



B.E/B.TECH DEGREE EXAMINATIONS: DEC-2022

(Regulation 2018)

Third Semester

IT/ CSE/ ISE

U18MAT3102: Discrete Mathematics

COURSE OUTCOMES

CO1:	Understand the concepts of set theory and apply them to situations involving inclusion and exclusion.
CO2:	Acquire the knowledge of relations and analyse equivalence relations and their properties.
CO3:	Understand and analyse the properties of different kinds of functions.
CO4:	Apply mathematical induction to prove mathematical facts, analyse and use the concept of permutation and combination and solve recurrence relations.
CO5:	Evaluate the validity of logical arguments and construct simple mathematical proofs.
CO6:	Determine whether given graphs are isomorphic and apply Dijkstra's algorithm to find the shortest path.

Time: Three Hours

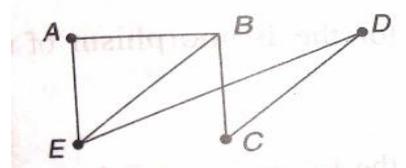
Maximum Marks: 100

Answer all the Questions: -

PART A (10 x 2 = 20 Marks)

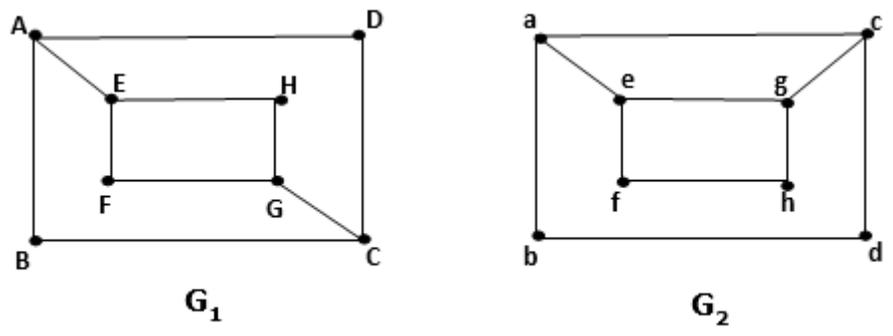
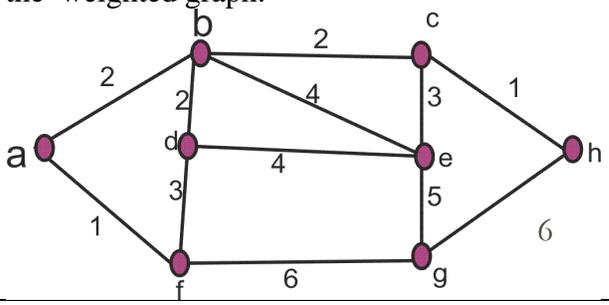
(Answer not more than 40 words)

1.	If $A = \{1, 2\}$ and $B = \{3, 4\}$, then find $(A \times B) \cup (B \times A)$.	CO1	[K ₂]
2.	If R and S be relations on a set A represented by the matrices $M_R = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}, M_S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ Find the matrices that represent (i) $R \cup S$ (ii) $R \cap S$.	CO2	[K ₃]
3.	If $f : R \rightarrow R$ and $g : R \rightarrow R$ are functions defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$, find $f \circ g$.	CO3	[K ₂]
4.	Define inverse of a function.	CO3	[K ₁]
5.	Find the number of ways in which 5 boys and 3 girls may be arranged in a row, so that all the girls may be together.	CO4	[K ₃]
6.	Obtain the number of permutations of all the letters of the word "ENGINEERING".	CO4	[K ₂]
7.	Write the statement in the symbolic form: "Every student in this school is either good at studies or good in sports".	CO5	[K ₃]
8.	Prove that $(P \rightarrow Q) \vee (Q \rightarrow P)$ is a tautology.	CO5	[K ₂]
9.	Define a complete graph and draw the complete graph K_5 .	CO6	[K ₁]

10.	Write the adjacency matrix for the given graph. 	CO6	[K ₂]
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**Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)**

11.	a) A survey of 500 television watchers produced the following information: 285 watch football games, 195 watch hockey games, 115 watch basketball games, 70 watch football and hockey games, 50 watch hockey and basketball games, 45 watch football and basketball games. 50 do not watch any of the three games. How many people watch exactly one of the three games?	8	CO1	[K ₄]
	b) If R is the relation on the set of integers such that $(a, b) \in R$ iff $3a+4b = 7n$ for some integer n, prove that R is an equivalence relation.	8	CO2	[K ₅]
12.	a) Let $A = \{1,2,3,4,5,6\}$ and $P_1 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 1 & 2 & 6 & 5 \end{bmatrix}$, $P_2 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 4 & 6 \end{bmatrix}$ $P_3 = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 3 & 2 & 5 & 4 & 1 \end{bmatrix}$ find (i) P_1^{-1} and P_2^{-1} (ii) $P_1^{-1} \circ P_2^{-1}$ (iii) $(P_2 \circ P_1) \circ P_3$ (iv) $P_3 \circ (P_2 \circ P_1)^{-1}$.	8	CO3	[K ₃]
	b) If $S = \{1,2,3,4,5\}$ and if $f, g, h: S \rightarrow S$ are given by $f = \{(1,2), (2,1), (3,4), (4,5), (5,3)\}$, $g = \{(1,3), (2,5), (3,1), (4,2), (5,4)\}$ and $h = \{(1,2), (2,2), (3,4), (4,3), (5,1)\}$, then prove that $h \circ (g \circ f) = (h \circ g) \circ f$.	8	CO3	[K ₅]
13.	a) Prove by Mathematical induction that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$	8	CO4	[K ₄]
	b) Find the formula for the general term F_n of the Fibonacci sequence 0,1,1,2,3,5,8,13	8	CO4	[K ₅]
14.	a) Obtain the PCNF and PDNF of $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$.	8	CO5	[K ₄]
	b) Give an argument which will establish the validity of the following inference. (Take the universe as the set of real numbers):	8	CO5	[K ₅]

		“All integers are rational numbers. Some integers are powers of 2. Therefore, some rational numbers are powers of 2.”			
15.	a)	Examine whether the two graphs are isomorphic. 	8	CO6	[K4]
	b)	Using Dijkstra’s Algorithm, find the shortest path between the vertex A and H in the weighted graph: 	8	CO6	[K5]
16.	a)	Prove that the premises $p \rightarrow q, q \rightarrow r, s \rightarrow \neg r$ and $p \wedge s$ are inconsistent.	8	CO5	[K4]
	b)	If $f: \mathbb{Z} \rightarrow \mathbb{N}$ is defined by $f(x) = \begin{cases} 2x - 1, & \text{if } x > 0 \\ -2x, & \text{if } x \leq 0 \end{cases}$. (i) Prove that f is one to one and onto. (ii) Determine f^{-1} .	8	CO3	[K3]
