Register Number: ……………………

M.C.A DEGREE EXAMINATIONS: JUNE 2010

Second Semester

MASTER OF COMPUTER APPLICATIONS

MAT509: Mathematical Foundations of Computer Science

Time: Three Hours **Maximum Marks: 100**

Answer All Questions:-

PART A (10 x 2 = 20 Marks)

1. State the condition in terms of ranks for a system of equations AX = B to possess a solution.

2. If 3 and 6 are the eigen values of A = $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, write down all the eigen values of A^{-1}.

3. State De-Morgan laws of set theory.

4. Write down the matrix of the relation R = $\{(1,1)(1,3)(2,2)(3,3)\}$ on A = {1, 2, 3}.

5. Construct the truth table for $P \to (P \wedge Q)$.

6. Define Universal quantifier with an example.

7. Construct a grammar generating the language $L = \omega \subset \omega^R$, where $\omega \in \{a, b\}^*\}$

8. What are the types of a phrase structure grammar?

9. Differentiate between Deterministic and Non-deterministic finite automata.

10. Construct a NFA for the language $L = \{a^n b / n \geq 1\}$.

PART B (5 x 16 = 80 Marks)

11. (a) (i) Test for consistency and solve if consistent, the set of equations

$x + 2y - z = 3$, $3x - y + 2z = 1$, $x + 2y + 3z = 2$, $x - y + z = -1$. (8)

(ii). Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$ (8)

(OR)

(b) (i) Find the eigen values and the corresponding eigen vectors of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ (10)

(ii) Find the rank of the matrix $A = \begin{bmatrix} 3 & 5 & 2 & 4 \\ 1 & 2 & 5 & 7 \\ 6 & 10 & 4 & 8 \\ 4 & 7 & 7 & 11 \end{bmatrix}$ (6)

12. (a) (i) Among 100 students, 32 study FORTRAN, 20 study PASCAL, 45 study C language, 15 study FORTRAN & C, 7 study FORTRAN & PASCAL, 10 study PASCAL & C and 30 do not study any of the three languages. Find the number of students studying all three languages. (8)

(ii) Prove that the relation $R = \{(1,1)(1,3)(2,2)(2,4)(3,1)(3,3)(4,2)(4,4)\}$ on A={1,2,3,4} is reflexive, symmetric and transitive. Also give the matrix of R. (8)

(OR)

(b) (i) If A,B,C are any three sets, show that $(A - B) - C = A - (B \cup C)$. (8)

(ii) If f(n)= n+2, $g(n)=n^2$, h(n) = 3n-1 defined on Z^+ then find (i) fog (ii) fo (goh) (iii) hoh (iv) hof. (8)

13. (a) (i) Prove that $(A \to B) \wedge (B \to C) \Rightarrow 7A \vee C$ (8)

(ii) Determine whether $(p \to q) \to r$ and $(p \wedge 7q) \to r$ are logically equivalent. (8)

(OR)

(b) (i) Obtain the PCNF and PDNF of $(P \leftrightarrow Q) \wedge (P \vee R)$ (8)

(ii)Using direct proof, show that $P \to Q, 7Q \vee R, 7(R \wedge 7S), P \Rightarrow S$ (8)

14. (a) (i) Derive the string 'aabbabba' given

$S \to aB/bA;\ A \to a/aS/bAA;\ B \to b/bS/aBB.$ (8)

(ii) State Pumping Lemma for Regular languages. Also prove that the set $\perp = \{a^n b a^n / n \geq 1$ is not regular. (8)

(OR)

(b) (i) Construct a phrase structure grammar G which generates the set of all strings over {0,1}containing an equal number of 0's and 1's. (8)

(ii) Show that the grammar G = (N, T, P, S) where N ={S,A}, T={0,1,2} and P = {S $\to$ 0SA2, S $\to$ 012, $2A \to A2$, $1A \to 11\}$ generates the language L(G) = $\{0^n 1^n 2^n / n \geq 1\}$. (8)

15. (a) (i) Construct deterministic finite automata accepting the set of all strings ending in 00 over the alphabet {0,1}. (8)

(ii) Draw the transition diagram of the NFA M = (Q, ∑, δ, q_0 , F) where

Q = {q_0, q_1, q_2, q_3, q_4}, ∑ = {0, 1}, F = {q_2, q_4} and δ is defined by

States	Inputs 0	Inputs 1
q_0	{q_0, q_3}	{q_0, q_1}
q_1	ϕ	{q_2}
q_2	{q_2}	{q_2}
q_3	{q_4}	ϕ
q_4	{q_4}	{q_4}

Also write the regular grammar accepted by this NFA (8)

(OR)

(b) (i) Construct DFA equivalent to the NFA M =({p,q,r,s}, {0,1}, δ, p, {s}) where δ is given by the following table. (10)

State \ Inputs	0	1
p	p,q	p
q	r	r
r	s	ϕ
s	s	s

(ii) Construct a NFA recognizing L(G), where G is the grammar

S ⟶ aS / bA / b : A⟶ aA / bS / a (6)

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