

**B.E/ B.TECH DEGREE EXAMINATIONS: JUNE 2010**

First Semester

**U07MA101: MATHEMATICS –I**

(Common to All Branches)

**Time: Three Hours****Maximum Marks: 100****Answer ALL Questions:-****PART A (10 x 1 = 10 Marks)**

1. If the eigenvalues of the matrix  $P = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$  are 2, 3 and 6 then the

eigenvalues of the matrix  $Q = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 4 & -1 \\ 1 & -1 & 2 \end{bmatrix}$  are

A. -2, -3 and -6      B. 1, 2 and 5      C. 3, 5 and 3      D. 2, 4 and 2.

2. The sum of the eigenvalues of the associated symmetric matrix corresponding to a quadratic form  $2xy + 2yz + 2zx$  is equal to

A. 6      B. 4      C. 0      D. -6

3. Any two points on the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$  other than the prominent point

(1, 2, 3) are

A. (3, 5, 7) and (-3, -5, -7)      B. (3, 5, 7) and (-1, -2, -3)  
C. (3, 5, 7) and (5, 8, 11)      D. (3, 5, 7) and (-5, -8, -11)

4. The equation of a sphere concentric with the sphere

$x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$  and passing through the point (1, 1, 1) is

A.  $x^2 + y^2 + z^2 - 2x + 3y + 4z - 8 = 0$       B.  $x^2 + y^2 + z^2 - 2x + 3y + 4z + 8 = 0$   
C.  $2(x^2 + y^2 + z^2) - 2x + 3y + 4z - 7 = 0$       D.  $2(x^2 + y^2 + z^2) - 2x + 3y + 4z - 8 = 0$

5. If the tangent drawn to a curve at a point is perpendicular to X-axis then the formula to be applied to find the curvature  $k$  of the curve is

A.  $k = \frac{d^2x}{dy^2}$       B.  $k = \frac{d^2y}{dx^2}$       C.  $k = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$       D.  $k = \frac{1}{\frac{d^2y}{dx^2}}$

6. The evolute of the curve  $x^2 + y^2 + 4x - 6y + 3 = 0$  is

- A. the point (2,-3)      B. the point (-2,-3)  
 C. the point (-2, 3)      D. the point (2, 3)

7. If  $y \sin x = x \cos y$  then the value of  $\frac{dy}{dx}$  is

A.  $\left(\frac{y \cos x - \cos y}{\sin x + x \sin y}\right)$       B.  $-\left(\frac{x \cos y - \cos x}{\sin y + y \sin x}\right)$   
 C.  $-\left(\frac{y \cos x - \cos y}{\sin x + x \sin y}\right)$       D.  $\left(\frac{x \cos y - \cos x}{\sin y + y \sin x}\right)$

8. Which one of the following equations is **true**?

A.  $\frac{d^2y}{dx^2} = \frac{1}{\left(\frac{d^2x}{dy^2}\right)}$       B.  $\frac{\partial x}{\partial r} = \frac{1}{\left(\frac{\partial r}{\partial x}\right)}$       C.  $\frac{dy}{dx} = \frac{1}{\left(\frac{dx}{dy}\right)}$       D.  $\frac{\partial^2 x}{\partial r^2} = \frac{1}{\left(\frac{\partial^2 r}{\partial x^2}\right)}$

9. If the roots of the auxiliary equation corresponding to a fourth order homogeneous linear differential equation with constant coefficients are  $1, -1, i, -i$  then the differential equation is

A.  $(D^4 + 1)y = 0$       B.  $(D^4 - D)y = 0$       C.  $(D^4 - 1)y = 0$       D.  $(D^4 + D)y = 0$

10. The solution of the differential equation  $\frac{dy}{dx} + Py = Q$  where  $P, Q$  are

functions of  $x$  is given by

A.  $y = \int Qe^{\int Pdx} dx + C$       B.  $y = \int Qe^{-\int Pdx} dx + C$   
 C.  $y = e^{-\int Pdx} \left[ \int \left( \frac{Q}{e^{-\int Pdx}} \right) dx + C \right]$       D.  $y = e^{\int Pdx} \left[ \int \left( \frac{Q}{e^{\int Pdx}} \right) dx + C \right]$

**PART B (10 x 2 = 20 Marks)**

11. Show that the vectors  $(2, 3, 0)$ ,  $(1, 2, 0)$  and  $(8, 13, 0)$  are linearly dependent.
12. State Cayley Hamilton theorem.
13. Find the angle between the straight lines  $x + y - 2z = 0 = x - y + z + 1$  and  $x + y + 2z - 5 = 0 = 8x + 5y + 12z - 1$
14. Find the equation of the sphere **OABC** passing through the points  $O(0,0,0)$ ,  $A(1,0,0)$ ,  $B(0,-2,0)$  and  $C(0,0,5)$  also find its centre.
15. Find the radius of curvature at the point  $(r, \theta)$  of the curve  $r = a\theta$ .
16. Find the envelope of  $x - y \sin \theta = a \cos \theta$  where  $\theta$  is the parameter.
17. If  $x^y + y^x = c$  find  $\frac{dy}{dx}$
18. Find the Taylor series expansion of  $e^x \sin y$  near the point  $\left(-1, \frac{\pi}{4}\right)$  upto the second degree terms.
19. Solve  $(D^2 + D + 1)y = 0$ .
20. Solve by the variation of parameters method the equation  $\frac{dy}{dx} + y = x$ .

**PART C (5 x 14 = 70 Marks)**

21. (a) (i) Find the eigenvalues and eigenvectors of the matrix  $adj(A)$

$$\text{given } A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}. \quad (7)$$

(ii) Diagonalise the matrix  $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$  by means of an

orthogonal transformation. (7)

(OR)

(b) (i) Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}. \quad (7)$$

(ii) Reduce the Quadratic form  $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$  into canonical form by means of an orthonormal transformation and hence find its rank, index, nature and signature. (7)

22. (a) (i) Find the equation of the plane through the line of intersection of the planes  $2x + y - 2z = 5$ ,  $3x + 7y - 12z + 6 = 0$  and parallel to the line  $\frac{x-1}{2} = \frac{y+3}{3} = \frac{z-4}{5}$ . (7)

(ii) Show that the circles  $x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$ ,  $5y + 6z + 1 = 0$  and  $x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0$ ,  $x + 2y - 7z = 0$  lie on the same sphere and find its equation. (7)

(OR)

(b) Find the length and equation of the line of shortest distance between the

$$\text{lines } \frac{x+1}{2} = \frac{y+1}{3} = \frac{z+1}{4} \text{ and } \frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}.$$

23. (a) (i) Find the radius of curvature at any point  $\theta$  on the

$$\text{curve } x = a(\theta - \sin \theta), y = a(1 - \cos \theta) \quad (7)$$

(ii) Find the envelope of the straight lines  $\frac{x}{a} + \frac{y}{b} = 1$  where  $a^2 + b^2 = c^2$  (7)

(OR)

(b) (i) Find the radius of curvature of  $r = a(1 - \cos \theta)$ . (7)

(ii) Find the evolute of the parabola  $y^2 = 4ax$ . (7)

24. (a) (i) Find the Jacobian  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$  of the transformation  $x = r \sin \theta \cos \phi$ ,

$$y = r \sin \theta \sin \phi, \quad z = r \cos \theta. \quad (7)$$

(ii) A rectangular box open at the top is to have a volume 108 cubic meters, find its dimensions if its total surface area is minimum.

(7)

**(OR)**

(b) (i) Expand  $\cos x \cos y$  as a Taylor series in powers of

$$\left(x - \frac{\pi}{2}\right) \text{ and } \left(y - \frac{\pi}{2}\right) \text{ up to third degree terms.} \quad (7)$$

(ii) Find the maximum and minimum values of the function

$$f(x, y) = xy(a - x - y). \quad (7)$$

25. (a) (i) Solve  $(D^2 + 2D + 1)y = e^{-x} \cos x$ . (7)

(ii) Solve the equation  $(x^2 D^2 - xD - 2)y = x^2 \log x$ . (7)

**(OR)**

(b) (i) Solve the equation  $y'' + y = \tan x$  using the method of variation of parameters. (7)

(ii) Solve the following simultaneous equations  $\frac{dx}{dt} - y = t$ ;  $\frac{dy}{dt} + x = t^2$  (7)

\*\*\*\*\*