Register Number:……..

KUMARAGURU
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B.E/B.TECH DEGREE EXAMINATIONS: NOV/DEC 2023

(Regulation 2018)

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

U18MAT4103: Probability and Random Processes

COURSE OUTCOMES

CO1: Analyze random or unpredictable experiments and investigate important features of random experiments and analyse various distributions.
CO2: Construct probabilistic models for observed phenomena through distributions.
CO3: Analyze various random processes with practical applications.
CO4: Analyze correlation related to various random processes and establish the properties of spectral densities.
CO5: Analyze linear time invariant systems performance for random inputs.

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions: -

PART A (10 x 2 = 20 Marks)

(Answer not more than 40 words)

1.	Find the chance that a non-leap year selected at random will contain 53 Tuesdays.	CO1	[K_2]
2.	Given example of a relation R on A = {1,2,3,4} which is neither symmetric nor antisymmetric.	CO1	[K_2]
3.	The mean of a binomial distribution is 20 and standard deviation is 4. Find the parameters of the distribution.	CO2	[K_2]
4.	If X & Y are two independent random variables with variance 2 & 3, then find Variance of 3X + 4Y.	CO2	[K_2]
5.	If the transition probability matrix of a Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$ find the steady-state distribution of the chain	CO3	[K_2]
6.	Determine whether the given matrix is irreducible or not $P = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8 \end{bmatrix}$.	CO3	[K_2]
7.	State any two properties of cross correlation function.	CO4	[K_1]
8.	Find the mean of the stationary process {X(t)} whose auto correlation function is given by	CO4	[K_2]

$R_{XX}(\tau) = 2 + 4e^{-2|\tau|}$.

9. Find the system transfer function, if a linear time invariant system has an impulse function CO5 [K_2]

$h(t) = \begin{cases} \frac{1}{2c}, & |t| \le c \\ 0, & |t| \ge c \end{cases}$

10. Define time-invariant system. CO5 [K_1]

Answer any FIVE Questions: -
PART B (5 x 16 = 80 Marks)
(Answer not more than 400 words)

11. a) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total output. Of their output 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product. Find the probability that it was manufactured by the machine B. 8 CO1 [K_3]

b) In a survey of 100 students, it was found that 40 studied Mathematics, 64 studied physics, 35 studied Chemistry. 1 studied all the three subjects. 25 studied mathematics and physics, 3 studied mathematics and chemistry and 20 studied physics and chemistry. Find the number of students who studied only chemistry and the number of students who studied none of these subjects. 8 CO1 [K_2]

12. a) A random variable X has the following probability distribution. 8 CO2 [K_2]

X	0	1	2	3	4	5	6	7
$P(x)$	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

Find (a) value of K (b) $P(X < 6)$ & $P(0 < x < 5)$ (c) CDF of X

b) The monthly breakdown of a machine is a random variable with Poisson distribution, with an average 1.8. Find the probability that the machine will function for a month (a) without breakdown (ii) with exactly one breakdown (c) With at least one breakdown. 8 CO2 [K3]

13. a) The joint distribution of X and Y is given by $P(x, y) = k(2x + 3y), x = 0,1,2$ & $y = 1,2,3$. Find the marginal distribution of X and Y, conditional distribution of X given Y=2 and P $(X + Y < 4)$. 8 CO2 [K2]

b) Trains arrive at a station at 15 minutes intervals starting at 4 a.m. If a passenger arrives at a station at a time that is uniformly distributed between 9.00 a.m. and 8 CO2 [K_3]

9.30 a.m. Find the probability that he has to wait for the train for (i) less than 6 minutes (ii) more than 10 minutes.

14.	a)	A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only 6 appeared. Find 1) The probability that he takes a train on the third day. 2) The probability that he droves to work in the long run.	8	CO3	[K_3]
	b)	Show that the random process $X(t) = A\cos(\omega t + \theta)$ is a wide sense stationary process if A and ω are constants and θ is a uniformly distributed random variable in $(0,2\pi)$.	8	CO3	[K_2]
15.	a)	Given the power spectral density of a continuous process as $S_{XX}(\omega) = \frac{\omega^2+9}{\omega^4+5\omega^2+4}$, find the mean square value of the process.	8	CO4	[K_2]
	b)	The auto correlation of the random binary transmission is given by $R_{XX}(\tau) = \begin{cases} 1-\frac{\lvert\tau\rvert}{T}, for\ \lvert\tau\rvert \le T \\ 0, \quad \lvert\tau\rvert > T \end{cases}$. Find the power spectrum.	8	CO4	[K_2]
16.	a)	A WSS process $X(t)$ with $R_{XX}(\tau) = Ae^{-a\lvert\tau\rvert}$ where A and a are real positive constants is applied to the input of system with system function $H(\omega) = \frac{1}{b+j\omega}$ where b is a real positive constant. Find the output spectral density of the system.	8	CO5	[K_2]
	b)	If $\{X(t)\}$ is a band limited process such that $S_{XX}(\omega) = 0$, when $\lvert\omega\rvert > \sigma$ prove that $2[R_{XX}(0) - R_{XX}(\tau)] \le \sigma^2\tau^2 R_{XX}(0)$.	8	CO5	[K_2]

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