KUMARAGURU
college of technology
character is life

B.E DEGREE EXAMINATIONS: NOV/DEC 2022

(Regulation 2018)

Third Semester

(AE/AUE/CE/ME/MCE/EEE)

U18MAT3101: Partial Differential Equations and Transforms

COURSE OUTCOMES

CO1:	Form partial differential equations and solve certain types of partial differential equations
CO2:	Determine the Fourier Series and half range Fourier Series of a function
CO3:	Solve one dimensional wave equation, one dimensional heat equation in steady state using Fourier series.
CO4:	Apply Fourier series to solve the steady state two-dimensional heat equation in cartesian coordinates.
CO5:	Identify Fourier transform, Fourier sine and cosine transform of certain functions and use Parseval's identity to evaluate integrals.
CO6:	Evaluate Z – transform of sequences and inverse Z – transform of functions and solve difference equations.

Time: Three Hours	**Maximum Marks: 100**

Answer all the Questions: -

PART A (10 x 2 = 20 Marks)
(Answer not more than 40 words)

1.	Form the partial differential equation by eliminating a and b from $z = ax^n + by^n$.	CO1	[K3]
2.	Find the complete solution of $pq = y$.	CO1	[K3]
3.	State Dirichlet's condition on a Fourier series.	CO2	[K2]
4.	Find the root mean square value for the function $f(x) = x^2$ defined in $(0, \pi)$.	CO2	[K3]
5.	A rod 30 cm long has its ends A and B kept at $20^0 C$ and $80^0 C$ respectively until steady state conditions prevail. Find the steady state temperature in the rod.	CO3	[K3]
6.	What are the possible solutions of two-dimensional heat equations?	CO4	[K2]
7.	Write the Fourier transform pair.	CO5	[K2]
8.	State the convolution theorem for Fourier transforms.	CO5	[K2]
9.	Find $Z\left(\frac{1}{n}\right)$.	CO6	[K3]
10.	If $Z\{f(n)\} = F(z)$ then P.T $Z\{a^n f(n)\} = F\left(\frac{z}{a}\right)$	CO6	[K3]

11.	a)	Solve $x(z^2 - y^2)p + y(x^2 - z^2)q = z(y^2 - x^2)$.	6	CO1	[K4]
	b)	Solve $(D^2 + 4DD' - 5D'^2)z = sin(x - 2y) + 3e^{2x--y}$	10	CO1	[K4
12.	a)	Obtain the half range cosine series of $f(x) = x$ in $(0, \pi)$. Deduce $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^4} = \frac{\pi^4}{96}$	6	CO2	[K4]
	b)	Expand f (x) in a Fourier series upto 2nd harmonic using the following table x 0 1 2 3 4 5 $f(x)$ 9 18 24 28 26 30	10	CO2	[K4]
13.	a)	A string is stretched and fastened to two points $'l'$ apart. Motion is started by displacing the string in the form $y = k \, sin\frac{\pi x}{l}$ from which it is released at time t = 0. Find the displacement of any point at a distance x from one end at time $'t'$.	10	CO3	[K4]
	b)	A rod length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0^0 C and are kept at that temperature. Obtain the most general solution.	6	CO3	[K3]
14.	a)	A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite length without introducing an appreciable error. If the temperature at short edge y = 0 is given by $$u = \begin{cases} 20x , & 0 \le x \le 5 \\ 20(10 - x) , & 5 \le x \le 10 \end{cases}$$ and all the other three edges are kept at $0^0 c$, find the steady state temperature at any point of the plate.	16	CO4	[K4]
15.	a)	Find the Fourier Transforms of $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$. Hence evaluate $\int_0^\infty \frac{sin \, s - s \, cos \, s}{s^2} cos\left(\frac{s}{2}\right) ds$	8	CO5	[K4]
	b)	Find the Fourier Sine Transform of $e^{-ax} \, and \, e^{-bx}$ and hence show that $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$, by using Parseval's identity	8	CO5	[K4]
16.	a)	Solve $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ given that $y_0 = y_1 = 0$	10	CO6	[K4]
	b)	Verify initial value theorem for $f(n) = n^2 - n + 1$	6	CO6	[K4]

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