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B.E. DEGREE EXAMINATIONS: APRIL / MAY 2010

Fourth Semester

U07MA401: NUMERICAL METHODS

(Common to B.E – Aeronautical Engineering, Civil Engineering, Electrical & Electronics Engineering, Mechatronics Engineering, Mechanical Engineering)

Time: Three Hours **Maximum marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 marks)

1. The rate of convergence of Newton-Raphson method is

 (a) 1 (b) 1.5 (c) 2 (d) 4

2. The method in which new value of the variable found is used immediately is

 (a) Gauss-Seidel (b) Gauss-Jacobi (c) Gauss-Jordan (d) Relaxation

3. The method of finding the unknown at the beginning of the table where the data is equally spaced is

 (a) Newton- forward (b) Newton-backward (c) Lagrange (d) Gauss-Seidel

4. When the data is unequally spaced, the unknown can be found by Lagrange's interpolation and

 (a) Newton-forward (b) Newton-backward (c) Newton-divided (d) Runge - Kutta 4

5. The method of finding integration in which number of intervals is even is

 (a) Trapezoidal (b) Simpson's $\frac{1}{3}$ (c) Simpson's $\frac{3}{8}$ (d) Romberg

6. The error in Simpson's $\frac{3}{8}$ rule is

 (a) 0 (b)h (c) h^2 (d) $\frac{3}{8}h^5$

7. The Euler's algorithm is

 (a) $y_{n+1} = y_n + hf(x_n, y_n)$ (b) $y_{n+1} = 2y_n + hf(x_n, y_n)$

 (c) $y_{n+1} = y_n - hf(x_n, y_n)$ (d) $y_{n+1} = y_n + \frac{3h}{8} f(x_n, y_n)$

8. In Runge-Kutta 4th order method formula, $y(x+h) = y(x) + \Delta y$, Δy is given by

 (a) $\Delta y = \frac{(k_1 + 2k_2 + 2k_3 + k_4)}{6}$ (b) $\Delta y = \frac{(k_1 + 2k_2 - 2k_3 + k_4)}{6}$

 (c) $\Delta y = \frac{(k_1 - 2k_2 + 2k_3 + k_4)}{6}$ (d) $\Delta y = \frac{(k_1 + 2k_2 + 2k_3 + k_4)}{8}$

9. The Bender-Schmidt formula is given by

(a) $u_{i,j+1}=\frac{(u_{i-1,j+1}+u_{i,j+1})}{2}$ (b) $u_{i,j+1}=\frac{(u_{i-1,j}+u_{i+1,j+1})}{2}$

(c) $u_{i,j+1}=\frac{(u_{i-1,j}+u_{i+1,j})}{2}$ (d) $u_{i,j+1}=\frac{(u_{i-1,j}+u_{i+1,j})}{6}$

10. The value of λ in the Crank-Nicolson's difference equation for the parabolic equation $u_t=\alpha^2 u_{xx}$ is given by

(a) $\frac{k\alpha^2}{h^2}$ (b) $\frac{k\alpha^2}{h}$ (c) $\frac{k\alpha^2}{h^3}$ (d) 0

PART B (10 x 2= 20 Marks)

11. By Newton's method, find an iterative formula to find $\sqrt{N}$ (where N is a positive number).

12. State fixed point theorem

13. Write the difference between Lagrange's formula and Newton's forward difference formula.

14. Define the terms interpolation and extrapolation.

15. Evaluate $\int_{-1}^{1}\frac{1}{1+x^2}dx$ by Gaussian two point formula.

16. What are the order of error in Trapezoidal and Simpson's rules of numerical integration?

17. Using Euler's method find y(0.2) from. $\frac{dy}{dx}=x+y$, y(0)=1 with h = 0.2

18. Write down the formula for Adam's-Bashforth method?

19. Name at least two numerical methods that are used to solve one dimensional heat equation.

20. Write down the standard five-point formula.

PART C (5 x 14 = 70 Marks)

21. (a) (i) Find a root of $x\log_{10}x-1.2=0$ by Newton Raphson method correct to 3 decimal places. (7)

(ii) Using Gauss Seidel method, solve the following system.

$$27x+6y-z=85$$
$$x+y+54z=110$$
$$6x+15y+2z=72$$
(7)

(OR)

(b) (i) Solve the following system of equations by Gauss elimination method.

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 5z = 7 \qquad (7)$$

(ii) Find the numerically largest eigen value of $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and the corresponding eigen vector. (7)

22. (a) (i) The following data are taken from the steam table (7)

Temp.(in deg)°	140	150	160	170	180
Pressure	3.685	4.854	6.302	8.076	10.225

Find the pressure of the steam for a temperature of $t = 142$.

(ii) Find $f(1)$ for the following data by Newton's divided difference formula. (7)

x	-4	-1	0	2	5
$f(x)$	1245	33	5	9	1335

(OR)

(b) (i) Use Newton's forward and backward formula to find $y(2)$ and $y(6)$ for the following data.

x	1	3	5	7
y	3	12	30	60

(7)

(ii) From the following table, compute $y(1.5)$, using cubic spline approximation. (7)

x	1	2	3
y	-8	-1	18

23. (a) (i) Obtain the value of $f'(0.5)$ using an appropriate formula for the given data. (7)

x	0	1	2	3	4
$f(x)$	1	1	15	40	85

(ii) Evaluate $\int_0^1 \frac{dx}{1+x}$ using the three point Gaussian Quadrature formula. (7)

(OR)

(b) (i) Evaluate $\int_0^1 \frac{dx}{1+x}$ by using Romberg's method. (7)

(ii) Evaluate $\int_1^2 \int_1^2 \frac{dxdy}{x^2+y^2}$ numerically with $h = 0.2$ along x-direction and along y-direction using Trapezoidal rule. (7)

24. (a) Given $y'' + xy' + y = 0$, $y(0) = 1$, $y'(0) = 0$, find the value of $y(0.1)$ using Runge-Kutta method of fourth order.

(OR)

(b) Determine the value of $y(0.4)$ using Milne's Predictor and Corrector method given $y' = xy + y^2$, $y(0) = 1$. Use Taylor series method to get the values of $y(0.1), y(0.2), y(0.3)$.

25. (a) (i) Solve $u_{xx} - 2u_t = 0$ given $u(0,t) = 0, u(4,t) = 0,$ $u(x,0) = x(4-x)$. Assume $h = 1$. Find the values of u up to $t = 5$. (7)

(ii) Solve $y_{tt} = y_{xx}$ up to $t = 0.5$ with a spacing of 0.1 subject to

$y(0,t) = 0, y(1,t) = 0, y_t(x,0) = 0$ and $y(x,0) = 10 + x(1-x)$ (7)

(OR)

(b) Solve the Poisson's equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = y = 0; x = y = 3$ with $u = 0$ on the boundary and mesh length 1 unit.

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