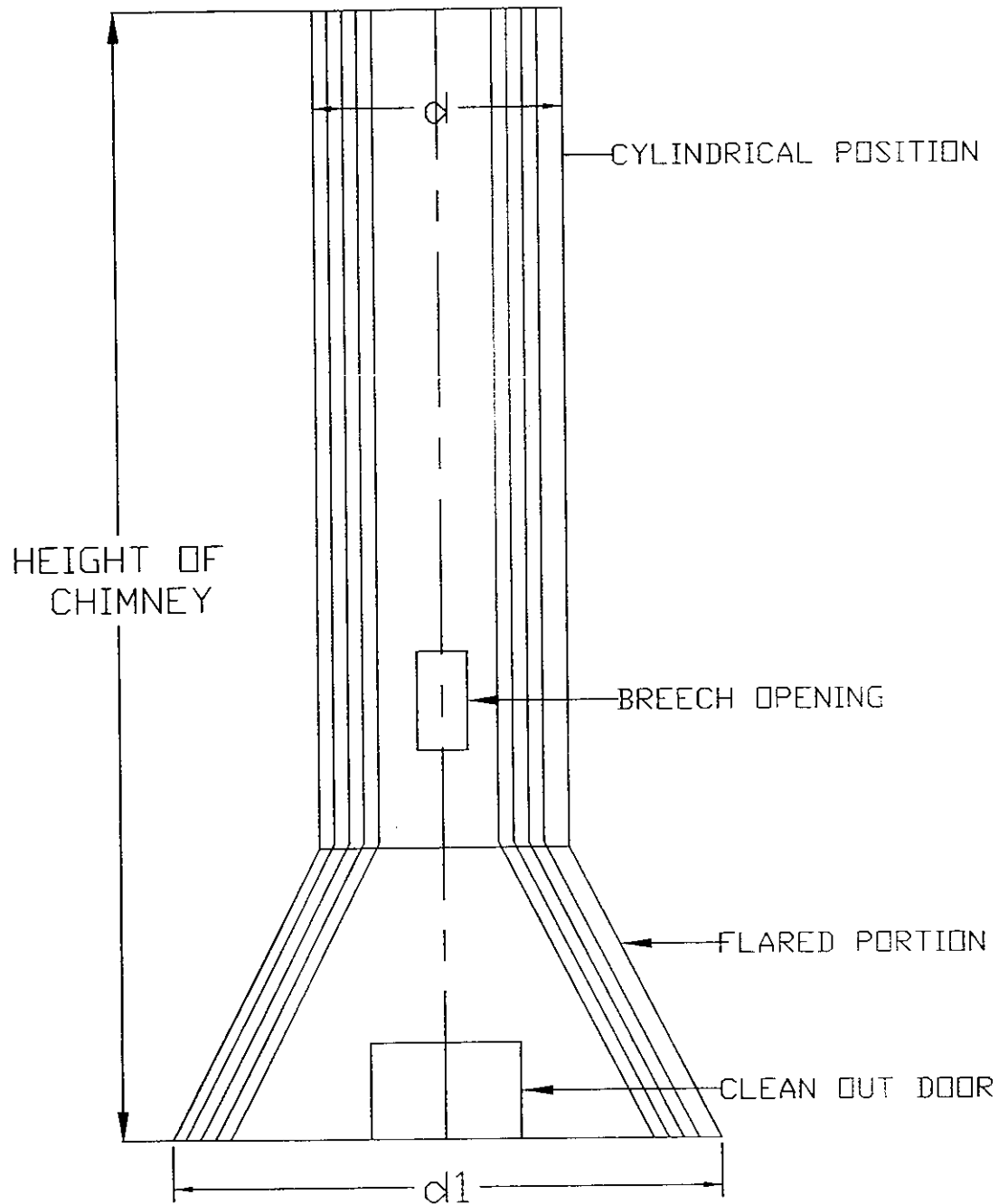
1. The height of steel chimney above sea level
2. The type of fuel to be burnt
3. The type of furnace and the temperature of burnt gases.

When the gases in a steel chimney are heated, then the gases expand. The hot gases occupy larger volume than before. The weight of gases per cubic metre becomes less. As a result of this, the unit pressure at the bottom of chimney due to weight of hot gases also becomes less than the unit pressure due to weight of cold air outside the chimney.

The difference between two pressure results in the flow of the burnt gases up the chimney. For the purpose of the structural design of the steel chimney, the height and diameter of chimney at the top are known data.

The steel chimney is made cylindrical in shape. The lower portion of steel chimney is widened or flared, in order to provide a large base and greater stability. The widened section of the chimney at the base reduces the unit stresses in the steel at the base of the chimney. The loads acting on the steel chimney are transferred to the foundation easily by the widened section

# SELF SUPPORTING STEEL CHIMNEY



## **1.2 FUNCTION OF A CHIMNEY:**

A chimney is a means by which waste gases are discharged at a high enough elevation so that after dilution due to atmospheric turbulence, their concentration and that of their entrained solid particulate is within acceptable limits on reaching the ground. A chimney achieves simultaneous reduction in concentration of a number of pollutants (SO<sub>2</sub>, fly ash, etc.) and being highly reliable it does not require a standby. While these are its distinct merits, it is well to remember that a chimney is not the complete solution to the problem of pollution control.

## **1.3 TYPES OF STEEL CHIMNEYS:**

The steel chimneys are two types:

1. Self-supporting steel chimneys.
2. Guyed steel chimneys.

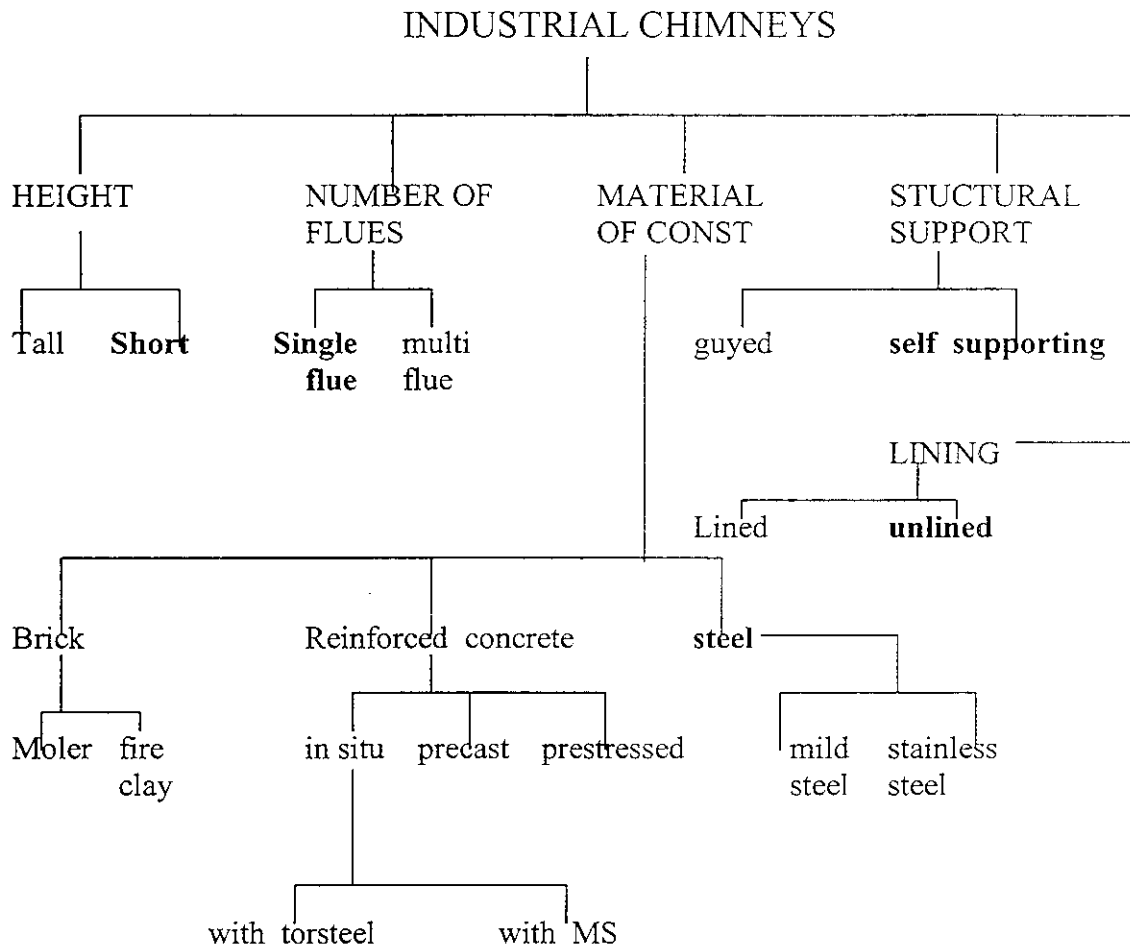
### **1.3.1 SELF-SUPPORTING STEEL CHIMNEYS.**

When the lateral forces (wind or seismic forces) are transmitted to the foundation by the cantilever action of the chimney, then the chimney is known as self-supporting chimney. The self-supporting chimney together with the foundation remains stable under all working conditions without any additional support.

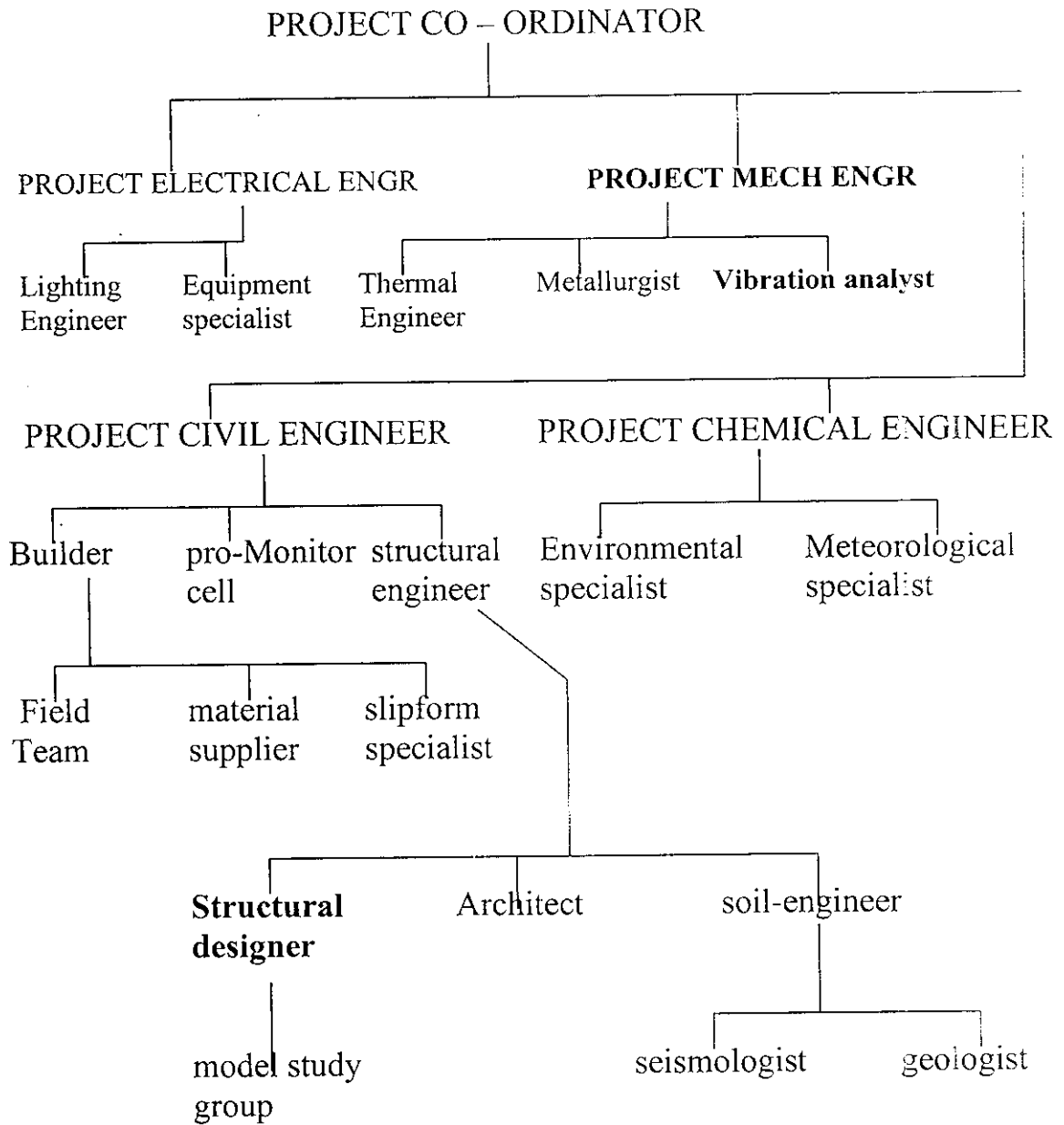
### **1.3.2 GUYED STEEL CHIMNEYS.**

In high steel chimneys, the mild steel wire ropes or guys are attached to transmit the lateral forces. Such steel chimneys are known as guyed steel chimneys. These guys or stays ensure the stability of the guyed steel chimney. The steel chimneys may provide with one, two or three sets of guys.

# CLASSIFICATION OF INDUSTRIAL CHIMNEYS



# CHIMNEY IMPLEMENTATION TEAM





## **1.4 FORCES ACTING ON STEEL CHIMNEY**

The various forces acting on the self- supporting steel chimney are as follows:

- 1.self-weight of the steel chimney.
- 2.wind pressure

### **1.4.1 SELF-WEIGHT OF THE STEEL CHIMNEY.**

The self-weight of steel chimney,  $W_s$  acts vertically. Consider a horizontal section XX as shown in fig. The thickness of steel plates of chimney above the section XX may be assumed constant. The self –weight of chimney is given by

$$W_s = \rho D t H$$

Where,

$\rho$  - unit weight of the steel

D - outer diameter of the cylindrical portion of the chimney

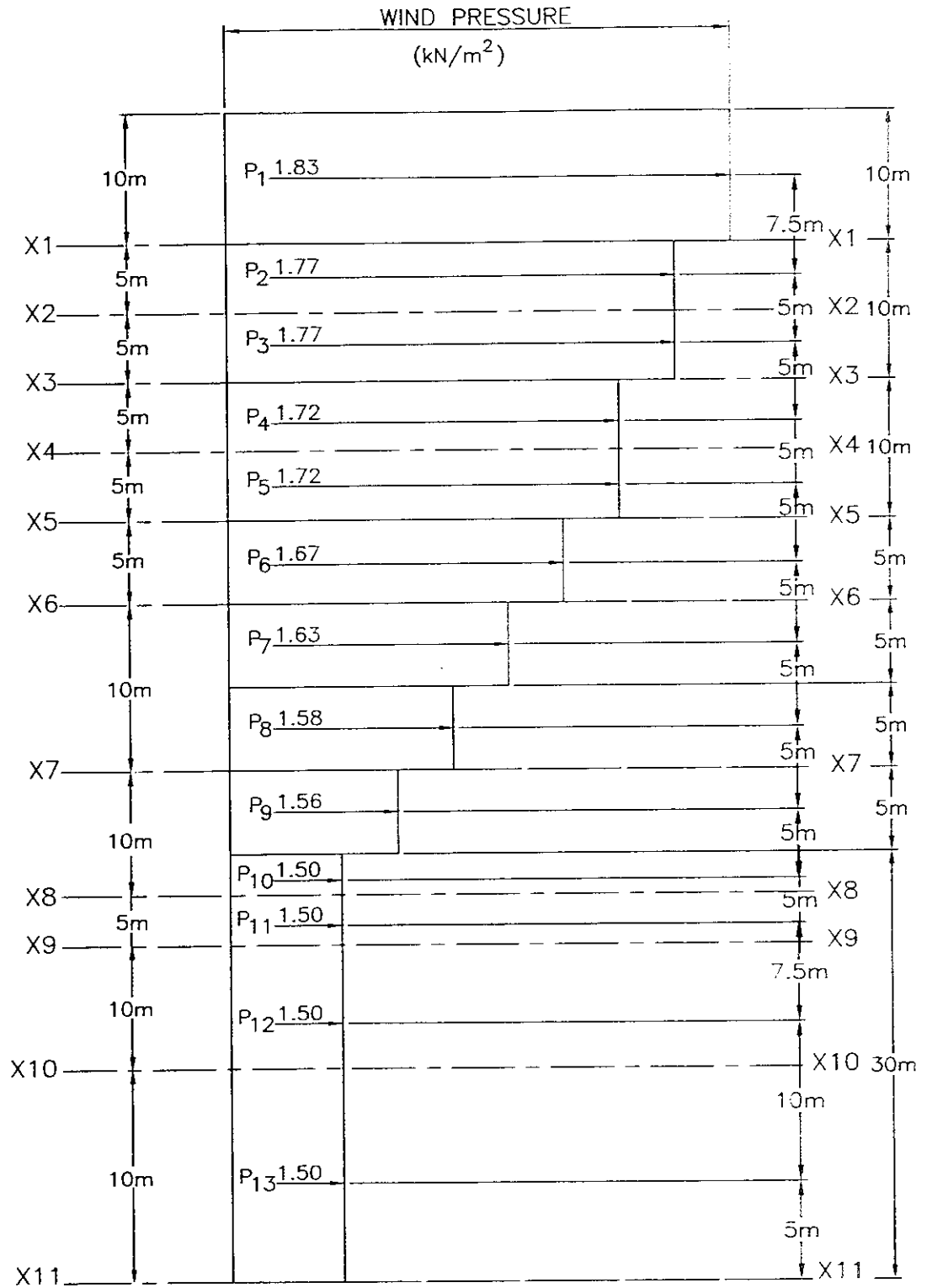
T - thickness of the steel plate

H - height of the steel portion of the chimney

### **1.4.2.WIND PRESSURE.**

The wind pressure acts horizontally. The wind pressure acting on a structure depends on the shape of the structure, the width of the structure, the height of the structure, the location of the structure, and the climatic conditions. The wind pressure per unit area increases with the height of the structure above the ground level. In order to simplify the design, the steel chimney is divided into number of segments of equal height. The height of each segment may be kept 10m. The intensity of wind pressure over each segment may be assumed as uniform. The intensity of wind pressure corresponding to the mid-height of each segment may be noted from IS: 875-1964.the wind pressure on the flared portion may be found by using average diameter. The wind pressure is assumed to act at the mid-height of each segment and as also in the flared portion. It has also been practice to take uniform wind pressure of the full height of chimney.

# WIND PRESSURE DISTRIBUTION



## **2. INTRODUCTION TO VIBRATION:**

When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a vibratory motion. This is due to the reason that, when a body is displaced, the internal forces in the form of elastic or strain energy are present in the body. At release these forces in the body to its original position. When the body reaches the equilibrium position, the whole of the elastic or strain energy is converted into kinetic energy due to which the body continues to move in the opposite direction. The whole of the kinetic energy is again converted into strain energy due to which the body again returns to the equilibrium position. In this way, the vibratory motion is repeated indefinitely.

### **2.1 TERMES USED IN VIBRATORY MOTION:**

The following terms are commonly used in connection with the vibratory motions:

#### **2.1.1 Period of vibration or time period.**

It is the time interval after which the motion is repeated itself. The period of vibration is usually expressed in seconds.

#### **2.1.2 Cycle.**

It is the motion completed during one time period.

#### **3. Frequency.**

It is the number of cycles described in one second. in S.I. units, the frequency is expressed in hertz which is equal to one cycle per second.

## **2.2 TYPE OF VIBRATORY MOTION:**

### **2.2.1.Free or Natural Vibrations.**

when no external force acts on the body, after giving it in an initial displacement, then the body is said to be under free or natural vibrations. The frequency of the free vibrations is called free or natural frequency.

### **2.2.2 Forced vibrations.**

When the body vibrates under the influence of external force, then the body is said to be under forced vibrations. The external force applied to the body is periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

### **2.2.3.Damped vibrations.**

When there is a reduction in amplitude over every cycle of vibration, the motion is said be damped vibration. This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistance to the motion.

## **2.3 TYPES OF FREE VIBRATIONS**

### **2.3.1 Longitudinal vibrations.**

When the particles of the system moves parallel to the axis of the system, then the vibrations is known as longitudinal vibrations. In this case, the system is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the system

### **2.3.2 Transverse vibrations.**

When the particles of the system move approximately perpendicular to the axis of the system, then the vibrations are known as transverse vibrations. In this case, the system is straight and bent alternately and bending stresses are induced in the system.

### **2.3.3 Torsional vibrations.**

When the particle of the system moves in a circle about the axis of the shaft, then the vibrations are known as torsional vibrations. In this case, the system is twisted and untwisted alternately and the torsional shear stresses are induced in the system.

## **2.4 VIBRATION THEORY**

As the stiffness is increased, in general it may be stated that amplitude of vibration decreases while the frequency of vibration increases.

The lowest natural frequency of the vibration is termed as the fundamental frequency.

A condition of resonance will occur if the frequency of the applied force system coincides with one of the natural frequencies of the body. At resonance condition the amplitude of vibration will approach infinity with time.

A node is defined as any point having zero amplitude on the unsupported length of a vibrating elastic body.

An elastic body vibrates of its fundamental frequency has no nodes. This second natural frequency has one node, third two etc.

As the number of nodes increases the amplitude of vibration decreases. As the number of nodes becomes infinitely large, the amplitude approaches zero in limits.

The amplitude of vibration does not affect the frequency of given mode vibration.

## **2.5 VIBRATION ANALYSIS PROCEDURE:**

A vibratory system is a dynamic system for which the variables such as the excitations (inputs) and responses (outputs) are time dependent. The response of a vibrating system generally depends on the initial conditions as well as the external excitations. Most practical vibrating systems are very complex, and it is impossible to consider all the details for a mathematical analysis. Only the most important features are considered in the analysis to predict the behavior of the system under the specified condition. Thus the analysis of a vibrating system usually involves mathematical modeling, derivations, of the governing equations, solution of the equation, and interpretation of the results.

### **Step 1: Mathematical Modeling.**

The purpose of the mathematical modeling is to represent all-important features of the system for the purpose of deriving the mathematical (or analytical) equations governing the behavior of the system. The mathematical model should include enough details to be able to describe the system in terms of equations. Sometimes the mathematical model is gradually improved to obtain more accurate results.

### **Step 2: Derivation Of Governing Equation.**

Once the mathematical model is available, we use the principle of dynamics and derive the equations that describe the vibration of the system. The equation of motion can be derived conveniently by drawing the free body diagram of all masses involved. The equations of motion of a vibrating system are usually in the form of a set of ordinary differential equations for a discrete system and partial differential equations for

a continuous system. Several approaches are commonly used to derive the governing equations. Among them are Newton's second law of motion, D'Alembert's principle, and the principle of conservation of energy.

### **Step 3: Solution of the governing equation.**

The equation of motion must be solved to find the response of the system depending on the nature of the problem; we can use standard methods such as Laplace transformation method, matrix method, and numerical method for finding the solution.

### **Step 4: Interpretation of the results.**

The solution of the governing equations gives the displacement, velocities, and acceleration of the various masses of the system. These results must be interpreted with a clear view of the purpose of the analysis and the possible design implications of the results.

## **2.6 MATHEMATICAL MODEL:**

1. Lumped parameter model or discrete system
2. Continuum mechanics based models or continuous system

### **2.6.1 Lumped parameter model:**

In a lumped parameter mathematical model, the actual system response is directly described by the solution of a finite number of state variables. There are some general procedures such as steady state, propagation, and eigenvalue problems.

#### **Continuum mechanics based mathematical model**

For a continuum mechanics based model, the formulation of the governing equations is achieved as for a lumped parameter model, but instead of a set of algebraic equations for the unknown state variables, differential equations govern the response. The

exact solution of the differential equations satisfying all boundary conditions is possible only for relatively simple mathematical models, and numerical procedures must in general be employed.

The free vibration of discrete and lumped mass systems have more than one degree of freedom and consequently require more than one coordinate to describe their motion. In general, discrete and lumped mass systems have finite numbers of degrees of freedom, while continuous systems theoretically have an infinite number of degrees of freedom.

An n degree of freedom system inherently has n natural frequencies and n normal modes of vibration describing its configurations as it vibrates at the different frequencies. Thus, when a system vibrates freely at a particular natural frequency, the normal mode corresponding to that frequency describes the configuration of the system for that particular mode, and there is a particular relationship between the amplitudes and the coordinates defining the motion of the system.

### 2.7 MASS, STIFFNESS, AND DAMPING MATRICES:

The matrix form of equations for multiple degree-of-freedom systems will generally involve a mass matrix M, a stiffness matrix K, and a damping matrix C.

The general form of the matrix equation for an n-degree-of-freedom system subjected to excitation forces and/or moments is

$$\begin{bmatrix} M_{11} & M_{12} & \dots & M_{1n} \\ M_{21} & M_{22} & \dots & M_{2n} \\ \dots & \dots & \dots & \dots \\ M_{n1} & M_{n2} & \dots & M_{nn} \end{bmatrix} \begin{bmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \dots \\ \ddot{X}_n \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \dots & \dots & \dots & \dots \\ C_{n1} & C_{n2} & \dots & C_{nn} \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dots \\ \dot{X}_n \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} & \dots & K_{1n} \\ K_{21} & K_{22} & \dots & K_{2n} \\ \dots & \dots & \dots & \dots \\ K_{n1} & K_{n2} & \dots & K_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \dots \\ X_n \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ \dots \\ F_n \end{bmatrix}$$

Where,

$X_1, X_2, \dots, X_n$  are displacements and  
 $F_1, F_2, \dots, F_n$  are excitation forces or moments.

The above equation can be written in simple, compact form as

$$M \ddot{X} + C \dot{X} + K X = F$$



Where,

F is a column matrix involving excitation forces.

It should represent a system with many degrees of freedom, has the general appearance as the differential equation of motion of a single degree of freedom system.

For undamped free vibration

$$M \ddot{X} + K X = 0$$

Where,

0 is a null column matrix containing only zero elements.

As we shall see in this equation is a fundamental to the formation of equations pertinent to determining the natural frequencies and normal mode configurations of n degree of freedom system.

A unique characteristic of the mass, stiffness and damping matrices of linear system with small displacements is that they are all symmetric.

That is,

$$m_{ij} = m_{ji}$$

$$c_{ij} = c_{ji}$$

$$k_{ij} = k_{ji}$$

The mass matrix is usually a diagonal matrix, in which all the elements are except for those on the diagonal.

The elements of such a matrix for an n degree of freedom system are generally zero everywhere, except for those which are on the diagonal and immediately above and below it. The stiffness matrix is thus referred to as a banded matrix.

We assume that the mass distribution of the chimney can be represented by the lumped masses at the different levels, shown in the figure.

The co ordinates  $x_1, x_2$ , and the velocities and acceleration associated with them are assumed positive to the right. Arbitrarily assuming that  $x_1 > x_2 > x_3$ , the free body diagrams show the forces that the columns exert on the masses. In applying Newton's second law, the forces having senses the same as the positive sense assumed for the acceleration are summed positive; the others are summed negative.

Applying Newton's second law to each free body of the three masses, we obtain

$$-K_1 X_1 + K_2(X_2 - X_1) = M_1 \ddot{X}_1$$

$$K_2(X_2 - X_1) + K_3(X_3 - X_2) = M_2 \ddot{X}_2$$

$$-K_3(X_3 - X_2) = M_3 \ddot{X}_3$$

Collecting and rearranging terms in each of the above equations, we get

$$M_1 \ddot{X}_1 + (K_1 + K_2)X_1 - K_2 X_2 = 0$$

$$M_2 \ddot{X}_2 - K_2 X_1 + (K_2 + K_3) X_2 - K_3 X_3 = 0$$

$$M_3 \ddot{X}_3 - K_3 X_2 + K_3 X_3 = 0$$

It should be apparent that the differential equations of motion for the undamped free vibration of the chimney can be written in matrix form as

$$\begin{pmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{pmatrix} \begin{pmatrix} \ddot{X}_1 \\ \ddot{X}_2 \\ \ddot{X}_3 \end{pmatrix} + \begin{pmatrix} (K_1 + K_2) & -K_2 & 0 \\ -K_2 & -(K_2 + K_3) & -K_3 \\ 0 & -K_3 & K_3 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} = 0$$

As observed before it can be written in the more compact form as

$$M \ddot{X} + K X = 0$$

Where,

the mass matrix M and the stiffness matrix K are shown above.

It should be noted that the stiffness matrix K is symmetrical and that the nonzero elements appear on the diagonal and immediately above and below it.

That is,

$$k_{13} = k_{31} = 0.$$

## **2.8 SOLUTION OF MATHEMATICAL MODELS**

### **2.8.1 DISCRETE SYSTEM MATHEMATICAL MODELS**

The essence of a lumped parameter mathematical model state of the system can be described adequate precision by the magnitudes of a finite number of state variables. The solution requires the following steps:

#### **1.System idealization:**

The ideal system is idealized as an assemblage of elements.

#### **2.Element equilibrium:**

The equilibrium requirements of each element are established in terms of state variables.

#### **3.Element assemblage:**

The element interconnection requirements are invoked to establish a set of simultaneous equations for the unknown state variables.

#### **4.Calculation of response:**

The simultaneous equations are solved for the state variables, and using the element equilibrium requirements, the response of each element is calculated.

#### **5.Steady state problems**

The main characteristic of a steady state problem is that the response of the system does not change with time. Thus, the state variables describing the response of the system under consideration can be obtained from the solution of a set equations that do not involve time as a variable.

#### **Example**

1. Elastic spring system

2. Heat transfer system
3. Hydraulic network
4. Dc network
5. Non linear elastic spring system.

## **6.Propagation problems**

The main characteristic of a propagation or dynamic problem is that the response of the system under consideration changes with time. For the analysis of a system, in principle, the same procedures as in analysis of a steady state problem are employed, but now the state variables and element equilibrium relations depend on time. The objective of the analysis is to calculate the state variables for all time t.

## **7.Eigenvalue problems:**

A main characteristic of an eigenvalue problem is that there is no unique solution to the response of the system, and the objective of the analysis is to calculate the various possible solutions. Eigenvalue value problems arise in both steady state and dynamic analysis.

The generalized eigenvalue problem in the form

$$K\phi = \lambda M\phi$$

Where,

K - Stiffness matrix

M - Mass matrix

$\lambda$  - Eigen value

$\phi$  - Eigen vectors

## **2.9 MULTIDEGREE OF FREEDOM SYSTEMS :**

There are  $n$  natural frequencies, each associated with its own mode shape, for a system having  $n$  degrees of freedom. The method of determining the natural frequencies from the characteristic equation obtained by equating the determinant to zero also applies to these systems. However, as the number of degrees of freedom increases, the solution of the characteristic equation becomes more complex. The mode shapes exhibit a property known as orthonality, which often enables us to simplify the analysis of multidegree of freedom systems.

The minimum number of coordinates necessary to describe the motion of the lumped masses defines the number of degrees of freedom of the system. Naturally, the larger the number of lumped masses used in the model, the higher the accuracy of the resulting analysis

## **2.10 INFLUENCE COEFFICIENTS:**

The equation of motion of a multidegree of freedom system can also be written in terms of influence coefficients, which are extensively used in structural engineering. Basically, one set of influence coefficients can be associated with each of the matrices involved in the equations of motion. The influence coefficients associated with the stiffness and mass matrices are, respectively, known as the stiffness and inertia influence coefficients. In some cases, it is more convenient to rewrite the equations of motion using the inverse of the stiffness matrix (known as the flexibility matrix) or the inverse of the mass matrix. The influence coefficients corresponding to the inverse stiffness matrix are called the flexibility influence coefficients, and those corresponding to the inverse mass matrix are known as the inverse inertia coefficients.

The following aspects of stiffness influence coefficients are to be noted:  
 since the force required at point I to cause a unit deflection at point j and zero deflection at all other points is the same as the force required at point j to cause a unit deflection at point I and zero deflection at all other points (maxwell's reciprocity theorem, we have

$$k_{ij} = k_{ji}.$$

1. The stiffness influence coefficients can be calculated by applying the principles of statics and solid mechanics.

2. The stiffness influence coefficients for torsional system can be defined in terms of unit angular displacement and the torque that causes the angular displacement. For example, in a multirotor torsional system,  $k_{ij}$  can be defined as the torque at point I (rotor I) due to a unit angular displacement at point j and zero angular displacement at all other points.

The stiffness influence coefficients of a multidegree of freedom system can be determined as follows:

1. assume a value of one for the displacement  $x_j$  ( $j = 1$  to start with) and a value of zero for all other displacements  $x_1, x_2, \dots, x_{j-1}, x_{j+1}, \dots, x_n$ . By definition, the set of forces  $k_{ij}$  ( $i = 1, 2, \dots, n$ ) will maintain the system in the assumed configuration ( $x_j = 1, x_1 = x_2 = \dots = x_{j-1} = \dots = x_n = 0$ ). then the static equilibrium equations are written for each mass and resulting set of n equations solved to find the n influence coefficients  $k_{ij}$  ( $i = 1, 2, \dots, n$ ). After completing step 1 for  $j = 1$ , the procedure is repeated for  $j = 2, 3,$

### **3. ABOUT FEM:**

The finite element method is a numerical method that can be used for accurate solution of the complex mechanical and structural vibration problems. In this project, the actual structure is replaced by the several pieces or elements, each of which is assumed to behave as a continuous structural member called a finite element. The elements are assumed to be interconnected at certain points known as joints or nodes. Since it is very difficult to find the exact solution (such as displacements) of the original structure under the specified loads, a convenient approximate solution is assumed in each finite element. The idea is that if the solutions of the various elements are selected properly, they can be made to converge to the exact solution of the total structure as the element size is reduced. During the solution process, the equilibrium of forces at the joints and the compatibility of displacements between the elements are satisfied so that the entire structure (assemblage of the elements) is made to behave as a single entity.

#### **3.1 WHY FEA:**

The question is often asked, “why should FEA be used to solve my problem(s). This is a valid question and one that deserves a rational answer. A preconception of those who do little analysis during a product design is that their existing design techniques have always worked in the past and the existing design techniques should work in the future. While this might have been and still might be the case, little thought is given to the real economics and subsequent optimization of the design. Every bit of material not necessary for the functioning of a particular product adds to the overall product cost in raw material cost, production cost (complexity of machinery required to build the product), shipping cost (on high volume items), and general over-head cost associated with the product. The bottom line for any product is cost and profitability. Today’s market, which includes not only local market areas, but also international market areas, is highly competitive. To be competitive, the manufacturer must produce the best

product at the lowest cost. FEA in the manufacturing environment is in order to produce the optimum product: a product that performs as intended, meets all of the specified environmental requirements, and is the least costly to produce.

### **3.2 APPLICATION OF FEA**

Every area of engineering can use the power of the FEA analysis.

#### **1. STRUCTURAL ANALYSIS**

From the history of the analytical technique, the most obvious application is structural analysis. The fields of civil aerospace engineering rely heavily on FEA methods to analyze various structures ranging from buildings to spacecraft design.

#### **2. THERMAL (steady and transient state)**

Many models contain thermal and fluid elements, which will allow the determination of temperature distributions in structures and such items as velocity, pressure, and concentration distributions in fluid flows. This type of analysis is very important in external atmospheric structure loading, piping analysis, etc.

3. Stress analysis

4. Vibration analysis (both linear and non linear)

5. Fluid Flow Problems (Laminar and Turbulent)

### **3.3 PARAMETERS REQUIRED FOR FEA:**

The Finite Element Method is an analytical tool, which requires the following information:

1. Nodal point spatial locations (geometry)
2. Elements connecting the nodal points
3. Mass properties
4. Boundary conditions or restraints



5. Loading or forcing function details
6. Analysis options

### **3.4 STEPS – in FINITE ELEMENT METHOD:**

#### **3.4.1 PREPROCESSING :**

Preprocessing is the initial part of any modeling process. It includes designing the model, assembling, the model selection of nodes, selection of elements, defining constraints (restraints), loads and defining other parameters.

##### **i) Designing the model:**

The required system, which is going to be analyzed, is first designed with the correct dimensions in the sketch mode of any FEA package.

##### **ii) Assembling the model:**

If the assumed model is very complex, then it is modeled separately and then assembled together to get the exact model.

##### **iii) Selection of nodes:**

Nodes are selected to represent the points in the boundaries, the surface(s) and any position interior to the physical problem. The number of nodes and element determines the size of the matrix while solving the problem. Increasing the number of nodes used for model increases the accuracy of the model.

##### **iv) Selection of element:**

There are several types of elements namely

1. Line element
2. 2D – element
3. 3D – element
  - a) Rectangular
  - b) Triangular

- c) Quadrilateral
  - d) Pipe element
  - e) Shell element
  - f) Square element etc.
4. Axi – symmetric rectangular element

Among these, Suitable types of element is selected for the given model

**v) Defining constraints**

The definition of the proper restraints for a problem is very important if the user assumptions are not correct, then the model may act as a rigid body. Normally, the FEA programs do not model the behavior of rigid bodies unless directed to consider this type of motion.

A restraint is applied to a node. This restraint restricts movement in a given direction. This will in turn require an output from the program of a reaction force at that node. Another term to become familiar with is a member end release. A member end release is the removal of a force-carrying ability on the member, thus allowing movement.

**vi) Defining loads:**

Each FEA program has the ability to apply forces at nodal locations in a model in order to stimulate the actual structure loading. These loads ( $F_x$ ,  $F_y$ ,  $F_z$ ,  $M_x$ ,  $M_y$ ,  $M_z$ , Temperature, Connective boundaries, Heat flows, and / or Heat fluxes) are input at a given node in and about the respective global coordinate directions. The loads at these respective global coordinate directions. The loads at these nodes can also be specified as applied nodal displacements.

#### **vii) Defining other parameters:**

Other constants must be defined in order for the model to run. These include, but are not limited to, the material properties (modulus of elasticity, material coefficient of thermal expansion, etc.), the cross-sectional areas, the moment of inertia(s) ( $I_{xx}$ ,  $I_{yy}$ ,  $I_{zz}$ ).

#### **3.4.2 EXECUTING THE MODEL or SOLUTION PHASE:**

With all parts of the model defined (nodes, elements, restraints, and loading), the analysis phase of the model is ready to begin. Stresses, deflections, temperatures, pressures, velocities, and vibration modes can be determined.

After executing the model, the response of the system is analyzed as per our requirement.

#### **3.4.3 POST PROCESSING**

The post processing of the data generated from an analysis is an important phase of any problem. The post processors associated with the FEA programs reviewed are quite good and offer much insight into the program-generated results.

In addition to organization of the results in an orderly fashion, the FEA post processors offer graphical output to the monitor, printer, and / or plotter.

Output from a vibration analysis is usually given as the deflected shape with the accelerations and stresses associated with frequency analysis. A particularly useful form of the output is the animation of the calculated mode shapes. Animation helps the user visualize exactly what is happening to the item modeled, and allows the user to assess the reality of the modeling process.

### **3.4.4 DESIGN OPTIMIZATION**

After the completion of a structural analysis, a question often asked is whether or not a part is over designed from either a weight or cost standpoint. Optimized designs are those designs, which meet all of the strength requirements while minimizing such factors as cost and weight.

#### **3.4.4.1 Criteria for optimum design:**

In order to achieve the optimum structural design, the following four criteria must be satisfied:

1. The design satisfies all necessary standard engineering practices.
2. All stress is below the allowable stress.
3. All vibration, natural frequency, and deflections (both static and dynamic) are within specifications.
4. The structural weight and / or cost are minimized.

Structural optimization is an iterative process. This process involves the repeated analysis of a structural system while changing the design variable values based on the previous analysis result.

#### **4. METHODOLOGY:**

We already know that every vibrating system has its own frequency known as natural frequency, whenever the natural frequency of the vibrating system coincides with the frequency of external excitation, there occurs a phenomenon known as resonance which leads to excessive deflection and failure.

When the fluid flows around the system, pressure wakes are formed with different intensities in the down stream side. The high pressure in the up-stream side will try to move the structure towards the down stream direction. So, the structure will exhibit some deflection towards the down stream direction. If this continues with different pressure intensities, the system will be set into vibration.

When the system vibrates under external loading condition it will have no motion at certain points and violent motion at intermediate points. The former ones are called nodes and later is called anti-nodes.

The loop between first two nodes is called as fundamental mode of vibration (or) first mode of vibration. When the frequency in the first mode of vibration exceeds 1 Hz, we are going for the dynamic analysis. If the period of natural oscillation for the chimney ( $T = 1 / f_n$ ) exceeds 0.25 seconds the design wind loads shall take into consideration the dynamic effect due to pulsation of thrust caused by wind velocity in addition to the static wind load.

When the fluid flows past a system with sufficient velocity vortices are formed in the wake of the fluid. Such vortices are frequently referred to as Von Karman vortices and shed in a regular pattern over a wide range of Reynolds numbers. ( $Re = v \rho D / \mu$ ). The vortices shed alternately from opposite sides of the system with the frequency 'F'. This causes an alternative pressure on each side of the system, which acts as a sinusoidally varying force 'F' perpendicular to the velocity of the fluid before its flow is disturbed.

Severe Von Karman type oscillation is not likely if the calculated velocity known as ‘Critical strouhal velocity’ is greater than the maximum design velocity. The strouhal critical velocity “V<sub>cr</sub>” for the chimney system is given by

$$V_{cr} = 5 D_t \times f_n$$

D<sub>t</sub> - Diameter at top of chimney.

F<sub>n</sub> - Natural frequency of system.

When V<sub>cr</sub> > D<sub>esign</sub>, we have to consider the vortex shedding effect and also the dynamic analysis in the design.

#### 4.1.To find natural frequency:

Consider the chimney as the cantilever structure so that it can be modeled as lumped mass system for the purposes of analyzing their vibration characteristics, lumped mass models can also be employed in the modal analysis method used to obtain the vibration response of systems subjected to various types of excitation.

The number of lumped masses selected to represent a distributed mass having a theoretically infinite number of degrees of freedom depends upon the no. of natural frequencies and mode shapes necessary to obtain results to some desired degree of accuracy. In general the lowest natural frequency and associated mode shape of an ‘n’ lumped mass model will be the most accurate obtained, with the second natural frequency and mode shape, the next most accurate and so on.

The total mass of the system can be distributed equally among ‘n’ lumped masses so that,

$$M_{ij} = A\gamma l / n$$

Where,

$$I = 1, 2, 3, \dots n$$

$$J = 1, 2, 3, \dots m$$

A - cross sectional area

γ - mass density material

l - length of the system.

#### 4.2 .To find the stiffness matrix (K):

Each mass in the lumped mass system has its own stiffness “K” which can be found out by,

$$K_{ij} = \frac{AE}{L/n}$$

Where,

$$I = 1, 2, 3, \dots n$$

$$J = 1, 2, 3, \dots m$$

E - Young's modulus of the material

A - cross sectional area

L - length of the system

#### 4.3. To find the influence coefficient (a):

An influence coefficient is denoted by “ $a_{12}$ ” and is defined as the static deflection of “1” due to unit force applied at position “2” when the unit force is the only force acting.

$$\text{i.e. } a_{ij} = a_{ji}$$

Where,

$a_{ij}$  - deflection at position “I” due to the unit force applied at “j”

$a_{ji}$  - deflection at position “j” due to the unit force applied at “I”

using the mass matrix(M), stiffness matrix(K), and influence coefficient( $a_{ij}$ ) the natural frequency of the system “ $f_n$ ” is found out by using the eigen value method.

#### 4.4.To find the critical condition or resonance condition:

From the basic wind speed available to the particular site, the design wind speed is calculated. From this design wind speed, the design wind load is found out. The self-weight of the chimney is also included along with the wind load.

Using this load, the bending moment diagram for the cantilever structure is drawn, by splitting the structure into several convenient zones such that no. of zones shall not be less than three and the zone height shall not exceed 10 cm the dynamic effect of wind is influenced by a number of factors such as mass and its disposition along chimney height, period and mode of natural oscillation, logarithmic decrement of damping, pulsation of velocity thrust etc. values of dynamic components of wind

load should be determined for each mode of oscillation of chimney as to system of inertia forces acting at the center of the zone being considered Inertia force "P<sub>dyn</sub>" acting at the centre of the "j" zone of the chimney in the '1' mode of natural oscillation is determined.

From the bending moment diagram, load(p) bending moment (M) and deflection ( $\gamma$ ) due to wind load are found out for both static & dynamic conditions at each zones.

Using all above values the total design lateral force, bending moment, and deflection due to wind load are computed for each zone. From the total design parameters and the static & dynamic parameters, the load, Bending moment, deflection are found out for each zone under resonance condition.

Since we have computed the natural frequency and the design parameters under resonance condition, we can modify the design in safe side for our requirements in the economical way.



## **5.DESIGN:**

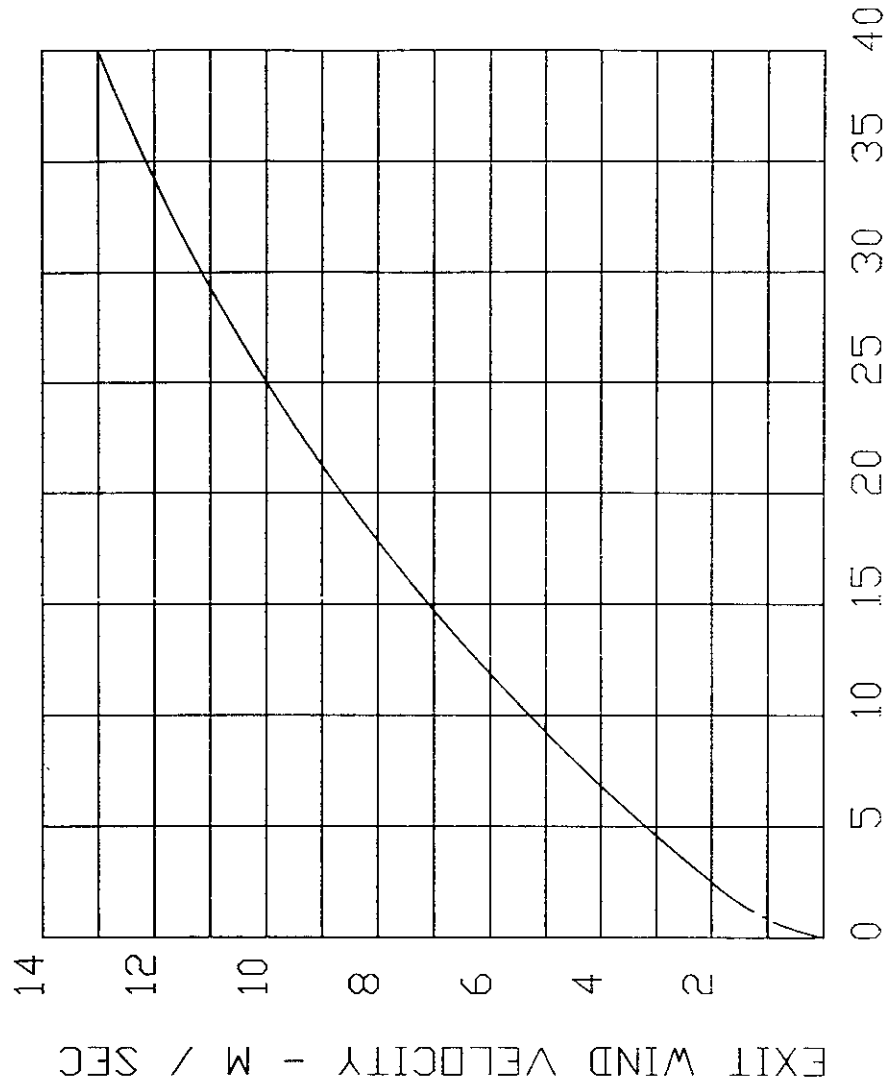
A chimney is a wind structure, i.e. a structure in whose design wind loads play dominant role. The vibration of chimneys due to vortex shedding was observed in the field, but an association between the periodicity of the wake of a cylinder and Benard stated vortex formation only in 1908. Thereafter, Von Karman stated the relationship of this periodicity with the formation of a street of stable vortices in 1912. The collapse of Tacoma's narrow bridge (1940) due to wind-induced oscillations followed by Ferrybridge cooling towers (1965) resulted in much research directed towards gaining more insight into the cause and magnitude of aerodynamic forces. For the satisfactory and economical design of such chimneys, it became necessary to critically reassess wind loads and improve the mathematical models for predicting structural responses. It was realized that such reassessment can best be done by studying the effect on and behaviour of scale models in wind tunnels coupled with field measurements of response on prototype structures.

### **5.1 EXIT VELOCITY**

If flue gases emanating from a chimney experience a field of increasing wind speed, soon a speed will be reached (termed critical wind speed) when the wind will shear off the emerging gas plume and this can lead to excessive pollutant deposition. Also, if the exit velocity is low, it can permit cold air to flow down a part of the chimney causing acids contained in flue gases to condense on the walls and cause damage.

Hence, the least flue gas velocity (i.e. when chimney is operating on part load) should be such that the corresponding critical wind speed is greater than the estimated

EXIT WIND VELOCITY TO AVOID DOWN WORK



WIND SPEED - KM / H

design wind speed at site .the chart are given the minimum exit velocities which will ensure, for different design wind speeds, that gases leave a chimney clearly. Apart from this requirement, exit velocity should be high enough so that a gas plume will escape the wake of neighbouring structures. For the above reason, a high exit velocity is preferred.

Wind is essentially the large-scale movement of free air due to thermal currents. It plays an important role in chimney design because of its capacity to transport and disperse pollutants and also because it exerts static and dynamic loads whose effects on a slender structure, such as a chimney, are significant.

The wind load exerted at any point on a chimney can be considered as the sum of quasistatic and a dynamic-load component. The static-load component is that force which wind will exert if it blows at a mean (time-average) steady speed and which will tend to produce a steady displacement in a structure.

The dynamic component, which can cause oscillations of a structure, is generated essentially due to the following three reasons:

**1.Gusts:**

They cause dynamic pressure changes initiating inline oscillations.

**2.Vortex shedding:**

This phenomenon creates crosswind dynamic forces, which can lead to transverse vibrations.

**3.Bufetting:**

A downwind chimney could oscillate due to the bufetting effect on up stream structure.

## 5.2 WAKE BUFETTING

This is the aerodynamic effect on a downstream chimney due to vortices shed from an upstream structure. Information on wind pressures exerted due to bufetting is limited in spite of the fact that many chimneys are built in groups and in the proximity of other tall structures.

In a chimney, oscillations of large amplitude can be caused if the predominant frequency of vortex shedding from an upstream obstruction coincides with its natural frequency. Thus, frequency mistuning is an important tool to reduce bufetting effects.

### Strouhal number

Critical wind velocity which can generate vortex shedding. Its magnitude depends on the natural frequency and diameter of the chimney.

It depends on Reynolds number,

$$Re = \rho v D / \mu$$

Where,

$\rho$  - Density of the fluid

$v$  - Velocity of the flowing fluid

$D$  - Outer diameter of the chimney

$\mu$  - Dynamic viscosity of the fluid

## 5.3 VORTEX FORMATION

When air flows past a chimney of circular cross-section, vortices are shed alternately from the sides causing a pressure drop at regular intervals across the chimney section.

Such pressure of vortices in the sub-critical Re range of (i.e.  $Re < 3 \cdot 10^5$ ) as well as in the ultra-critical range ( $Re > 3.5 \cdot 10^6$ ). in the super-critical range ( $3 \cdot 10^5 < Re < 3.5 \cdot 10^6$ ) the point of vortex separation oscillates causing an instability and as a result, vortex shedding become irregular.

With increasing wind speed, as the frequency of vortex shedding approaches the chimney's natural frequency, the former jumps to lock-in with the latter. Thereafter, the chimney structure controls the frequency of vortex shedding, for a range of wind velocities close to the velocity at which vortex-shedding frequency coincides with the chimney's natural frequency. While on this subject, it may be mentioned that in order to maintain vortex oscillations steady unidirectional wind is required instead of a gusty wind.

## **6.DESIGN PROCEDURE:**

### **6.1 DESIGN OF CHIMNEY DIMENSIONS:**

#### **I .To find top diameter of cylindrical portion of chimney (D1)**

$$\text{Inside diameter } \sqrt{D1 = 4Q / \pi V_{\text{exit}}}$$

Q-Quantity of gas discharged (m<sup>3</sup>/sec)

V<sub>exit</sub>-velocity of flue gas at the exit point of the chimney (m/sec)

As per code of practice for design loads for buildings structures

IS: 875 (part 3)-1987

Basic wind speed for Coimbatore = 39 m/sec

From the graph the exit velocity of flue gas for the wind speed of 40 km/hr,

$$V_{\text{exit}} = 13.2 \text{ m/sec}$$

i.e. for 1 m/sec,

$$V_{\text{exit}} = 1.188 \text{ m/sec}$$

Now for Wind velocity of 39 m/sec,

$$V_{\text{exit}} = 46.33 \text{ m/sec}$$

Assume the required discharge of flue gas from the chimney as

$$Q = 36 \text{ m}^3/\text{sec}$$

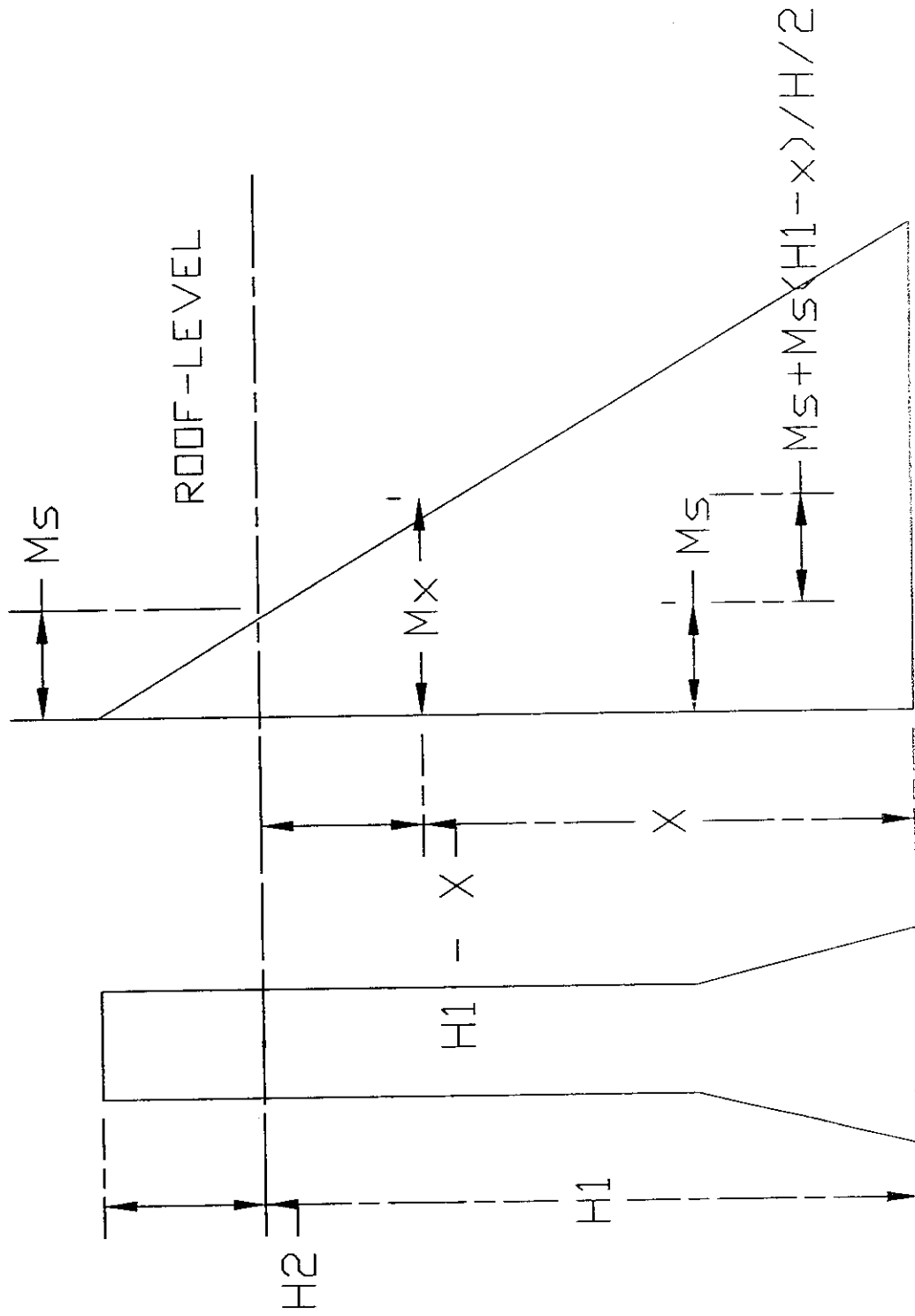
The required inside diameter of the chimney is,

$$D2 = \sqrt{4*Q / \pi V_{\text{exit}}} = 1.501\text{m} = 1.5\text{m}$$

#### **2.To fix the dimensions of flared portion:**

A self – supporting chimney of height 40m and above shall be provided with the flare at the base to achieve better stability i.e.bottom portion of chimney in the form of truncated cone .

BENDING MOMENT DIAGRAM



As per IS 6533,

min. Outside diameter of the conical flared portion of its bottom,

$$D_3] \min = 1.6D_1 \\ = 1.6 * 1500 = 2400\text{mm}$$

The thickness of the steel plate of the flared portion should not be less than the thickness of the steel plate in the upper cylindrical portion.

$$T_2 = 56\text{mm}$$

Height of the flared portion:

Assume  $H=80\text{m}$  (as per pollution control)

For self-supporting chimney, min-height of flared portion ( $H_2$ ) should not be less than one third of the total height of chimney ( $H$ )

$$\text{i.e. } H_2 = H/3 = 80/3 = 26\text{m.}$$

Take it as 30m

$$H = H_1 + H_2$$

$$H_1 = H - H_2$$

$$= 80 - 30$$

$$H_1 = 50\text{m.}$$

**3.To find the bending moment( $M_w$ ) due to wind load:**

$$M_w = 0.7 * (P_1 * D_1 * H * H/2)$$

Where,

$P_1$  - Wind pressure value

$D_1$  - Inner diameter of the chimney

$H$  - Height over which wind load is acting



#### 4. To find the thickness of the steel plate of the chimney:

Let  $t$  be the initially assumed thickness value

Corrosion allowance = 4mm

Find  $D_1/t$  ratio value

Find  $K = 0.7 \cdot D_1/2$

Find  $h_1 = 2 \cdot L$

Find  $h_1/K$  ratio value

Where,

$D_1$  - Inner diameter of the chimney

$K$  - Radius of gyration

$H_1$  - Effective height

$L$  - Length over which wind load acting

From this ratio find the values of tensile stress and compressive stress

from the tables 1&2

##### i). Under tension :

$$t = (M_w / (250 \cdot \pi \cdot D_1 \cdot D_1)) \cdot (0.875 \cdot F_t + 0.0079 \cdot H)$$

##### ii). Under Compression:

$$t = (1 / (1.25 F_c - 0.079 H)) \cdot (M_w / ((785 \cdot D_1 \cdot D_1) + (0.002 \cdot H)))$$

Where,

$M_w$  - Bending moment due to wind load

$F_t$  - Tensile stress value

$F_c$  - Compressive stress value

$H$  - Height over which wind load acting

$D_1$  - Inner diameter of the chimney

From this take the greater value of ' $t$ '.

Now,

$$T_{\text{actual}} = t + 4$$

As per standard data, the thickness of the steel plates should not be less

than 8mm. to resist more corrosion likely at the top of the chimney.

The steel plates are available in the standard thickness of

5,6,8,10,12,14,16,18,20,22,25,28,32,36,40,45,50,56,63mm.

**5.To find the self weight of the chimney:**

$$m = \rho * \Pi * D_1 * t * H$$

Where,

- $\rho$  - Density of the steel
- $D_1$  - Inner diameter of the chimney
- $T$  - Thickness of the steel plate
- $H$  - Height considered

**TABLE : 1**

H <sub>1</sub> /K RATIO	ALLOWABLE STRESS IN AXIAL COMPRESSION (N/mm <sup>2</sup> )													
	D/t RATIO													
	100 AND LESS	125	150	175	200	225	250	300	350	400	450	500	550	600
0	125	125	125	125	125	125	125	125	124	107	92.8	85.8	78.3	71.8
10	125													
20	125													
30	125	125							*					
40	125	125	125	125										
50	125		125	125	125	125	125	125						
60	125				125	125	125	125	124					
70	116								116	107				
80	103.5									103.5	92.8			
90	90.6										90.6	85.8		
100	78.6											78.6	78.3	71.8
110	68.0													68
120	58.9													58.9
130	51.3													51.3
140														45.0
150														39.7
160														35.3
170														31.4
180									***					28.4
190														25.5
200														23.2
210														21.1
220														19.2
230														17.6

**TABLE : 1 CONTINUED**

H <sub>1</sub> /K RATIO	ALLOWABLE STRESS IN COMPRESSION (N/mm <sup>2</sup> )													
	D/t RATIO													
	100 AND LESS	125	150	175	200	225	250	300	350	400	450	500	550	600
240	16.2													16.2
250	15.0													15.0
300	10.6													10.6
350	7.72													7.7

**NOTE :**

- \* For ratio of d/t and h<sub>1</sub>/k in the zone above zig – zag line, the stresses from the top may be read.
- \*\* For ratio of d/t and h<sub>1</sub>/k in the zone below zig – zag line, the stresses from the column headed 100 and less may be read.
- \*\*\* Above values of stresses have been converted in S.I unit from those given in M.K.S units.

**TABLE : 2**

H <sub>r</sub> /K RATIO	ALLOWABLE STRESS IN TENSION (N/mm <sup>2</sup> )													
	D/t RATIO													
	100 AND LESS	125	150	175	200	225	250	300	350	400	450	500	550	600
UPTO 130	165.0	157.5	153.0	149.5	146.5	145.0	143.5	140.0	137	118.0	102.5	94.5	86.6	78.7
140	159.0								*					
150	151.0	157.5	153.0	149.5	146.5									
160	145.0	151.0	151.0	145.0	145.0	145.0	143.5	140.0	137					
170	138.5					138.0	138.5	138.5	132.0					
180	132.0								132					
190	126.0				**				118					
200	118.0													
210	116.5									116.5				
220	113.5									113.5				
230	112.0			***						112.0				
240	108.5									108.5				
250	107.0									107.0				
260	105.5									105.0				
270	104.0									104.0				
280	102.5									102.5				
290	101.0										101.0			
300	99.2										99.2	94.5	86.6	78.7

## 6.2 DESIGN CALCULATIONS

The corrosion allowance 4mm for slight internal corrosion.

Height of the chimney is 80 metre

### To find the top diameter of the chimney

$$q = v \times \frac{\pi}{4} d_1^2$$

where  $q$  - discharge of the fuel gas.  
 $V$  - exit velocity of the fuel gas.

$$d_1 = \sqrt{\frac{q \times 4}{v \times \pi}}$$

$$d_1 = \sqrt{\frac{81.4 \times 4}{46 \times 3.14}}$$

$$d_1 = 1.5m$$

### To find the height of the flared portion

A conical flared portion is provided in the lower portion of the steel chimney. The recommended height of flared portion.

$$= \frac{4}{3} = \frac{80}{3} = 26.67m$$

$$= 30m$$

### To find the diameter of conical flared portion at its bottom

$$d_2 = 1.6 \times d_1$$

$$= 1.6 \times 1.5$$

$$= 2.4m$$

### Intensity of wind pressure

The intensity of wind pressure at higher heights as per is .875 are as as follows.

$$0 \text{ to } 30m = 1.50 \text{ km/m}^2$$

$$\text{at } 35m = 1.50 \text{ km/m}^2$$

$$\text{at } 40m = 1.58 \text{ km/m}^2$$

$$\text{at } 45m = 1.63 \text{ km/m}^2$$

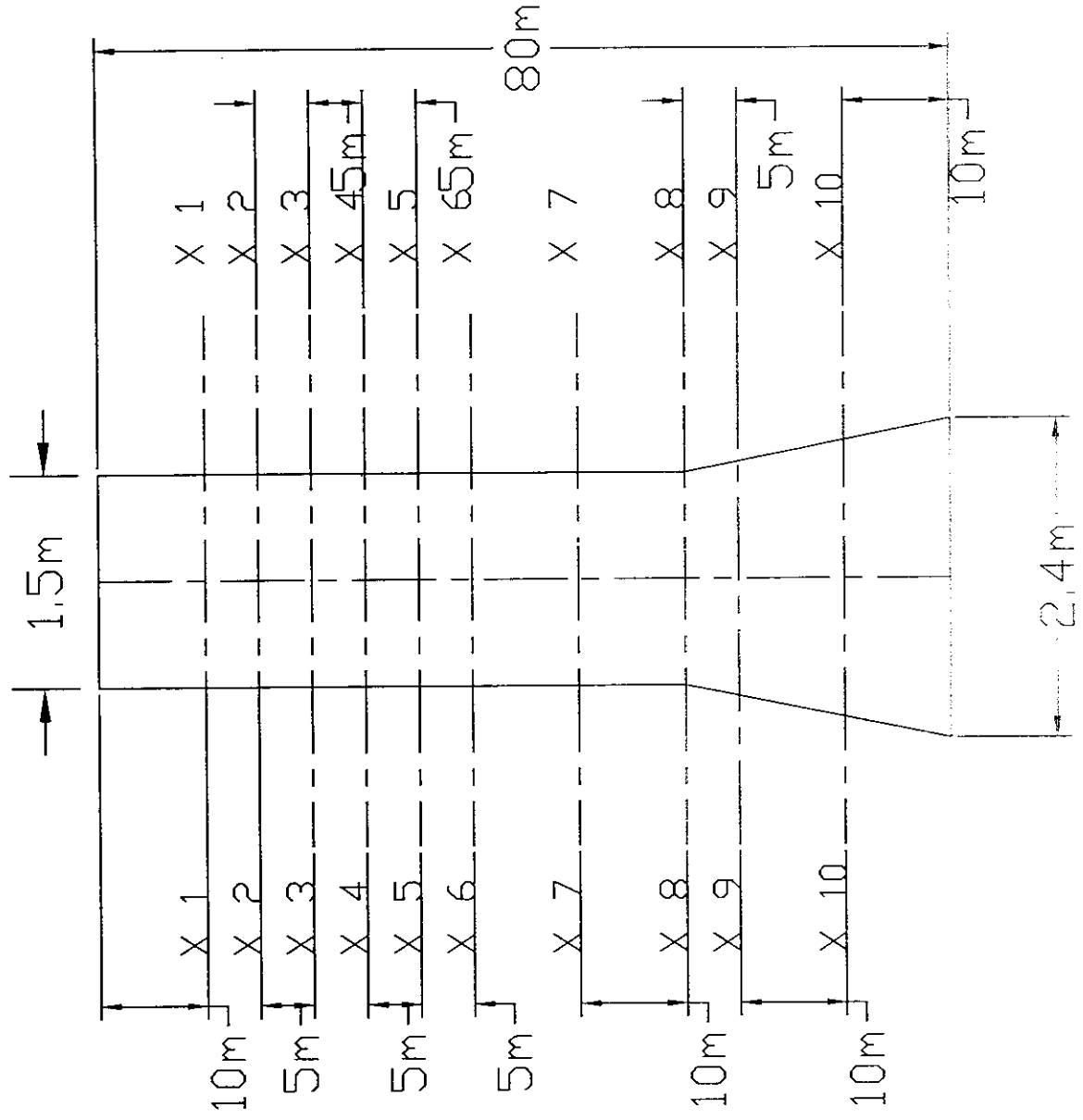
$$\text{at } 50m = 1.67 \text{ km/m}^2$$

$$\text{at } 60m = 1.72 \text{ km/m}^2$$

$$\text{at } 70m = 1.77 \text{ km/m}^2$$

$$\text{at } 80m = 1.83 \text{ km/m}^2$$

# ZONES IN CHIMNEY



### Over - turning moment due to wind (Mw)

The intensity of wind pressure is adopted as the average value of these at upper and lower section. For simplicity the maximum intensity of wind pressure (ie at the top) has been used in calculation.

The over - turning movement about the respective sections are as under.  $h_1, h_2, h_3, \dots$  etc. They represent the height of respective forces  $p_1, p_2, p_3, \dots$  etc.

At Zone 1

$$\begin{aligned} M_w &= 0.7 \times p_1 \times d_1 \times h_1 \times h_1 / 2 \\ &= 0.7 \times 1.83 \times 1.5 \times 10 \times \frac{10}{2} \\ &= 96.08 \text{ kn - m} \end{aligned}$$

At Zone 2

$$\begin{aligned} M_w &= 0.7 \times d_1 \left[ p_1 \times \left( h_2 + h \frac{1}{2} \right) + p_2 \times h_2 \times h \frac{2}{2} \right] \\ &= 0.7 \times 1.5 \left[ 1.83 \times 10 \times \left[ 10 + \frac{10}{2} \right] + 1.77 \times 10 \times \frac{10}{2} \right] \\ &= 381.15 \text{ kn - m} \end{aligned}$$

### To find thickness of chimney plate

At Zone 1

Diameter of the chimney of steel plate as 3mm without corrosion.

$$\frac{d}{t} \text{ ratio} = \frac{1.5 \times 1000}{3} = 500$$

$$\text{Ratios of gyration, } k = 0.7 \times \frac{1500}{2} = 525 \text{ mm}$$

Effective height for axial stress

$$h_1 = 2 \times 5 \times 1000 = 10,000 \text{ mm}$$

$$\text{Ratio } \frac{h_1}{k} = \frac{10,000}{525} = 19.04$$

For bending stress.

$$\frac{\frac{1}{2} h_1}{k} = \frac{1}{2} \times 19.04 = 9.52$$



Allowable stress in axial compression

$$= 71.8 \frac{\text{N}}{\text{mm}^2} \quad \text{from table 1}$$

$$= 78.7 \frac{\text{N}}{\text{mm}^2} \quad \text{from table 2}$$

These stresses are increase by 25 percent for wind. The maximum tensile stress on windword.

$$\frac{Mw}{250 \pi d^2 t} - 0.079h_1 \leq \eta_1 f_t$$

where  $\eta_1$  - efficiency of teh joint on the tension side = 70%

$$\left[ \frac{96.08}{250 \times \pi \times (1.5^2) \times t} - 0.079 \times 10 \right] = 0.7 \times 1.25 \times 71.8$$

$$t = 0.00085\text{m} = 0.85\text{m}$$

The maximum compressive stress on the leew are side of the steel chimney.

$$\left( \frac{Mw}{250 \pi d^2 t} + 0.079h + 0.002 \frac{h}{t} \right) \leq \eta_2 x f_1$$

$$\frac{96.08}{250 \times \pi \times (1.5^2 t)} + 0.079 \times 0.002 \times \frac{10}{t} = 1.00 \times 1.25 \times 71.8$$

There fore, total thickness with corrosion allowance is

$$(0.85 + 4)\text{mm} = 4.85\text{mm}$$

Thickness of steel plate  $t_1 = 5\text{mm}$

similarly after during the above calculations for other zones, the thickness for the respective zones are.

t2	=	8mm
t3	=	12mm
t4	=	14mm
t5	=	18mm
t6	=	20mm
t7	=	25mm
t8	=	50mm

### 6.3 DESIGN OF CHIMNEY DIMENSIONS IN C++ SOFTWARE:

The following program has been developed in C++ software for the design calculations. The inputs to this software are wind pressure values at several zones, the required discharge from the chimney, exit velocity of flue gas from the chimney, allowable stresses in tension and compression. Then, the outputs from this software will be internal diameter of the chimney, Bending moments at each zone, corresponding thickness of steel plate for this zone.

```
/* ***** DESIGN OF CHIMNEY IN C++ SOFTWARE *****
```

```
*****
```

p1,p2,p3,p4,p5,p6,p7,p8 - ARE THE WIND PRESSURE  
INTENSITY AT VARIOUS HEIGHT

h1,h2,h3,h4,h5,h6,h7,h8 - ARE THE HEIGHT OF THE EACH ZONE

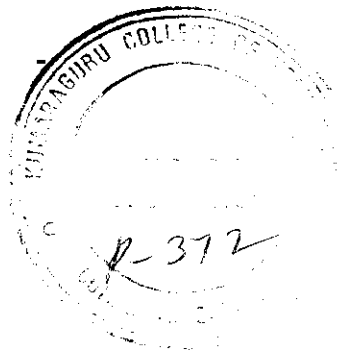
m1,m2,m3,m4,m5,m6,m7,m8- ARE THE TURNING MOMENT DUE TO  
WIND AT EACH ZONE

d1 - INTERNAL DIAMETER OF THE CHIMNEY

t3 - THICKNESS OF THE CHIMNEY  
PLATE AT EACH ZONE

Ft - ALLOWABLE BENDING STRESS

Fc - ALLOWABLE COMPRESSION STRESS



# WIND PRESSURE DISTRIBUTION IN EACH ZONE

\*\*\*\*\*

|  
p1	h1
p2|h2  
--|--  
p3|h3  
--|--  
p4|h4  
--|--  
p5|h5  
--|--  
p6|h6  
--|--  
p7|h7  
/ --|--\  
/ p8|h8 \  
/-----\  
\*/

```

#include<stdio.h>

#include<math.h>

#include<conio.h>

void main()

{

float
p1,p2,p3,p4,p5,p6,p7,p8,m1,m2,m3,m4,m5,m6,m7,m8,h1,h2,h3,h4
,h5,h6,h7,

h8,ft,fc, t1,t2,t3,Q,V,d1,p;

clrscr();

printf("\n ENTER THE VALUE OF WIND PRESSURE
1,p2,p3,p4,p5,p6,p7,p8 in KN/m2\n");

scanf("%f%f%f%f%f%f%f%f",&p1,&p2,&p3,&p4,&p5,&p6,&p7,&p
8);

printf("\n ENTRER THE VALUE OF HEIGHTS
h1,h2,h3,h4,h5,h6,h7,h8 in metre \n");

scanf("%f%f%f%f%f%f%f%f",&h1,&h2,&h3,&h4,&h5,&h6,&h7,&h
8);

printf("\n ENTER THE VALUE OF ft,fc in N/mm2 \n");

scanf("%f%f",&ft,&fc);

printf("\n ENTER THE VALUE OF DISCHARGE IN M3/sec \n");

scanf("%f",&Q);

printf("\n ENTER THE VALUE OF EXIT VELOCITY \n");

```

```

scanf("%f",&V);

d1=sqrt(((Q*4)/(V*3.142)));

m1=0.7*d1*p1*h1*(h1/2);

t1=(m1/((785*d1*d1)*((0.875*ft)+(0.079*h1))));

t2=(1/((1.25*fc)-(0.079*h1))*((m1/(785*d1*d1))+0.002*h1);

if(t2>t1)

t3=t2;

else t3=t1;

t3=t3+0.004;

printf("\n BENDING MOMENT DUE WIND IN KN-M is: \n");

printf("%5.2f",m1);

printf("\n THICNESS OF THE CHIMNEY PLATE IN metre : \n");

printf("%1.5f",t3);

m2=0.7*d1*((p1*h1*(h2+h1/2))+p2*h2*h2/2));

t1=(m2/((785*d1*d1)*((0.875*ft)+(0.079*h2))));

t2=(1/((1.25*fc)-(0.079*h2))*((m2/(785*d1*d1))+0.002*h2);

if(t2>t1)

t3=t2;

else t3=t1;

t3=t3+0.004;

printf("\n BENDING MOMENT DUE TO WIND IN KN-M is: \n");

```

```

printf("%5.2f",m2);

printf("\n THICNESS OF THE CHIMNEY PLATE IN metre: \n");

printf("%1.5f",t3);

m3=0.7*d1*((p1*h1*(h3+h2+h1/2))+(p2*h2*(h3+h2/2))+(p3*h3*h3/2)
);
t1=(m3/((785*d1*d1)*((0.875*ft)+(0.079*h3))));
t2=(1/((1.25*fc)-(0.079*h3))*((m3/(785*d1*d1))+0.002*h3);
if(t2>t1)
t3=t2;
else t3=t1;
t3=t3+0.004;

printf("\n BENDING MOMENT DUE TO WIND IN KN-M is: \n");

printf("%5.2f",m3);

printf("\n THICNESS OF THE CHIMNEY PLATE IN metre: \n");

printf("%1.5f",t3);

m4=0.7*d1*((p1*h1*(h4+h3+h2+h1/2))+(p2*h2*(h4+h3+h2/2))
+(p3*h3*(h4+h3/2))+(p4*h4*h4/2));
t1=(m4/((785*d1*d1)*((0.875*ft)+(0.079*h4))));
t2=(1/((1.25*fc)-(0.079*h4))*((m4/(785*d1*d1))+0.002*h4);
if(t2>t1)
t3=t2;

```

```
else t3=t1;
```

```
t3=t3+0.004;
```

```
printf("\n BENDING MOMENT DUE TO WIND IN KN-M is: \n");
```

```
printf("%5.2f",m4);
```

```
printf("\n THICNESS OF THE CHIMNEY PLATE IN metre: \n");
```

```
printf("%1.5f",t3);
```

```
m5=0.7*d1*((p1*h1*(h5+h4+h3+h2+h1/2))+(p2*h2*(h5+h4+h3+h2/2)))+
```

```
(p3*h3*(h5+h4+h3/2))+(p4*h4*(h5+h4/2))+(p5*h5*(h5/2));
```

```
t1=(m5/((785*d1*d1)*((0.875*ft)+(0.079*h5))));
```

```
t2=(1/((1.25*fc)-(0.079*h5))*((m5/(785*d1*d1))+0.002*h5);
```

```
if(t2>t1)
```

```
t3=t2;
```

```
else t3=t1;
```

```
t3=t3+0.004;
```

```
printf("\n BENDING MOMENT DUE TO WIND IN KN-M is: \n");
```

```
printf("%5.2f",m5);
```

```
printf("\n THICNESS OF THE CHIMNEY PLATE IN metre: \n" );
```

```
printf("%1.5f",t3);
```

```
m6=0.7*d1*((p1*h1*(h6+h5+h4+h3+h2+h1/2))+(p2*h2*(h6+h5+h4+h3+h2/2))
```

$$+(p3 \cdot h3 \cdot (h6+h5+h4+h3/2))+(p4 \cdot h4 \cdot (h6+h5+h4/2))+(p5 \cdot h5 \cdot (h6+h5/2))$$

$$+(p6 \cdot h6 \cdot h6/2));$$

$$t1=(m6/((785 \cdot d1 \cdot d1) \cdot ((0.875 \cdot ft)+(0.079 \cdot h6))));$$

$$t2=(1/((1.25 \cdot fc)-(0.079 \cdot h6))) \cdot ((m6/(785 \cdot d1 \cdot d1))+0.002 \cdot h6);$$

if( $t2 > t1$ )

$t3=t2;$

else  $t3=t1;$

$t3=t3+0.004;$

printf("\n BENDING MOMENT DUE TO WIND IN KN-M is: \n");

printf("%5.2f", $m6$ );

printf("\n THICNESS OF THE CHIMNEY PLATE IN metre: \n");

printf("%1.5f", $t3$ );

$$m7=0.7 \cdot d1 \cdot ((p1 \cdot h1 \cdot (h7+h6+h5+h4+h3+h2+h1/2))+(p2 \cdot h2 \cdot (h7+h6+h5+h4+h3+h2/2))$$

$$+(p3 \cdot h3 \cdot (h7+h6+h5+h4+h3/2))+(p4 \cdot h4 \cdot (h7+h6+h5+h4/2))+(p5 \cdot h5 \cdot (h7+h6+h5/2))$$

$$+(p6 \cdot h6 \cdot (h7+h6/2))+(p7 \cdot h7 \cdot h7/2));$$

$$t1=(m7/((785 \cdot d1 \cdot d1) \cdot ((0.875 \cdot ft)+(0.079 \cdot h7))));$$

$$t2=(1/((1.25 \cdot fc)-(0.079 \cdot h7))) \cdot ((m7/(785 \cdot d1 \cdot d1))+0.002 \cdot h7);$$

if( $t2 > t1$ )



```

t3=t2;

else t3=t1;

t3=t3+0.004;

printf("\n BENDING MOMENT DUE TO WIND IN KN-M is: \n");

printf("%5.2f",m7);

printf("\n THICNESS OF THE CHIMNEY PLATE IN metre: \n");

printf("%1.5f",t3);

m8=0.7*d1*((p1*h1*(h8+h7+h6+h5+h4+h3+h2+(h1/2)))+(p2*h2*(h8
+h7+h6+h5+h4+h3+(h2/2)))

+(p3*h3*(h8+h7+h6+h5+h4+(h3/2)))+(p4*h4*(h8+h7+h6+h5+(h4/2))
)

+(p5*h5*(h8+h7+h6+(h5/2)))+(p6*h6*(h8+h7+h6/2)))+(p7*h7*(h8+h7
/2)))+(p8*h8*(h8/2));

t1 =(m8/((785*d1*d1)*((0.875*ft)+(0.079*h8))));

t2=(1/((1.25*fc)-(0.079*h8))*((m8/(785*d1*d1))+0.002*h8);

if(t2>t1)

t3=t2;

else t3=t1;

t3=t3+0.004;

printf("\n BENDING MOMENT DUE TO WIND IN KN-M is: \n");

printf("%5.2f",m8);

```

```
printf("\n THICNESS OF THE CHIMNEY PLATE IN metre: \n");  
printf("%1.5f",t3);  
printf("\n INTERNAL DIAMETER OF THE CHIMNEY IN metre:\n");  
printf("%2.2f",d1);  
  
getch();  
getch();  
  
}
```

Enter the value of wind pressure  $p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8$  in  $\text{KN/m}^2$  :

1.83

1.77

1.72

1.67

1.63

1.58

1.56

1.50

Enter the value of Height  $h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8$  in metre :

10

10

10

5

5

5

5

30

Enter the value of  $f_t, f_c$  in  $\text{N/mm}^2$  :

78.7

71.8

Enter the value of discharge in  $\text{m}^3/\text{sec}$  :

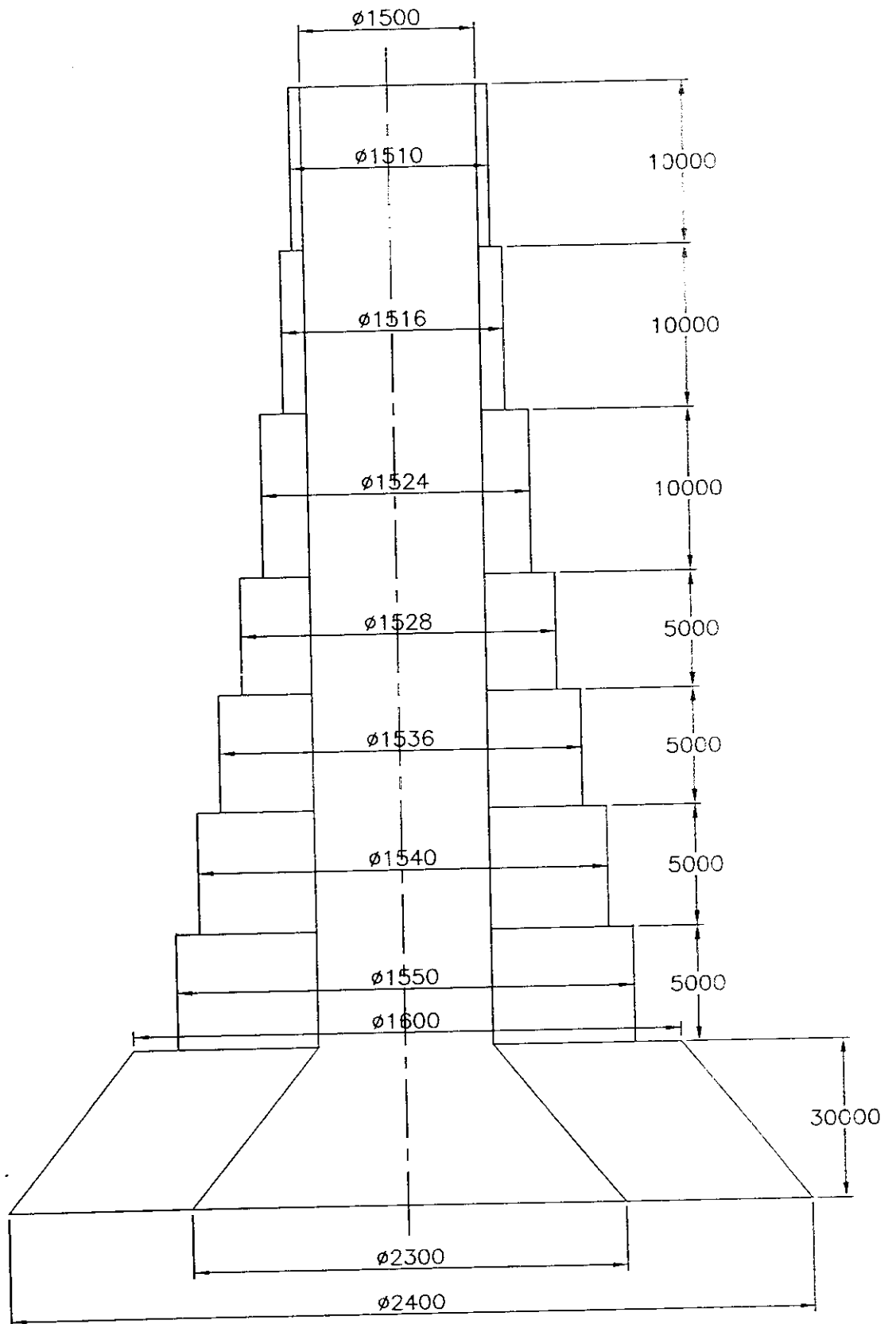
81.4

Enter the value of exit velocity :

46

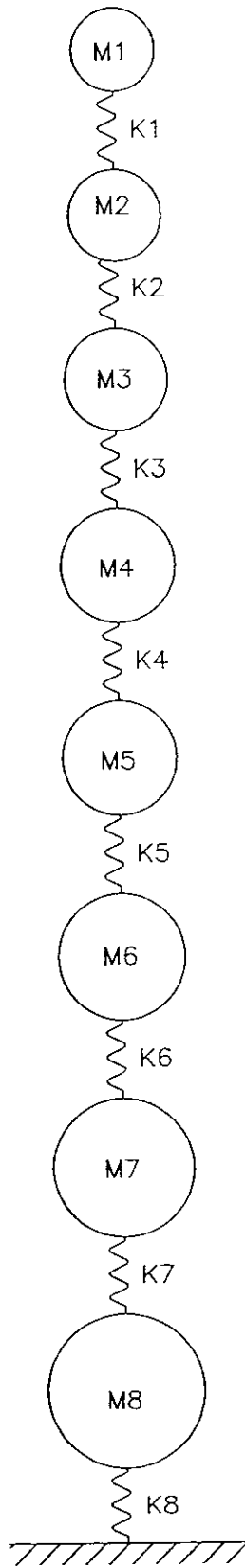
Bending moment due to wind in KN-M is:	-	-
96.07		
Thickness of the chimney plate in metre:	-	-
0.0048		
Bending moment due to wind in KN-M is:	-	-
381.25		
Thickness of the chimney plate in metre:	-	-
0.0734		
Bending moment due to wind in KN-M is:	-	-
849.98		
Thickness of the chimney plate in metre:	-	-
0.01090		
Bending moment due to wind in KN-M is:	-	-
1151.38		
Thickness of the chimney plate in metre:	-	-
0.01340		
Bending moment due to wind in KN-M is:	-	-
1496.13		
Thickness of the chimney plate in metre:	-	-
0.01622		
Bending moment due to wind in KN-M is:	-	-
1883.03		
Thickness of the chimney plate in metre:	-	-
0.01937		
Bending moment due to wind in KN-M is:	-	-
2311.17		
Thickness of the chimney plate in metre:	-	-
0.02287		
Bending moment due to wind in KN-M is:	-	-
5712.12		
Thickness of the chimney plate in metre:	-	-
0.0479		
The internal diameter of chimney in m is :		
1.500		

# CROSS SECTION OF CHIMNEY



NATURAL FREQUENCY  
CALCULATION

SYSTEM IDEALIZATION



## 7.1. FORMATION OF STIFFNESS MATRIX :(K)

We know that the deflection of the cantilever beam under uniformly distributed load is

$$\delta = \frac{WL^3}{8EI}$$

where

- W - Total acting to the structure
- l - Length of the beam
- E - Young's modulus of the beam
- I - Moment of inertia

$$\Rightarrow K = \frac{W}{\delta} = \frac{8EI}{L^3}$$

Now the stiffness for each zone of the chimney is as follows :

For Zone (1) :

$$E = 2.15 \times 10^6 \text{ N/mm}^2$$

$$I = 10,000 \text{ mm}^4$$

$$I = \frac{\pi}{64} [d_{out}^4 - d_{in}^4]$$

$$= \frac{\pi}{64} [1510^4 - 1500^4]$$

$$= 6.689 \times 10^9 \text{ mm}^4$$

$$\therefore K_1 = \frac{8 \times 2.15 \times 10^6 \times 6.689 \times 10^9}{(10,000)^3}$$

$$= 115.07 \times 10^3 \text{ N/mm}$$

similarly for other respective zones, the stiffness are,

$$K_2 = 185.21 \times 10^3 \text{ N/mm}$$

$$K_3 = 280.05 \times 10^3 \text{ N/mm}$$

$$K_4 = 2624.22 \times 10^3 \text{ N/mm}$$

$$K_5 = 3401 \times 10^3 \text{ N/mm}$$



$$K_6 = 3793.97 \times 10^3 \text{ N/mm}$$

$$K_7 = 4789.88 \times 10^3 \text{ N/mm}$$

$$K_8 = 92.76 \times 10^3 \text{ N/mm}$$

$$K_{eq} = \begin{bmatrix} 300.28 & -185.21 & 0 & 0 & 0 & 0 & 0 & 0 \\ -185.21 & 465.26 & -280.05 & 0 & 0 & 0 & 0 & 0 \\ 0 & -280.05 & 2905.27 & -2624.22 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2624.22 & 6025.22 & -3401 & 0 & 0 & 0 \\ 0 & 0 & 0 & -3401 & 7194.97 & -3793.97 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3793.97 & 8583.85 & -4789.88 & 0 \\ 0 & 0 & 0 & 0 & 0 & -4789.88 & 488.64 & -92.76 \end{bmatrix}$$

## 7.2 Formation of mass matrix : (m)

Mass of each zone is calculated as follows.

$$\begin{aligned} m_1 &= \rho \times \frac{\pi}{4} [d_{out}^2 - d_{in}^2] \times h_1 \\ &= 7850 \times \frac{\pi}{4} [(1.510)^2 - (1.500)^2] \times 10 \\ &= 1855 \text{ kg} \end{aligned}$$

Similarly the mass for other zones are

$$\begin{aligned} m_2 &= 2975 \text{ kg} \\ m_3 &= 4474.58 \text{ kg} \\ m_4 &= 2613.62 \text{ kg} \\ m_5 &= 3369.25 \text{ kg} \\ m_6 &= 3748.55 \text{ kg} \\ m_7 &= 4701.09 \text{ kg} \\ m_8 &= 72098.41 \text{ kg} \end{aligned}$$

$$m = \begin{bmatrix} 1855 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2975 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4474 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2613 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3369 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3748 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4701 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 72098 \end{bmatrix}$$

### 7.3. ABOUT MATLAB SOFTWARE:

#### INV

Matrix inverse.

INV(X) is the inverse of the square matrix X.

A warning message is printed if X is badly scaled or

Nearly singular.

#### EIG

##### **Eigenvalues and eigenvectors.**

EIG(X) is a vector containing the eigenvalues of a square matrix X.

[V,D] = EIG(X) produces a diagonal matrix D of eigenvalues and a full matrix V whose columns are the corresponding eigenvectors so that  $X*V = V*D$ .

[V,D] = EIG(X,'nobalance') performs the computation with balancing disabled, which sometimes gives more accurate results for certain problems with unusual scaling.

##### Generalized eigenvalues and eigenvectors.

EIG(A,B) is a vector containing the generalized eigenvalues of square matrices A and B.

[V,D] = EIG(A,B) produces a diagonal matrix D of generalized eigenvalues and a full matrix V whose columns are the corresponding eigenvectors so that  $A*V = B*V*D$ .

## 7.4. SOLUTION PHASE IN MATLAB:

To find the eigen values and eigen vectors from mass & stiffness matrix, do the following procedure:

1. First find the inverse for the mass matrix(k).
2. Multiply this inverted stiffness matrix with the mass matrix(m).
3. Then find the eigen values and eigen vectors for this matrix product.
4. This eigen value will represent the natural frequency of the system and eigen vector will represent the mode shape of the vibrating system.

Unavailable directories have been removed from MATLAB's search path:  
\\comp2 : Permission denied.

Problem opening demo file for reading.  
??? Demo file positioning error.

```
DEMO» k = [300280 -185210 0 0 0 0 0 0;-185210 465260 -280050 0 0 0 0 0;C -280050
2904270 -2624220 0 0 0 0;0 0 -2624220 6025220 -3401000 0 0 0;0 0 0 -3401000
7194970 -3793970 0 0;0 0 0 0 -3793970 8583850 -4789880 0;0 0 0 0 0 -4789880
4882640 -92760;0 0 0 0 0 0 -92760 92760]
```

k =

Columns 1 through 6

```
300280    -185210         0         0         0         0
-185210    465260    -280050         0         0         0
  0    -280050    2904270    -2624220         0         0
  0         0    -2624220    6025220    -3401000         0
  0         0         0    -3401000    7194970    -3793970
  0         0         0         0    -3793970    8583850
  0         0         0         0         0    -4789880
  0         0         0         0         0         0
```

Columns 7 through 8

```
  0         0
  0         0
  0         0
  0         0
  0         0
-4789880         0
4882640    -92760
-92760     92760
```

DEMO» k' = inv(k)

??? k' =

|

Missing operator, comma, or semi-colon.

DEMO» s = inv(k)

s =

1.0e-004 \*

Columns 1 through 7

0.0869	0.0869	0.0869	0.0869	0.0869	0.0869	0.0869
0.0869	0.1409	0.1409	0.1409	0.1409	0.1409	0.1409
0.0869	0.1409	0.1766	0.1766	0.1766	0.1766	0.1766
0.0869	0.1409	0.1766	0.1804	0.1804	0.1804	0.1804
0.0869	0.1409	0.1766	0.1804	0.1834	0.1834	0.1834
0.0869	0.1409	0.1766	0.1804	0.1834	0.1860	0.1860
0.0869	0.1409	0.1766	0.1804	0.1834	0.1860	0.1881
0.0869	0.1409	0.1766	0.1804	0.1834	0.1860	0.1881

Column 8

0.0869  
0.1409  
0.1766  
0.1804  
0.1834  
0.1860  
0.1881  
0.2959

DEMO» m = [1855 0 0 0 0 0 0 0;0 2975 0 0 0 0 0 0;0 0 4474 0 0 0 0 0;0 0 0 2613 0 0 0 0;  
0;0 0 0 0 3369 0 0 0;0 0 0 0 0 3748 0 0;0 0 0 0 0 0 4701 0;0 0 0 0 0 0 0 72098]

m =

Columns 1 through 6

1855	0	0	0	0	0
0	2975	0	0	0	0
0	0	4474	0	0	0
0	0	0	2613	0	0
0	0	0	0	3369	0
0	0	0	0	0	3748
0	0	0	0	0	0
0	0	0	0	0	0

Columns 7 through 8

```
0      0
0      0
0      0
0      0
0      0
0      0
4701   0
0      72098
```

DEMO» a = s\*m

a =

Columns 1 through 7

```
0.0161  0.0259  0.0389  0.0227  0.0293  0.0326  0.0409
0.0161  0.0419  0.0630  0.0368  0.0475  0.0528  0.0662
0.0161  0.0419  0.0790  0.0461  0.0595  0.0662  0.0830
0.0161  0.0419  0.0790  0.0471  0.0608  0.0676  0.0848
0.0161  0.0419  0.0790  0.0471  0.0618  0.0687  0.0862
0.0161  0.0419  0.0790  0.0471  0.0618  0.0697  0.0874
0.0161  0.0419  0.0790  0.0471  0.0618  0.0697  0.0884
0.0161  0.0419  0.0790  0.0471  0.0618  0.0697  0.0884
```

Column 8

```
0.6266
1.0158
1.2733
1.3008
1.3220
1.3410
1.3560
2.1333
```

DEMO» b = eig(a)

b =

```
4.396
5.652
6.908
7.065
7.203
7.988
8.157
3.956
```

DEMO» [v,d] = eig(s,m)

v =

Columns 1 through 7

0.3877	-0.9239	0.4700	-0.0182	0.0086	-0.0014	0.0002
0.3697	-0.2242	-0.8528	0.1616	-0.1048	0.0449	-0.0110
0.2928	0.0814	-0.0113	-0.5193	0.4928	-0.6116	0.2575
0.5085	0.1765	0.0848	-0.5910	0.4216	0.4812	-0.6636
0.3973	0.1526	0.1166	0.0011	-0.1945	0.5130	0.6022
0.3586	0.1452	0.1329	0.3708	-0.4951	-0.0919	0.1412
0.2863	0.1182	0.1146	0.4395	-0.4364	-0.3472	-0.3326
0.0188	0.0082	0.0097	0.1547	0.3081	0.0129	0.0060

Column 8

0.0000  
0.0006  
-0.0239  
0.1387  
-0.4244  
0.7662  
-0.4616  
0.0044

d =

1.0e-007 \*

Columns 1 through 7

0.3166	0	0	0	0	0	0
0	0.0236	0	0	0	0	0
0	0	0.0064	0	0	0	0
0	0	0	0.0018	0	0	0
0	0	0	0	0.0014	0	0
0	0	0	0	0	0.0005	0
0	0	0	0	0	0	0.0003
0	0	0	0	0	0	0

Column 8

0  
0  
0  
0  
0  
0  
0  
0  
0.0002

The following table gives the frequencies obtained from the MATLAB software.

ZONE LEVEL	NATURAL FREQUENCY In (rad/sec)	NATURAL FREQUENCY In (Hz)
1	4.396	0.70
2	5.652	0.90
3	6.908	1.10
4	7.065	1.125
5	7.203	1.147
6	7.988	1.272
7	8.157	1.299
8	3.956	0.630

## 8.1. DESIGN PROCEDURE IN ANSYS SOFTWARE:

The following log file explains the design procedure in the ANSYS software.

```
/BATCH
/COM,ANSYS REVISION 5.2  UP020996      18:51:37  27/2/1999

```



```

/VIEW, 1, -6770 , -5392 , .5010
/ANG, 1, -11.83
/VIEW, 1, -8482 , -1216 , .5155
/ANG, 1, -10.02
/REPLOT
/VIEW, 1, -8104 , .2093 , .5472
/ANG, 1, -9.868
/REPLOT
/VIEW, 1, -4401 , .7031 , .5585
/ANG, 1, -1.124
/VIEW, 1, -.2792E-01, .8533 , .5207
/ANG, 1, 26.01
/VIEW, 1 ,1,1,1
FLST,2,910,1,ORDE,2
FITEM,2,7821
FITEM,2,-8730
FDELE,P51X,ALL
NSEL,S,LOC,Z,25
THE FOLLOWING SELECT COMMANDS WERE GENERATED BY THE
ALLSEL COMMAND
SELECT COMMANDS WERE GENERATED BY THE ALLSEL COMMAND
VSEL,ALL
ASEL,ALL
/VIEW, 1 ,1,1,1
/VIEW, 1, .5153 , .3615 , .7771
/ANG, 1, -5.674
/LIG, 1,1,1.000, -.52931E-01, .70494 , .70729 , .00000
/VIEW, 1, .3933 , .1323 , .9098
/ANG, 1, -7.845
/LIG, 1,1,1.000, -.13213 , .53696 , .83320 , .00000
/VIEW, 1 ,,1

```

```

/VIEW, 1, 1
/VIEW, 1, 1, 1, 1
/COM, ANSYS REVISION 5.2  UP020996      23:39:25  07/13/1996
/input, menust, tmp  ,,1
wpstyle, 0.05, 0.1, -1, 1, 0.003, 0, 1, ,, 5
/DIST, 1, 1.371742, 1
/DIST, 1, 1.371742, 1
/DIST, 1, 1.371742, 1
/DIST, 1, 1.371742, 1
ET, 1, SOLID92
wpstyle, .1, 0.1, -1, 1, 0.003, 0, 1, ,, 5
/ZOOM, 1, RECT, 0.155786, -0.013437, -0.174461, -0.343684
wpstyle, .05, .05, -1, 1, 0.003, 0, 1, ,, 5
FITEM, 3, -.3, .25, 0
KMODIF, P51X, P51X
LARC, 11, 13, 12
/DIST, 1, 0.729000, 1
/DIST, 1, 0.729000, 1
/DIST, 1, 0.729000, 1
/ZOOM, 1, RECT, -0.804235, 0.309049, 0.052104, -0.547289
wpstyle, 0.05, 0.05, -1, 1, 0.003, 0, 1, ,, 5
/VIEW, 1, .9886 , .3667E-01, .1460
/ANG, 1, -.3625E-01
/VIEW, 1, .9508 , .1141 , .2880
/ANG, 1, -.7150
WPCSYS, -1, 0
/VIEW, 1, ,, 1
THE FOLLOWING SELECT COMMANDS WERE GENERATED BY THE
ALLSEL COMMAND
NSEL, ALL
/VIEW, 1, .2052 , .5123 , .8339

```

```

/ANG, 1, -17.56
/LIG, 1,1,1.000, .74587 , .18381 , .64023 , .00000
/REPLOT
/LIG, 1,1,1.000, .74587 , .18381 , .64023 , .00000
/REPLOT
FLST,2,49,4,ORDE,2
FITEM,2,1
FITEM,2,-49
LDELE,P51X,,1
/VIEW, 1, .1273 , .6884 , .7141
/ANG, 1, -19.67
/LIG, 1,1,1.000, .69848 , .37042 , .61231 , .00000
ASBA, 17, 18
/FOC, 1, -.2831 , .5506 , .5077
/VIEW, 1, .5774 , .5774 , .5774
/LIG, 1,1,1.000, .95081 , .11413 , .28800 , .00000
/REPLOT
/VIEW, 1, .3898 , .5557 , .7343
/ANG, 1, -9.740
/LIG, 1,1,1.000, .85942 , .16191 , .48495 , .00000
/REPLOT
/VIEW, 1, .1206 , .4515 , .8841
/ANG, 1, -20.27
/LIG, 1,1,1.000, .68515 , .15221 , .71232 , .00000
/REPLOT
/VIEW, 1, .5643 , .4297 , .7049
/ANG, 1, -5.517
/LIG, 1,1,1.000, .93413 , .54874E-02, .35689 , .00000
FLST,2,48,1,ORDE,8
FITEM,2,1
FITEM,2,6

```

! THE FOLLOWING SELECT COMMANDS WERE GENERATED BY THE  
ALLSEL COMMAND

```
V/VIEW,1,,1  
//DIST,1,1.371742,1  
FITEM,2,592  
ALLSEL,ALL
```

! THE FOLLOWING SELECT COMMANDS WERE GENERATED BY THE  
ALLSEL COMMAND

```
VSEL,ALL  
/VIEW,1, .2100 , .4205 , .8827  
/ANG, 1, -16.73  
/LIG, 1,1,1.000, .72108 , -.32726E-01, .69208 , .00000  
/REPLOT  
/VIEW,1, -.1347 , .1106 , .9847  
/ANG, 1, -23.04  
/LIG, 1,1,1.000, .46916 , -.23643 , .85088 , .00000  
/REPLOT  
/VIEW,1, .2695 , .1587 , .9498  
/ANG, 1, -19.74  
/LIG, 1,1,1.000, .75646 , -.23079 , .61196 , .00000  
/REPLOT  
/VIEW,1, .5328 , .3334E-03, .8462  
/ANG, 1, -17.60  
/LIG, 1,1,1.000, .86182 , -.37453 , .34203 , .00000  
/REPLOT  
/VIEW,1, .5529 , .5592 , .6177  
/ANG, 1, -19.51  
/LIG, 1,1,1.000, .96347 , .12324 , .23778 , .00000  
/REPLOT  
PLDISP,1  
/VIEW,1, .2285 , .2601 , .9381
```

```

/ANG, 1, -32.44
/LIG, 1,1,1.000, .79184 ,-.19667E-01, .61042 , .00000
/REPLOT
/VIEW, 1, -.1435 ,-.5383E-01, .9882
/ANG, 1, -35.30
/LIG, 1,1,1.000, .52267 ,-.23061 , .82076 , .00000
/REPLOT

/VIEW, 1, .3254 , .3870 , .8627
/ANG, 1, -12.27
/LIG, 1,1,1.000, .26732 ,-.45645 , .84864 , .00000
/REPLOT
/VIEW, 1, .2939 , .7712E-01, .9527
/ANG, 1, -13.69
/LIG, 1,1,1.000, .17716 ,-.71386 , .67751 , .00000
/REPLOT
/VIEW, 1, -.8683E-01, .4607 , .8833
/ANG, 1, -18.53
/LIG, 1,1,1.000, -.56104E-01, -.38348 , .92184 , .00000
/REPLOT
/VIEW, 1, .1702 , .1138 , .9788
/ANG, 1, -12.10
/LIG, 1,1,1.000, .73418E-01, -.68649 , .72342
/VIEW, 1, .5147 , .5164 , .6844
/ANG, 1, -4.499
/LIG, 1,1,1.000, .74065E-02, -.75424 , .65656 , .00000
/REPLOT
/VIEW, 1, .4841 , .3364 , .8078
/ANG, 1, -7.370
/LIG, 1,1,1.000, -.10060 ,-.86143 , .49782 , .00000
/REPLOT

```

```

/VIEW,1, .4726 , .1839 , .8619
/ANG, 1, -7.786
/LIG, 1,1,1.000, -.18088 , -.91219 , .36769 , .00000
/REPLOT
/ZOOM,1,RECT,-0.746875,0.597561,0.751670,-0.900984
VIEW,1, .4742 , .5583 , .6808
/ANG, 1, -5.769
/LIG, 1,1,1.000, -.17442 , -.94390 , .28041 , .00000
/REPLOT
/VIEW,1, .1954 , .6186 , .7611
/ANG, 1, -17.80
/LIG, 1,1,1.000, -.57973E-01, -.96647 , .25014 , .00000
/REPLOT
/VIEW,1, .1998 , .5639 , .8013
/ANG, 1, -18.02
/LIG, 1,1,1.000, -.72230E-01, -.98007 , .18506 , .00000
/REPLOT
/VIEW,1, .3170 , .4535 , .8330
/ANG, 1, -14.40
/LIG, 1,1,1.000, -.14923 , -.98469 , .90098E-01, .00000
/REPLOT
/VIEW,1, .4781 , .4237 , .7693
/ANG, 1, -9.468
/LIG, 1,1,1.000, -.21508 , -.96883 , .12289 , .00000
/REPLOT

/VIEW,1, -.2671 , .4495 , .8524
/ANG, 1, -32.45
/LIG, 1,1,1.000, .25419 , -.95785 , -.13386 , .00000
/REPLOT
/VIEW,1, -.2468 , .5008 , .8297

```

```
/ANG, 1, -33.38
/LIG, 1,1,1.000, .24857 , -.96515 , -.81831E-01, .00000
/REPLOT
/VIEW, 1, -.1779 , .7897 , .5871
/ANG, 1, -28.57
/LIG, 1,1,1.000, .75155E-01, -.95980 , .27042 , .00000
/REPLOT
/VIEW, 1, .4212 , .5736 , .7026
/ANG, 1, 3.619
/LIG, 1,1,1.000, -.28613 , -.95491 , .79200E-01, .00000
/REPLOT
/VIEW, 1, .5052 , .3742 , .7777
/ANG, 1, 2.188
/LIG, 1,1,1.000, -.38233 , -.91889 , -.97238E-01, .00000
/REPLOT
/VIEW, 1, -.7038 , .3271 , .6306
/ANG, 1, -18.09
/LIG, 1,1,1.000, .24304 , -.94728 , -.20879 , .00000
/VIEW, 1, -.4920 , .3552 , .7948
/ANG, 1, -12.36
/LIG, 1,1,1.000, .92826E-01, -.95337 , -.28716 , .00000
/REPLOT
/VIEW, 1, -.3545 , .3574 , .8640
/ANG, 1, -8.932
/LIG, 1,1,1.000, .94084E-03, -.94871 , -.31615 , .00000
/VIEW, 1, -.1260 , .3474 , .9292
/ANG, 1, -3.611
/LIG, 1,1,1.000, -.14797 , -.93200 , -.33089 , .00000
/REPLOT
/VIEW, 1, -.3051 , .3696 , .8777
/ANG, 1, -7.642
```

```

/LIG, 1,1,1.000, -.36693E-01, -.94978 , -.31075 , .00000
/REPLOT
/REPLOT,RESIZE
/REPLOT,RESIZE
/DIST, 1 ,1.371742,1
/ZOOM,1,RECT,-0.742606,0.610366,0.747401,-0.879641
/REPLOT,RESIZE
/REPLOT,RESIZE
/VIEW, 1 ,1,1,1
/COM,ANSYS REVISION 5.2  UP020996      18:20:14  07/14/1996
/input,menust,tmp      ,,1
/PREP7
ET,1,PLANE82
DELE,P51X, , ,1
KEYW,PR_SET,1
KEYW,PR_STRUC,1
KEYW,PR_THERM,0
/PMETH,OFF
FINISH
/CLEAR,START
/COM,ANSYS REVISION 5.2  UP020996      19:45:53  07/14/1996
/input,start,ans      ,C:\ANSYS52\docu\,,,,,1
/REPLOT,RESIZE
/COM, ANSYS Material Library.
/COM, SI units. (Meter, kilogram, second)
/NOPR
/COM, Typical properties of STEEL AISI C1020
mp,ex,ARG1,2.07e11*EX_FAC  ! modulus of elasticity (Pascals)
mp,alpx,ARG1,15.1e-6*ALP_FAC ! coeff. of expansion (meters/meter/K)
mp,nuxy,ARG1,.29      ! poisson's ratio (No units)
mp,dens,ARG1,7850*DEN_FAC  ! mass density (kilograms/meter**3)

```



```

mp,kxx,ARG1,46.7*K_FAC    ! thermal conductivity (Watts/meter/K)
mp,c,ARG1,419*C_FAC      ! specific heat (Joules/kilogram/K)
FITEM,3,,35505417538,,276908142248,
KL,  3,P51X
KL,  3, .4348327297399635
ESIZE,0,1000,
FLST,2,4,4,ORDE,2
FITEM,2,1
FITEM,2,-4
LMESH,P51X
/SOLU
FINISH
/COM,ANSYS REVISION 5.2  UP020996    09:25:16  07/15/1996
/input,menust,tmp  ,,1
ABBRESU,NEW,xxx, ,
*SET,ARG1,1
/INPUT,ansuitmp,,,1
! /INPUT,ansuitmp
/COM, Typical properties of STEEL AISI C1020
mp,ex,ARG1,2.07e11*EX_FAC  ! modulus of elasticity (Pascals)
mp,alpx,ARG1,15.1e-6*ALP_FAC ! coeff. of expansion (meters/meter/K)
mp,nuxy,ARG1,.29          ! poisson's ratio (No units)
mp,dens,ARG1,7850*DEN_FAC  ! mass density (kilograms/meter**3)
mp,kxx,ARG1,46.7*K_FAC    ! thermal conductivity (Watts/meter/K)
mp,c,ARG1,419*C_FAC      ! specific heat (Joules/kilogram/K)
/COM,ANSYS REVISION 5.2  UP020996    09:32:36  07/15/1996
/input,start,ans  ,C:\ANSYS52\docu\,,,1
/ZOOM,2,RECT,-0.125090,0.595240,-0.418158,0.302173
/ZOOM, 2 ,BACK
/ZOOM, 2, .22500  , .17500  , .00000  , .24750
/ZOOM, 2 ,BACK

```

/ZOOM,2,RECT,-0.195848,0.489129,-0.493969,0.191009  
/VIEW, 2 ,1,1,1  
/ZOOM,5,RECT,0.926173,-0.329438,0.754332,-0.637665  
/ZOOM,4,RECT,0.895848,-0.334491,0.557305,-0.673035  
BLC4,0.45,0.2,-0.15,0.15  
/ZOOM,1,RECT,0.926173,0.691245,0.587630,0.352701  
BLC4,1.1,0.6,0.05,0.05  
BLC4,0.45,-0.15,0.45,0.4  
BLC4,0.65,0.5,0.3,0.25  
BLC4,0.35,0.1,0.05,0.05  
BLC4,0.05,-0.45,0.95,0.75  
BLC4,-0.2,-0.75,0.65,0.55  
BLC4,1.05,0.55,0.1,0.05  
BLC4,0.65,0.35,0.4,0.25  
BLC4,0.3,0.15,0.1,0.05  
BLC4,-0.2,-1.25,-0.65,0.65  
BLC4,0.3,0.1,-0.05,-0.05  
BLC4,0.6,-0.85,-0.85,-1  
BLC4,1.85,-0.7,-0.7,-1  
BLC4,2.2,1.3,-0.85,-1.2  
BLC4,0.4,0.1,-0.15,0.1  
BLC4,-1.9,-3.4,3.4,5  
BLC4,0.35,0.1,0.05,0.1  
/VIEW,ALL,1,1,1  
/ANG,ALL  
/WIN,1,-.746590759264,.702801451092E-01,.155555548491,.863333294127  
/REPLOT  
/WIN,1,.179196265692,.51372435034,-.139999993642,.233333322737  
/REPLOT  
/WIN,1,-.878846048543,-.225349325045,-.54444441972,.816666629579  
/REPLOT

```

/WIN,1,-1,1.67,-1,1
/REPLOT
WIN1,-1,1.67,-1,1
/REPLOT
/ZOOM,1,RECT,1.392833,0.264444,0.739336,-0.225066
/DIST,ALL,1.371742,1
/REP
/DIST,ALL,1.371742,1
/REP
/DIST,ALL,1.371742,1
/REP
/DIST,ALL,1.371742,1
/REP
COM,ANSYS REVISION 5.2  UP020996      11:17:33  07/15/1996
/input,start,ans  ,C:\ANSYS52\docu\,,,,,,,,,,,,,1
!*
wpstyle,0.05,1,-1,10,0.003,0,2,,5
/DIST,ALL,1.371742,1
/REP
CYL4,7.2,-0.15,0.2
LPLOT
/ZOOM,1,RECT,0.265952,0.425799,0.141112,0.300959
FLST,2,768,4,ORDE,2
FITEM,2,1
12:12 PM 4/1/9012:12 PM 4/1/9012:12 PM 4/1/90FITEM,2,-768
LGLUE,P51X
/AUTO,ALL
/REP
FLST,2,772,4,ORDE,2
FITEM,2,1
FITEM,2,-772

```

LOVLAP,P51X

/BATCH

/COM,ANSYS REVISION 5.2 UP020996 11:31:31 07/15/1996

/input,menust,tmp,,,,,,,,,,,,,1

/REPLOT

/REPLOT

wpstyle,0.05,0.1,-1,1,0.003,0,1,,5

/DIST, 1 ,1.371742,1

/REP

IN ANSYS SOFTWARE THE LOADS FOR EACH NODE DEFINED AS FOLLOWS :

LIST ELEMENT SURFACE LOAD PRES FOR ALL SELECTED ELEMENTS

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
1	2	1	1	1830.0	.00000
			3	1830.0	.00000
2	2	3	3	1830.0	.00000
			4	1830.0	.00000
3	2	4	4	1830.0	.00000
			5	1830.0	.00000
4	2	5	5	1830.0	.00000
			6	1830.0	.00000
5	2	6	6	1830.0	.00000
			7	1830.0	.00000
6	2	7	7	1830.0	.00000
			8	1830.0	.00000
7	2	8	8	1830.0	.00000
			9	1830.0	.00000
8	2	9	9	1830.0	.00000
			10	1830.0	.00000
9	2	10	10	1830.0	.00000
			11	1830.0	.00000
10	2	11	11	1830.0	.00000
			2	1830.0	.00000

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
11	2	12	12	1770.0	.00000
			13	1770.0	.00000
12	2	13	13	1770.0	.00000
			14	1770.0	.00000
13	2	14	14	1770.0	.00000
			15	1770.0	.00000
14	2	15	15	1770.0	.00000
			16	1770.0	.00000
15	2	16	16	1770.0	.00000
			17	1770.0	.00000
16	2	17	17	1770.0	.00000
			18	1770.0	.00000
17	2	18	18	1770.0	.00000
			19	1770.0	.00000

Pressure

18	2	19	1770.0	.00000
		20	1770.0	.00000
19	2	20	1770.0	.00000
		21	1770.0	.00000
20	2	21	1770.0	.00000
		1	1770.0	.00000

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
21	2	22	22	1720.0	.00000
		23	23	1720.0	.00000
22	2	23	23	1720.0	.00000
		24	24	1720.0	.00000
23	2	24	24	1720.0	.00000
		25	25	1720.0	.00000
24	2	25	25	1720.0	.00000
		26	26	1720.0	.00000
25	2	26	26	1720.0	.00000
		27	27	1720.0	.00000
26	2	27	27	1720.0	.00000
		28	28	1720.0	.00000
27	2	28	28	1720.0	.00000
		29	29	1720.0	.00000
28	2	29	29	1720.0	.00000
		30	30	1720.0	.00000
29	2	30	30	1720.0	.00000
		31	31	1720.0	.00000
30	2	31	31	1720.0	.00000
		12	12	1720.0	.00000

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
31	2	32	32	1670.0	.00000
		33	33	1670.0	.00000
32	2	33	33	1670.0	.00000
		34	34	1670.0	.00000
33	2	34	34	1670.0	.00000
		35	35	1670.0	.00000
34	2	35	35	1670.0	.00000
		36	36	1670.0	.00000
35	2	36	36	1670.0	.00000
		22	22	1670.0	.00000
36	2	37	37	1630.0	.00000

		38	1630.0	.00000
37	2	38	1630.0	.00000
		39	1630.0	.00000
38	2	39	1630.0	.00000
		40	1630.0	.00000
39	2	40	1630.0	.00000
		41	1630.0	.00000
40	2	41	1630.0	.00000
		32	1630.0	.00000

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
41	2	42	1580.0	.00000	
		43	1580.0	.00000	
42	2	43	1580.0	.00000	
		44	1580.0	.00000	
43	2	44	1580.0	.00000	
		45	1580.0	.00000	
44	2	45	1580.0	.00000	
		46	1580.0	.00000	
45	2	46	1580.0	.00000	
		37	1580.0	.00000	
46	2	47	1560.0	.00000	
		48	1560.0	.00000	
47	2	48	1560.0	.00000	
		49	1560.0	.00000	
48	2	49	1560.0	.00000	
		50	1560.0	.00000	
49	2	50	1560.0	.00000	
		51	1560.0	.00000	
50	2	51	1560.0	.00000	
		42	1560.0	.00000	

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
51	2	52	1500.0	.00000	
		53	1500.0	.00000	
52	2	53	1500.0	.00000	
		54	1500.0	.00000	
53	2	54	1500.0	.00000	
		55	1500.0	.00000	
54	2	55	1500.0	.00000	
		56	1500.0	.00000	

Pressure

55	2	56	1500.0	.00000
		57	1500.0	.00000
56	2	57	1500.0	.00000
		58	1500.0	.00000
57	2	58	1500.0	.00000
		59	1500.0	.00000
58	2	59	1500.0	.00000
		60	1500.0	.00000
59	2	60	1500.0	.00000
		61	1500.0	.00000
60	2	61	1500.0	.00000
		62	1500.0	.00000

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
61	2	62	62	1500.0	.00000
		63	63	1500.0	.00000
62	2	63	63	1500.0	.00000
		64	64	1500.0	.00000
63	2	64	64	1500.0	.00000
		65	65	1500.0	.00000
64	2	65	65	1500.0	.00000
		66	66	1500.0	.00000
65	2	66	66	1500.0	.00000
		67	67	1500.0	.00000
66	2	67	67	1500.0	.00000
		68	68	1500.0	.00000
67	2	68	68	1500.0	.00000
		69	69	1500.0	.00000
68	2	69	69	1500.0	.00000
		70	70	1500.0	.00000
69	2	70	70	1500.0	.00000
		71	71	1500.0	.00000
70	2	71	71	1500.0	.00000
		72	72	1500.0	.00000

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
71	2	72	72	1500.0	.00000
		73	73	1500.0	.00000
72	2	73	73	1500.0	.00000
		74	74	1500.0	.00000
73	2	74	74	1500.0	.00000



Pressure

		75	1500.0	.00000
74	2	75	1500.0	.00000
		76	1500.0	.00000
75	2	76	1500.0	.00000
		77	1500.0	.00000
76	2	77	1500.0	.00000
		78	1500.0	.00000
77	2	78	1500.0	.00000
		79	1500.0	.00000
78	2	79	1500.0	.00000
		80	1500.0	.00000
79	2	80	1500.0	.00000
		81	1500.0	.00000
80	2	81	1500.0	.00000
		82	1500.0	.00000

ELEMENT	LKEY	FACE	NODES	REAL	IMAGINARY
81	2		82	1500.0	.00000

INPUTS GIVEN IN ANSYS SOFTWARE :

MATERIAL : STEEL  
DENSITY : 7850 Kg/mm<sup>3</sup>  
POISSONS RATIO : 0.25

ELEMENT LKEY FACE NODES REAL IMAGINARY  
81 2 82 1500.0 .00000 LOAD STEP OPTIONS

LOAD STEP NUMBER..... 1  
TIME AT END OF THE LOAD STEP..... 1.0000  
NUMBER OF SUBSTEPS..... 1  
STEP CHANGE BOUNDARY CONDITIONS..... NO  
AXISYMMETRIC HARMONIC LOADING PARAMETERS  
MODE..... 0  
ISYM..... 1  
PRINT OUTPUT CONTROLS.....NO PRINTOUT  
DATABASE OUTPUT CONTROLS.....ALL DATA WRITTEN  
FOR THE LAST SUBSTEP

Element Formation Element= 10 Cum. Iter.= 1 CP= 5511.000  
Time= 1.0000 Load Step= 1 Substep= 1 Equilibrium Iteration= 1.

\*\*\*\*\* CENTROID, MASS, AND MASS MOMENTS OF INERTIA \*\*\*\*\*

CALCULATIONS ASSUME ELEMENT MASS AT ELEMENT CENTROID

TOTAL MASS = 93184.

MASS AND MOMENT VALUES ARE EVALUATED FOR A 360. DEGREE MODEL.

THE BELOW TABLE IS BASED ON A WEIGHTED(MASS PROPORTIONAL TO RADIUS) AXISYMMETRIC SECTION.

	FIRST MOMENT/MASS MOMENT	FIRST MOMENT/MASS (CENTROID)	SQUARE ROOT OF MOMENT	SECOND (RADIUS OF GYR.)
RADIAL	.8484E+08	910.41	.7895E+11	920.5
AXIAL	.2096E+10	22490.		

\*\*\* MASS SUMMARY BY ELEMENT TYPE \*\*\*

TYPE	MASS
1	93184.3

Range of element maximum matrix coefficients in global coordinates

Maximum= 8.275495503E+12 at element 81.

Minimum= 8.676933452E+11 at element 10.

\*\*\* ELEMENT MATRIX FORMULATION TIMES

TYPE	NUMBER	ENAME	TOTAL CP	AVE CP
1	81	SHELL51	4.940	.061

Time at end of element matrix formulation CP= 5512.32.

Estimated number of active DOF= 244.

Maximum wavefront= 11.

Time at end of matrix triangularization CP= 5514.19.  
Equation solver maximum pivot= 1.312504486E+13 at node 82 ROTZ.  
Equation solver minimum pivot= 738054.512 at node 2 UZ.

**\*\*\* ELEMENT RESULT CALCULATION TIMES**

TYPE	NUMBER	ENAME	TOTAL CP	AVE CP
------	--------	-------	----------	--------

1	81	SHELL51	1.700	.021
---	----	---------	-------	------

**\*\*\* NODAL LOAD CALCULATION TIMES**

TYPE	NUMBER	ENAME	TOTAL CP	AVE CP
------	--------	-------	----------	--------

1	81	SHELL51	.110	.001
---	----	---------	------	------

**\*\*\* LOAD STEP 1 SUBSTEP 1 COMPLETED. CUM ITER = 1**

**\*\*\* TIME = 1.00000 TIME INC = 1.00000 NEW TRIANG MATRIX**

**\*\*\* PROBLEM STATISTICS**

**ACTUAL NO. OF ACTIVE DEGREES OF FREEDOM = 244**

**R.M.S. WAVEFRONT SIZE = 7.6**

**\*\*\* ANSYS BINARY FILE STATISTICS**

**BUFFER SIZE USED= 4096**

**.078 MB WRITTEN ON ELEMENT MATRIX FILE: file.emat**

**.219 MB WRITTEN ON ELEMENT SAVED DATA FILE: file.esav**

**.031 MB WRITTEN ON TRIANGULARIZED MATRIX FILE: file.tri**

**.109 MB WRITTEN ON RESULTS FILE: file.rst**

## 8.2. SOLUTION PHASE :

### FREQUENCY DISTRIBUTION UNDER LOADING CONDITION

\*\*\*\*\* INDEX OF DATA SETS ON RESULTS FILE \*\*\*\*\*

SET	TIME/FREQ	LOAD	STEP	SUBSTEP	CUMULATIVE
-----	-----------	------	------	---------	------------

1	2.023	1	1	1	
---	-------	---	---	---	--

2	2.033	1	2	2	
---	-------	---	---	---	--

3	2.078	1	3	3	
---	-------	---	---	---	--

4	2.103	1	4	4	
---	-------	---	---	---	--

5	2.276	1	5	5	
---	-------	---	---	---	--

6	2.298	1	6	6	
---	-------	---	---	---	--

7	2.341	1	7	7	
---	-------	---	---	---	--

8	2.472	1	8	8	
---	-------	---	---	---	--

9	2.741	1	9	9	
---	-------	---	---	---	--

10 2.856 1 10 10

11 3.032 1 11 11

12 3.077 1 12 12

13 3.143 1 13 13

14 3.254 1 14 14

15 3.367 1 15 15

16 3.476 1 16 16

17 3.598 1 17 17

18 3.645 1 18 18

19 3.798 1 19 19

20 3.912 1 20 20

21 4.012 1 21 21

22 4.098 1 22 22

23 4.145 1 23 23

24 4.201 1 24 24

25 4.299 1 25 25

26 4.312 1 26 26

27 4.397 1 27 27

28 4.401 1 28 28

29 4.445 1 29 29

30	4.497	1	30	30
31	4.691	1	31	31
32	4.701	1	32	32
33	4.741	1	33	33
34	4.773	1	34	34
35	4.985	1	35	35
36	5.034	1	36	36
37	5.371	1	37	37
38	5.435	1	38	38
39	5.567	1	39	39
40	5.675	1	40	40



41 5.734 1 41 41

42 5.857 1 42 42

43 5.956 1 43 43

44 5.987 1 44 44

45 6.094 1 45 45

46 6.143 1 46 46

47 6.278 1 47 47

48 6.456 1 48 48

49 6.499 1 49 49

50 6.545 1 50 50

51 6.634 1 51 51

52 6.687 1 52 52

53 6.712 1 53 53

54 6.734 1 54 54

55 6.746 1 55 55

56 6.758 1 56 56

57 6.776 1 57 57

58 6.789 1 58 58

59 6.804 1 59 59

60 6.812 1 60 60

61	6.877	1	61	61
62	6.894	1	62	62
63	6.906	1	63	63
64	6.923	1	64	64
65	7.032	1	65	65
66	7.134	1	66	66
67	7.342	1	67	67
68	7.476	1	68	68
69	7.563	1	69	69
70	7.698	1	70	70
71	7.723	1	71	71

72 7.765 1 72 72

73 7.798 1 73 73

74 7.856 1 74 74

75 7.866 1 75 75

76 7.957 1 76 76

77 8.276 1 77 77

78 8.356 1 78 78

79 8.467 1 79 79

80 8.598 1 80 80

81 8.675 1 81 81

STRESS VALUES OBTAINED IN VARIOUS ZONES :

PRINT ELEMENT TABLE ITEMS PER ELEMENT

\*\*\*\*\* POST1 ELEMENT TABLE LISTING \*\*\*\*\*

STAT	CURRENT	CURRENT	CURRENT	CURRENT
ELEM	shearforce	shearforce	bendingmoment	bendingmoment
1	.61762E-04	.61523E-04	.32886E+10	-.50112E+10
2	.53808E-04	.47041E-04	-.50112E+10	-.18864E+10
3	.41243E-04	.38290E-04	-.18864E+10	-.69193E+09
4	.34165E-04	.36942E-04	-.69193E+09	-.25601E+09
5	.36804E-04	.38827E-04	-.25601E+09	-.97052E+08
6	.43303E-04	.36574E-04	-.97052E+08	-.38985E+08
7	.32746E-04	.30988E-04	-.38985E+08	-.17472E+08
8	.17102E-04	.94129E-05	-.17472E+08	-.86828E+07
9	.74720E-05	.55041E-05	-.86828E+07	-.29341E+07
10	.57137E-05	-.29299E-05	-.29341E+07	.00000
11	.54212E-04	.52344E-04	.24388E+10	-.43662E+10
12	.60070E-04	.60070E-04	-.43662E+10	-.16351E+10
13	.67999E-04	.67050E-04	-.16351E+10	-.58450E+09
14	.82907E-04	.81955E-04	-.58450E+09	-.16910E+09
15	.73720E-04	.73720E-04	-.16910E+09	.73984E+08
16	.55578E-04	.53673E-04	.73984E+08	.41628E+09
17	.58029E-04	.58029E-04	.41628E+09	.12402E+10
18	.49196E-04	.47289E-04	.12402E+10	.34642E+10
19	.56408E-04	.56408E-04	.34642E+10	.93722E+10
20	.56446E-04	.55539E-04	.93722E+10	.32886E+10
21	.70214E-04	.68365E-04	.31466E+09	-.21786E+10
22	.62450E-04	.60591E-04	-.21786E+10	-.82075E+09
23	.58301E-04	.57373E-04	-.82075E+09	-.30202E+09

24	.57422E-04	.55563E-04	-.30202E+09	-.89093E+08
25	.61028E-04	.59171E-04	-.89093E+08	.59446E+08
26	.73713E-04	.75749E-04	.59446E+08	.31236E+09
27	.71446E-04	.61958E-04	.31236E+09	.95700E+09
28	.56994E-04	.56060E-04	.95700E+09	.27243E+10
29	.54769E-04	.52904E-04	.27243E+10	.74595E+10
30	.59858E-04	.57904E-04	.74595E+10	.24388E+10
31	.62648E-04	.61748E-04	.96458E+09	-.27883E+10
32	.60799E-04	.61705E-04	-.27883E+10	-.75543E+09
33	.61508E-04	.61508E-04	-.75543E+09	.53868E+09
34	.66996E-04	.66996E-04	.53868E+09	.24397E+10
35	.72345E-04	.72345E-04	.24397E+10	.31466E+09
36	.48313E-04	.50408E-04	.16760E+09	-.14078E+10
37	.51836E-04	.52720E-04	-.14078E+10	-.10543E+09
38	.49803E-04	.49803E-04	-.10543E+09	.11932E+10
39	.50988E-04	.50988E-04	.11932E+10	.38889E+10
40	.56986E-04	.56986E-04	.38889E+10	.96458E+09
41	.86530E-04	.77851E-04	.68630E+09	-.23080E+10

\*\*\*\*\* POST1 ELEMENT TABLE LISTING \*\*\*\*\*

STAT	CURRENT	CURRENT	CURRENT	CURRENT
ELEM	SMIS1	SMIS7	SMIS6	SMIS12
42	.78273E-04	.73124E-04	-.23080E+10	-.69191E+09
43	.72286E-04	.63978E-04	-.69191E+09	.25917E+09
44	.61646E-04	.49323E-04	.25917E+09	.15782E+10
45	.51100E-04	.45081E-04	.15782E+10	.16760E+09
46	.82757E-04	.81936E-04	.98545E+10	-.36819E+10
47	.78039E-04	.78886E-04	-.36819E+10	-.10810E+10
48	.80261E-04	.80261E-04	-.10810E+10	.50800E+09
49	.82329E-04	.82329E-04	.50800E+09	.28225E+10

50 .84430E-04 .84430E-04 .28225E+10 .68630E+09  
51 -.36353E+10 -.35320E+10 .36706E+12 .48359E+11  
52 -.35320E+10 -.33930E+10 .48359E+11 .86694E+10  
53 -.33930E+10 -.32543E+10 .86694E+10 .75331E+09  
54 -.32543E+10 -.31169E+10 .75331E+09 -.86817E+09  
55 -.31169E+10 -.29811E+10 -.86817E+09 -.11875E+10  
56 -.29811E+10 -.28468E+10 -.11875E+10 -.12337E+10  
57 -.28468E+10 -.27141E+10 -.12337E+10 -.12219E+10  
58 -.27141E+10 -.25830E+10 -.12219E+10 -.11974E+10  
59 -.25830E+10 -.24534E+10 -.11974E+10 -.11700E+10  
60 -.24534E+10 -.23254E+10 -.11700E+10 -.11419E+10  
61 -.23254E+10 -.21990E+10 -.11419E+10 -.11134E+10  
62 -.21990E+10 -.20741E+10 -.11134E+10 -.10848E+10  
63 -.20741E+10 -.19508E+10 -.10848E+10 -.10559E+10  
64 -.19508E+10 -.18291E+10 -.10559E+10 -.10268E+10  
65 -.18291E+10 -.17090E+10 -.10268E+10 -.99749E+09  
66 -.17090E+10 -.15904E+10 -.99749E+09 -.96796E+09  
67 -.15904E+10 -.14734E+10 -.96796E+09 -.93820E+09  
68 -.14734E+10 -.13579E+10 -.93820E+09 -.90819E+09  
69 -.13579E+10 -.12441E+10 -.90819E+09 -.87793E+09  
70 -.12441E+10 -.11318E+10 -.87793E+09 -.84740E+09  
71 -.11318E+10 -.10210E+10 -.84740E+09 -.81655E+09  
72 -.10210E+10 -.91186E+09 -.81655E+09 -.78529E+09  
73 -.91186E+09 -.80426E+09 -.78529E+09 -.75321E+09  
74 -.80426E+09 -.69824E+09 -.75321E+09 -.71886E+09  
75 -.69824E+09 -.59378E+09 -.71886E+09 -.67681E+09  
76 -.59378E+09 -.49090E+09 -.67681E+09 -.60682E+09  
77 -.49090E+09 -.38959E+09 -.60682E+09 -.43410E+09  
78 -.38959E+09 -.28987E+09 -.43410E+09 .11526E+09  
79 -.28987E+09 -.19179E+09 .11526E+09 .20343E+10  
80 -.19179E+09 -.95503E+08 .20343E+10 .87090E+10

81 -.95503E+08 .26619E+07 .87090E+10 .98545E+10

\*\*\*\*\* POST1 ELEMENT TABLE LISTING \*\*\*\*\*

STAT	CURRENT	CURRENT	CURRENT	CURRENT
ELEM	SMIS1	SMIS7	SMIS6	SMIS12

MINIMUM VALUES

ELEM	51	51	2	1
VALUE	-.36353E+10	-.35320E+10	-.50112E+10	-.50112E+10

MAXIMUM VALUES

ELEM	41	81	51	51
------	----	----	----	----



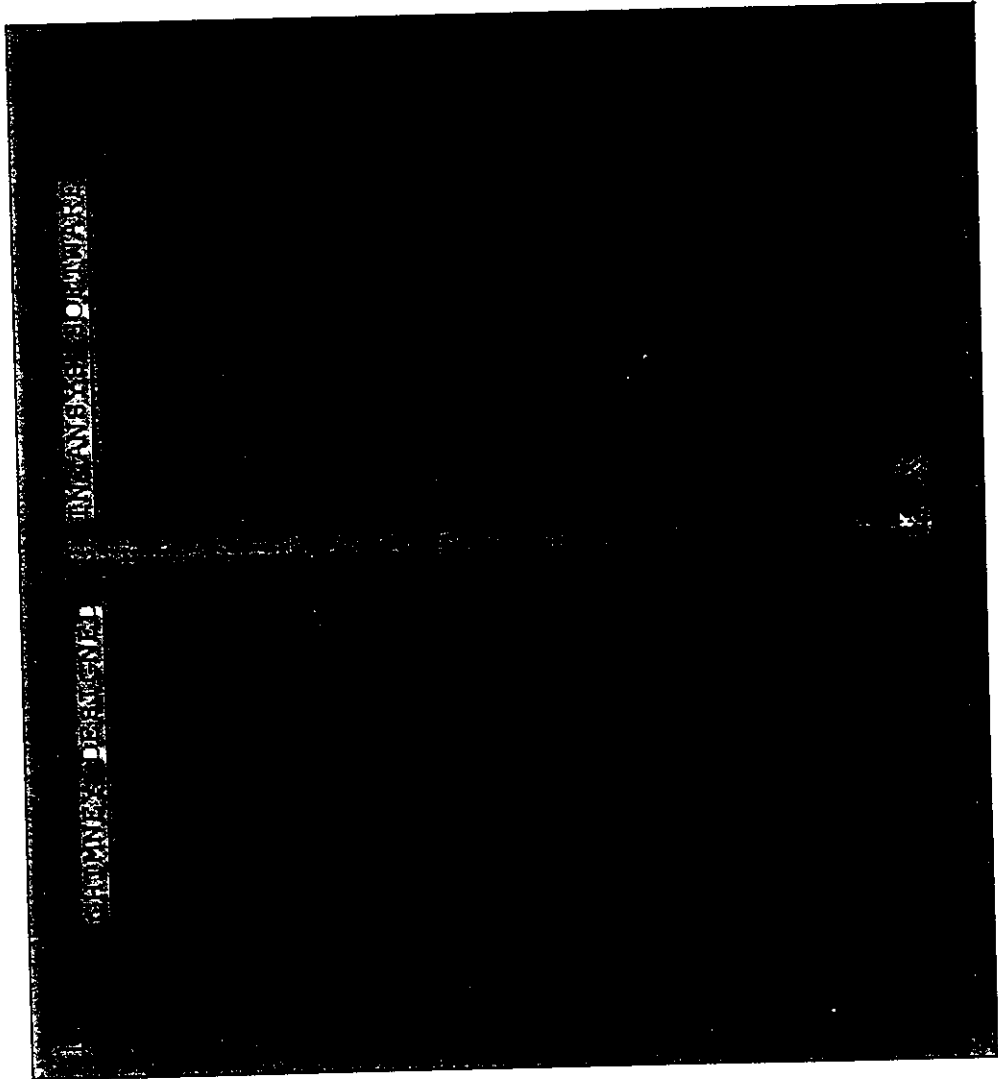
ANSYS  
27 FEB 1999  
21:00:38  
ELEMENTS  
TYPE NUM

ZV =1  
\*DIST=126.9.  
\*XF =754.1  
\*YF =69982  
Z-BUFFER

<u>ELEMENT FORMATION</u>	<u>IN ANSYS SOFTWARE</u>
1	

ANSYS 5.2  
27 FEB 1999  
20:50:47  
AREAS  
TYPE NUM

ZV =1  
DIST=49500  
YF =35000  
CAPPED HID



DEFLECTION UNDER LOADING CONDITION

ANSYS 5.2  
27 FEB 1999  
21:40:18  
DISPLACEMENT  
STEP=1  
SUB =1  
TIME=1  
RSYS=0  
DMX =466.162

DSCA=8.581  
XV =1  
YV =1  
ZV =1  
\*DIST=4242  
\*XF =493.89  
\*YF =4308  
\*ZF =-1939  
Z-BUFFER  
EDGE

VIBRATION MODE SHAPE - ELEMENT NO : 1

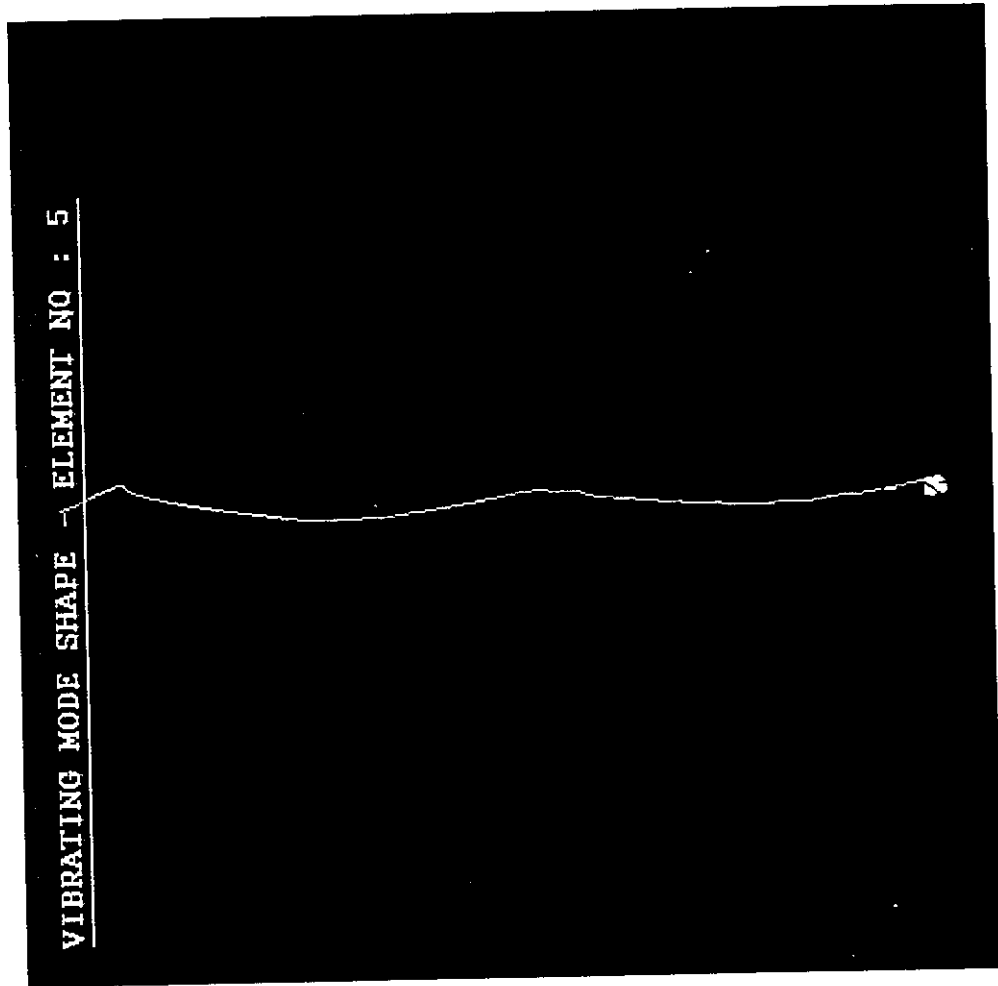
ANSYS 5.2  
27 FEB 1999  
21:29:03  
DISPLACEMENT  
STEP=1  
SUB =1  
FREQ= 2.023  
RSYS=0  
DMX =.009201  
  
DSCA=434742  
ZV =1  
DIST=41800  
XF =1049  
YF =38000  
Z-BUFFER

VIBRATING MODE SHAPE - ELEMENT NO : 4

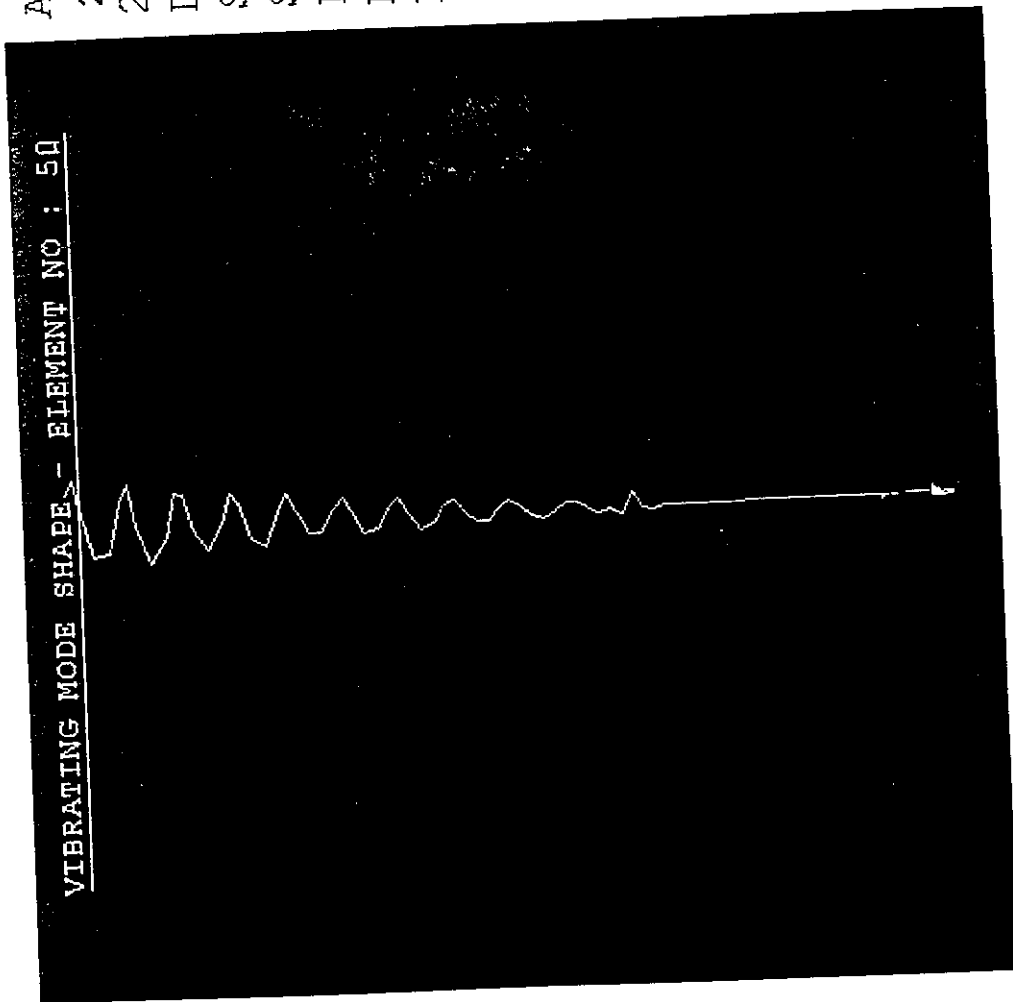
ANSYS  
27 FEB 1999  
21:10:06  
DISPLACEMENT  
SUB =1  
FREQ= 2.103  
RSYS=0  
DMX =.01069:  
  
DSCA=374063  
ZV =1  
DIST=41800  
XF =1088  
YF =38000  
Z-BUFFER

VIBRATING MODE SHAPE - ELEMENT NO : 5

ANSYS 5.2  
27 FEB 1999  
21.23:47  
DISPLACEMENT:  
STEP=1  
SUB =5  
FREQ= 2.276  
RSYS=0  
DMX =.010848  
  
DSCA=368732  
ZV =1  
DIST=41800  
XF =1131  
YF =38000  
Z-BUFFER

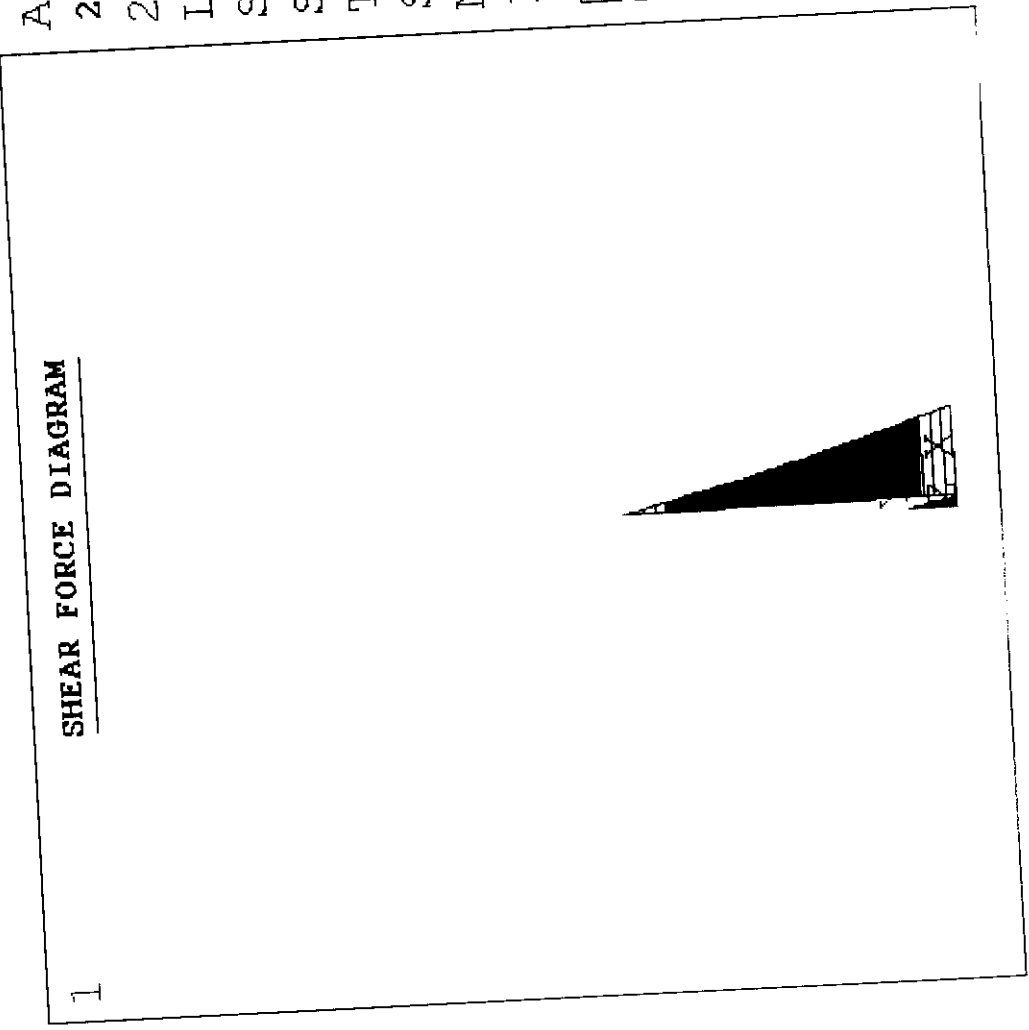


ANSYS 5.2  
27 FEB 1999  
21:27:02  
DISPLACEMENT  
STEP=1  
SUB =50  
FREQ=6.545  
RSYS=0  
DMX =.02662  
  
DSCA=150261  
ZV =1  
DIST=43781  
XF =882.69  
YF =39801  
Z-BUFFER



ANSYS 5.2  
27 FEB 1999  
22:36:30  
LINE STRES.  
STEP=1  
SUB =1  
TIME=1  
SMIS1 SM  
MAX =.266E  
ELEM=81

- .364  
- .323  
- .283  
- .242  
- .806  
- .402  
.266E

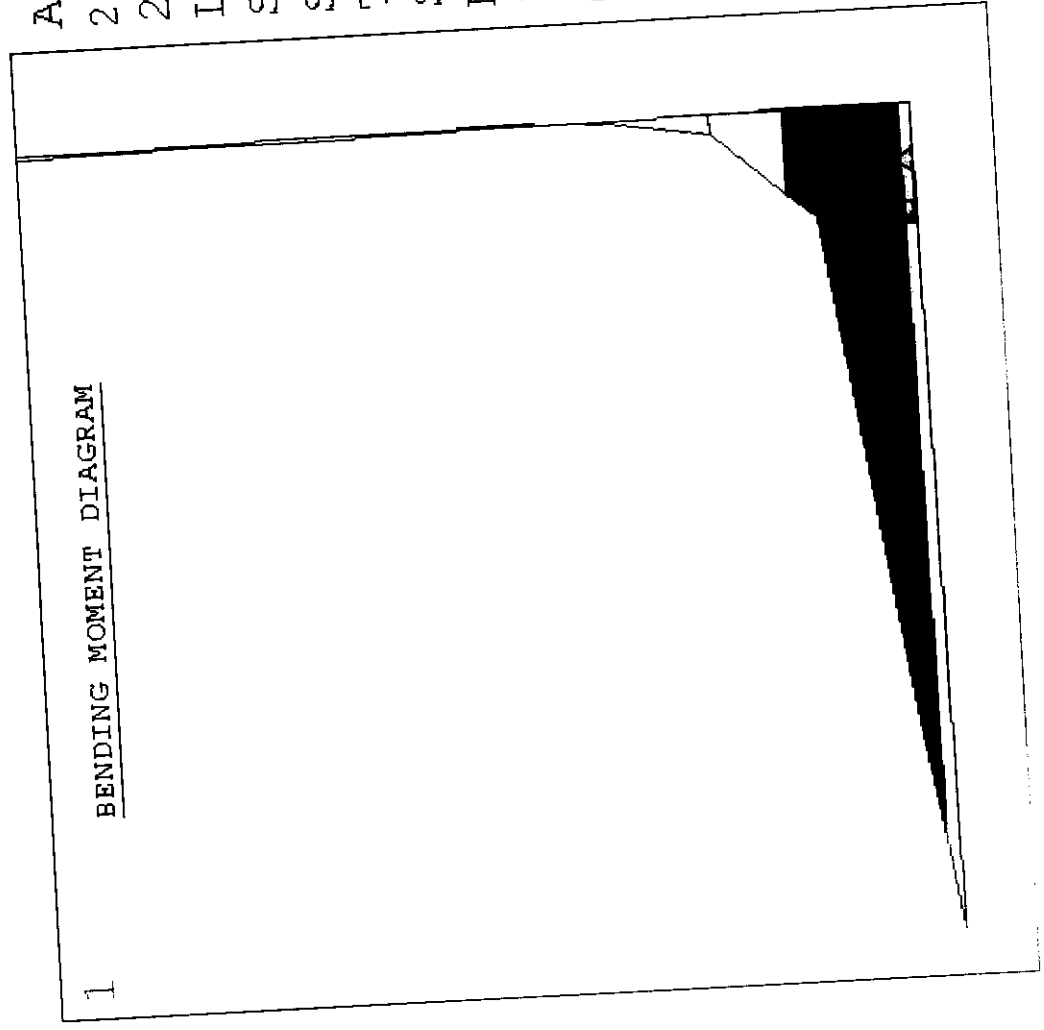




ANSYS 5.2  
27 FEB 1999  
22:33:13  
LINE STRESS  
STEP=1  
SUB =1  
TIME=1  
SMIS6 SMI  
MAX =.367E+  
ELEM=51  
-.501  
.363E  
.777E  
.119E  
.284E  
.326E  
.367E



BENDING MOMENT DIAGRAM



1

ANSYS 5.2

21:50:58

LINE STRESS

STEP=1

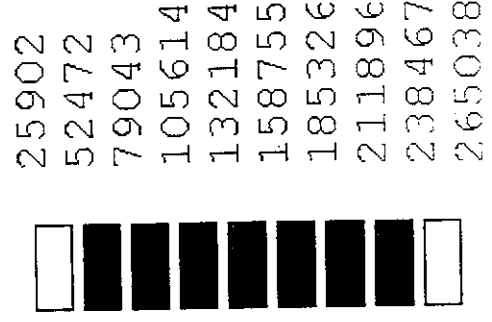
SUB =1

TIME=1

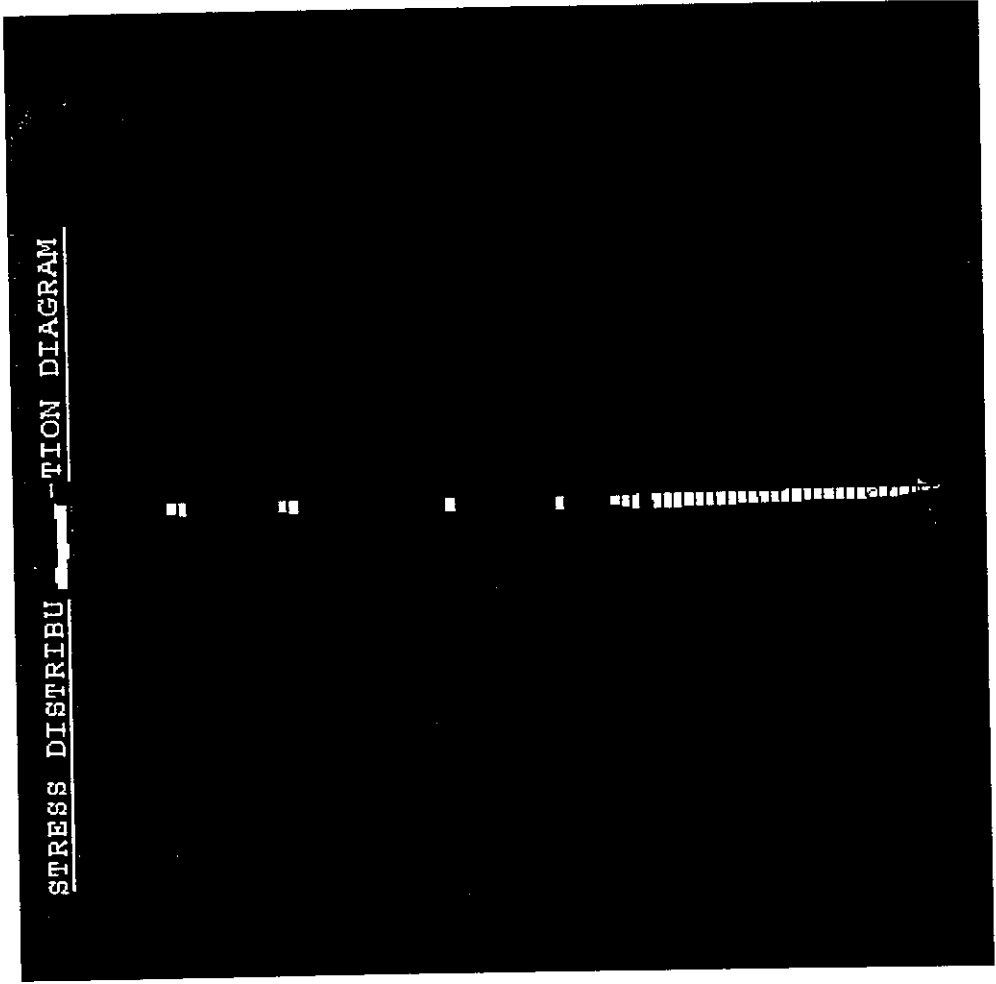
NMIS5 NMIS

MAX =265038

ELEM=10



STRESS DISTRIBUTION DIAGRAM



## **9. RESULT DISCUSSION AND CONCLUSION:**

In this project, we have designed the chimney structure in economical way which is not existing in practise ie. we have reduced the thickness of the steel plate used for the chimney based on the bending moment calculations. For this optimized structural design, we have conducted the stress analysis which also comes within the design criteria. Since we have developed C++ software for design calculation, it will be more easier, convenient and less time consuming rather than manual calculation.

Then, for this designed chimney structure, vibration test has been conducted. From the test analysis we found that the frequency of the vibrating structure increases as the deflection of the structure under loading increases and also it increases when the stiffness of the vibration structure increases.

So, from the analysis we conclude that the natural frequency of the system and the frequency under excited condition are not very much closely related. Hence there is no possibility for the occurrence of resonance in this designed structure under this condition.

## 10. REFERENCES AND BIBLIOGRAPHY:

1. Tall chimneys – Design and Construction  
by S. N. Manohar.
1. Design of steel structures - II  
by Ram Chandra.
2. Mechanical vibration  
by Singiresu S. Rao.
3. Engineering vibrations  
by Lydin S. Jacobsen  
Robert S. Ayne.
4. Finite element procedures  
by Klaus – Jurgen Bathe.
5. Finite element analysis in manufacturing engineering  
by Edward P. Champion, Jr.
6. Code of practice for design loads for buildings and structures
7. Theory of machines  
By - R.S. Khurumi