Register Number……………………

B.E. DEGREE EXAMINATIONS: APRIL / MAY 2010

Fifth Semester

COMPUTER SCIENCE AND ENGINEERING

U07MA404: Probability and Queuing Theory

Time: Three Hours **Maximum Marks: 100**

Answer ALL the Questions:-

PART A (10 x 1 = 10 Marks)

1. A coin is tossed four times, what is the probability that 'head' appears all the four times?

   a. $\frac{1}{2}$ b. $\frac{1}{4}$ c. $\frac{1}{8}$ d. $\frac{1}{16}$

2. If X is a continuous random variable with p.d.f. f(x) then the rth moment $E(x^r) =$

   a. $\int_0^{\infty} x^r f(x)\,dx$ b. $\int_0^{1} x^r f(x)\,dx$ c. $\int_{-\infty}^{\infty} x^r f(x)dx$ d. $\int_{-\infty}^{\infty} f(x)\,d(x)$

3. The mean of a binomial distribution is 4 and the variance is 2. The number of trails is

   a. 4 b. 8 c.16 d.32

4. If the mean of a poisson distribution is m, then the s.d. is m, then the s.d. is

   a. m b. m^2 c. $m^{\frac{1}{2}}$ d. $\frac{1}{m}$

5. If f(x,y)=2, 0≤x≤y≤1, then $f_x^{(x)}$ is given by

   a. $\int_0^1 f(x,y)dy$ b. $\int_0^y f(x,y)dx$ c. $\int_x^1 f(x,y)dy$ d. $\int_0^1 f(x,y)dx$

6. If X and Y are independent random variables with variances 2 and 3 respectively, then var (3X+4Y) =

   a. 18 b. 66 c. 17 d.59

7. If s and t are fixed then $\{X(s,t)\}$ is

   a. a variable b. a number c. a random variable d. none of these

8. If T is continuous and S is discrete then the random process is called

   a. Continuous random Sequence b. Discrete random process

   b. Continuous random process d. Discrete random sequence

9. The Traffic intensity is given by

   a. ρ b. $\frac{1}{\rho}$ c. $1-\rho$ d. $1+\rho$

10. In the multichannel system with limited capacity N, expected number of customers in the system is

    a. $E(m)+\frac{\lambda}{\mu}(1-P_N)$ b. $\frac{E(m)}{\lambda(1-P_N)}$ c. $\frac{E(m)}{\lambda(1+P_N)}$ d. $\frac{E(m)}{1-P_N}$

    where $E(m) =$ expected number of customers in the queue.

PART B (10 x 2 = 20 Marks)

11. What is the probability of throwing a six a least once in two throws of a single die?
12. Define moment generating function of a random variable X.
13. If 10 Coins are tossed 100 times, how many times would you expect 7 coins to fall head upwards.
14. Define Exponential distribution.
15. Define joint pdf for continuous random variables X and Y.
16. Define regression analysis.
17. Define wide-sense stationary process.
18. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train, but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Find the TPM of the Markov chain.
19. What is meant by Utilisation factor?
20. In M/M/1 Model, the arrival rate is 4/hr and the service rate is 5 per hr. What is the probability that there are at least 3 customers in the system.

PART C (5 x 14 = 70 Marks)

21. a. (i) A bag contains 8 red balls and 5 white balls. Two successive draws of 3 balls are made without replacement. Find the probability that the first draw will give three white balls and the second three red balls.

(ii) A continuous random variable has a pdf $f(x)=3x^2,\ \ 0 \le x \le 1$,

find a and b such that

(a) $P(x \le a) = P(x > a)$, and (b) $P(x > b) = 0.05$

(OR)

b. (i) In a bolt factory machines A, B and C manufacture respectively 25%, 35% and 40% bolts. Of the total of their output 5,4, and 2 present are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that it was manufactured by A, B and C?

(ii) Find the m.g.f of the random variable X has the pdf

$$f(x)=\begin{cases} x \ \ for \ \ 0 < x < 1 \\ 2-x \ \ \ for \ 1 < x < 2 \\ 0 \ \ otherwise \end{cases}$$

22. a. (i) Seven coins are tossed and number of heads noted. The experiment is repeated 128 times and the following distribution is obtained.

No. of heads:	0	1	2	3	4	5	6	7
Frequencies:	7	6	19	35	30	23	7	1

Fit a binomial distribution assuming that the coins are unbiased.

(ii) The probability that an individual suffers bad reaction from a certain serum is 0.001. Determine the probability that out of 2000 individuals. (a) exactly 3. (b) morethan 2 individuals will suffer a bad reaction. Use Poisson distribution.

(OR)

b. (i) X is a normal variate with $\mu=30$ and $\sigma=5$. Find

(i) $P(26 \le X \le 40)$ (ii) $P(X \ge 45)$

(ii) The mileage which car owners get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40,000 km. Find the probabilities that one of these tyres will last (a) at least 20,000 km and (b) at most 30,000 km.

23. a. (i) If (X, Y) is a two-dimensional random variable uniformly distributed over the triangular region R bounded by y=0, x=3 and $y=\frac{4}{3}x$. Find $f_x(x),\ f_y(y),\ E(x)\ E(y)$.

(ii) The lifetime of a certain brand of an electric bulb may be considered a random variable with mean 1200h. and standard deviation 250h. Find the probability, using central limit theorem, that the average life time of 60 bulbs exceeds 1250h.

(OR)

b. (i) Calculate the correlation coefficient between the height (inches) of father and son from the data given below:

Height of Father :	64	65	66	67	68	69	70
Height of Son :	66	67	65	68	70	68	72

(ii) For the following data, obtain the regression equations.

X	1	2	3	4	5	6	7
Y	4	5	8	6	4	7	8

24. a. (i) The process $\{X(f)\}$ whose probability distribution under certain conditions is given by

$$P\{x(t)\} = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,\ldots \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$ Show that it not stationary.

(ii) Let the two random processes $\{X(t)\}$ *and* $Y\{(t)\}$ be defined as $X(t) = A\cos wt + B\sin wt,\ Y(t) = B\cos wt - A\sin wt$, where A and B are random variables, w is a constant. If $E(A) = E(B) = 0,\ E(AB) = 0$ and $E(A^2) = E(B^2)$. Prove that $\{X(t)\}$ and $\{Y(t)\}$ are jointly wide-sense stationary.

(OR)

b. (i) Show that the process $\{X(t)\},\ X(t) = A\cos(wt + \theta)$ where A, w are constants and θ uniformly distributed in $[-\pi, \pi]$, is wide sense stationary.

(ii) Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

25. a. In a telephone booth, the arrivals follow Poission distribution with an average of 9 minutes between two consecutive arrivals. The duration of a telephone call is exponential with an average of 3 min. Find

(i) the probability that a person arriving at the booth has to wait.

(ii) The average queue length

(iii) The fraction of the day, the phone will be in use.

(iv) The company will install a second booth if a customer has wait for phone, for at least 4 minutes. If so, find the increase in the flow of arrivals in order that another booth will be installed.

(OR)

b. A supermarket has two sales girls. The service time for each customer is 4 minutes on the average and the arrival rate is 10 per hour. Find

(i) the probability that an arrival has to wait

(ii) The expected percentage of idle time for each girl.

(iii) The expected waiting time of a customer in the system.

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