Register Number:……………..

KUMARAGURU
college of technology
character is life

B.TECH. DEGREE EXAMINATIONS: APRIL / MAY 2023

(Regulation 2018)

Second Semester

ARTIFICIAL INTELLIGENCE AND DATA SCIENCE

U18MAT2001: Discrete Mathematics

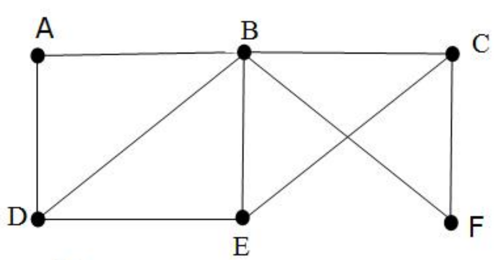
COURSE OUTCOMES

CO1: Understand the concepts of set theory and apply them to situations involving inclusion and exclusion.
CO2: Acquire the knowledge of relations, and analyse equivalence relations and their properties
CO3: Understand and analyse the properties of different kinds of functions and solve recurrence relations
CO4: Evaluate the validity of logical arguments and construct simple mathematical proofs.
CO5: Determine whether given graphs are isomorphic and apply Dijkstra's algorithm to find the shortest path

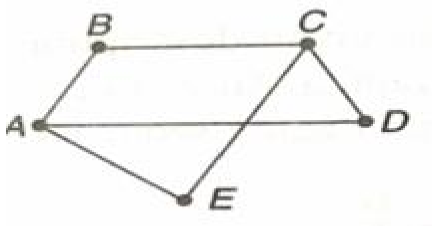
Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-
PART A (10 x 2 = 20 Marks)
(Answer not more than 40 words)

1. Determine the number of positive integers between 1 and 500 both inclusive, that are divisible by 2, 3 and 5 . CO1 [K2]
2. Let $A=\{1,2\}$, $B=\{2,3\}$, $C=\{4,5\}$, $D=\{5,6\}$. Determine $(A\times C)\cup(B\times D)$. CO1 [K2]
3. Check whether $R=\{(1,2),(2,3),(1,3),(2,1)\}$ is transitive. Justify your answer. CO2 [K2]
4. Let A ={2,3,5}, B = {6,8,10} and R be the relation defined by aRb if and only if '*a divides b*' find R and R^{-1} . CO2 [K2]
5. If $f:R\to R$ and $g:R\to R$, where R is the set of all real numbers, are defined by $f(x)=x-4,\ g(x)=x^3$. Find $f\circ g$. CO3 [K3]
6. Write down the Fibonacci sequence of numbers and the respective recurrence relation. CO3 [K2]
7. Check whether $(P\vee\neg P)\Leftrightarrow(Q\vee\neg Q)$ is a tautology or contradiction. CO4 [K2]
8. Symbolise the argument "All integers are rational numbers. Some integers are powers of 2. Therefore, some rational numbers are powers of 2". CO4 [K2]
9. Verify handshaking theorem for the graph CO5 [K3]



10. Is the following graph bipartite? Justify. CO5 [K4]



Answer any FIVE Questions:-
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) In a survey of 100 students, it was found that 40 studied Mathematics, 64 studied physics, 35 studied Chemistry. 1 studied all the three subjects. 25 studied mathematics and physics, 3 studied mathematics and chemistry and 20 studied physics and chemistry. Find the number of students who studied 8 CO1 [K3]

i) only one subject ii) only two subjects

iii) atleast one subject iv) none of these subjects.

b) If $A=\{x\in R/x\neq 1/2\}$ and $f:A\to R$ defined by $f(x)=\dfrac{4x}{2x-1}$. Check whether the function f is an invertible function. If so define f^{-1}. 8 CO3 [K3]

12. a) Prove that the relation 'Congruence modulo m' defined by $R=\{(x,y):x-y\,\text{is divisible by}\,m\}$ is an equivalence relation over the set of integers. 8 CO2 [K3]

b) Let $A=\{1,2,3\}$ and R, S be defined on A such that 8 CO2 [K3]

$$M_R=\begin{pmatrix}1&0&1\\0&1&1\\0&0&0\end{pmatrix},\quad M_S=\begin{pmatrix}0&1&1\\1&1&0\\0&1&0\end{pmatrix}$$ Find the matrices of

1) $R\cup S$ 2) $R\cap S$ 3) $\overline{R}$, R^{-1} 4) $R\circ S$

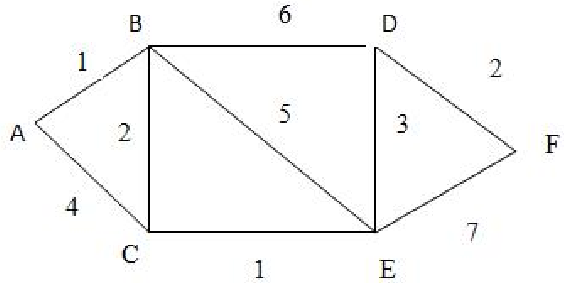
13. a) Solve the recurrence relation : $a_{n+1}-2a_n=5, n\geq 0, a_0=1$ 8 CO3 [K3]

b) Let A = {a,b,c,d,e,f} and P_1, P_2, P_3 be permutations defined from A to A 8 CO3 [K3]

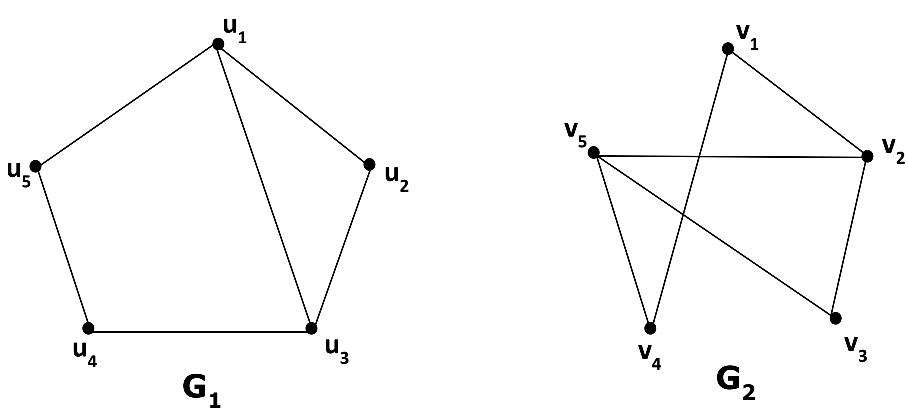
$$p_1=\begin{bmatrix}a&b&c&d&e&f\\c&d&a&b&f&e\end{bmatrix},\ p_2=\begin{bmatrix}a&b&c&d&e&f\\b&c&a&e&d&f\end{bmatrix},\ p_3=\begin{bmatrix}a&b&c&d&e&f\\f&c&b&e&d&a\end{bmatrix}$$

i) p_1^{-1} and p_2^{-1} ii) $p_1^{-1}\circ p_2^{-1}$ iii) $(p_2\circ p_1)\circ p_3$ iv) $p_3\circ(p_2\circ p_1)^{-1}$

14.	a)	Prove that $\forall x\big(P(x) \rightarrow (Q(y) \wedge R(x))\big), \exists x P(x) \Rightarrow Q(y) \wedge \exists x(P(x) \wedge R(x))$	8	CO4	[K_3]
	b)	Obtain the principal disjunctive and conjunctive normal forms: $$(p \rightarrow (q \wedge r)) \wedge (\neg p \rightarrow (\neg q \wedge \neg r))$$	8	CO4	[K_3]
15.	a)	Find the shortest path between the vertex A and F in the given Weighted Graph by using Dijkstra's Algorithm.	8	CO5	[K_4]



	b)	Can 5 houses be connected to 2 utilities without cross over of edges? If so draw the graph state and verify Euler's formula.	8	CO5	[K_4]
16.	a)	If R is the relation on the set of integers such that (a, b)∈R iff 3a+4b = 7n for some integer n, prove that R is an equivalence relation.	8	CO2	[K_3]
	b)	Examine whether the following two graphs are isomorphic.	8	CO5	[K_3]



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