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M.E. DEGREE EXAMINATIONS: DECEMBER 2009

First Semester

COMPUTER SCIENCE AND ENGINEERING

MAT507: Stochastic Models and Simulation

Time: Three Hours **Maximum Marks: 100**

Answer All the Questions:-

PART A (10 x 2 = 20 Marks)

1. Prove that P ($\bar{A}$) =1-P (A)
2. If P (A∪ B) =3/4, P (A∩ B) =1/4, P ($\bar{A}$) =2/3, find P ($\bar{A}$/B).
3. A continuous random variable X has p.d.f f(x) =kx^4, $1 \leq X \leq 2$ find k.
4. A binomial variable X satisfies the relation P(x=1) = P(x= 2), n=4, find the mean.
5. Define a Markov chain.
6. Define the term branching processes.
7. Define the term counting process.
8. Write two postulates for Poisson process.
9. What is renewal period of the process?
10. What are the basic characteristics of a queuing system?

PART B (5 x 16 = 80 Marks)

11\. a. (i) If A and B are any two events, then show that

P (A∪ B) =P (A) +P (B) – P (A∩ B). (8)

(ii) Three urns contain 1 white, 2 black balls; 3 white, 1 black balls; 2 white, 3 black balls respectively. One ball is taken from each urn. What is the probability that among the balls drawn, there are 2 white and 1 black balls? (8)

(OR)

b. (i) If A and B are two independent events, then show that $\bar{A}$ and B are also independent. (8)

(ii) An urn contains 5 balls. Two balls are drawn and are found to be white. What is the probability that all of the balls being white. (8)

12\. a. (i) A continuous r.v x has the p.d.f: f(x) = C $e^{-|x|}$: $-\infty < x < \infty$.Find C and C.D.F of f(x). (8)

(ii) Find the M.G.F, mean and variance of the Poisson distribution. (8)

(OR)

b. (i) A continuous r.v X has the distribution function $F(x) = \begin{cases} 0 & : x \leq 1 \\ K(x-1)^4 & : 1 \leq x \leq 3 \\ 1 & : x > 3 \end{cases}$

Find (i) K (ii) p.d.f f(x) (iii) P(x<2) (8)

(ii) The daily consumption of milk in a city in excess of 20,000 liters is approximately exponentially distributed. The average excess in consumption of milk is 3000 liters. The city has a daily stock of 35,000 liters. What is the probability that, of two days selected at random, the stock is insufficient for both the days. (8)

13. a. Explain the concept of Gambler's ruin problem, through an example.

(OR)

b. (i) If $\{X_n, n = 0,1,2 \ldots\}$, $X_0 = 1$ is a branching process with offspring distribution p.g.f P(s), Show that $\{X_{rk}, r = 0, 1, 2 \ldots\}$, where k is a fixed positive integer, is also a branching process.

(ii) Derive the Chapman- kolmogorov equation for the transition density $f_x(x_3/x_1)$. (8)

14. a. Given that only one occurrence of a Poisson process N (t) has occurred by epoch T, show that $P\{t < \gamma < t + dt \mid N(T) = 1\} = dt/T,\ 0 < t < T$.

(OR)

b. Suppose that the customers arrive at a bank according to Poisson process with mean rate of 'a 'per minute. Then the number of customers N (t) arriving in an interval of duration t minutes follows Poisson distribution with a mean 'at'. If the rate of arrival is 3 per minute, then in an arrival of 2 minutes, find the probability that the number of customers arriving is (i) exactly 4 (ii) greater than 4 (iii) less than 4.

15. a. Show that the renewal functions of a renewal process $\{N(t), t \geq 0\}$ generated by X_n, n =1, 2, 3 … is given by $E(N(t)) = M(t) = \frac{t}{\mu} + \frac{\sigma^2 - \mu^2}{2\mu^2} + O(1)$, where μ ($< \infty$) and σ^2 are the mean and the variance of X_n. Further, show that $Var(N(t)) = (\frac{\sigma^2}{\mu^3})t + O(t)$.

(OR)

b. Customers arrive at a shoe shine shop according to a Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes.

(i) Find the average number of customers L_s in the shop

(ii) Find the average time a customer spends in the shop W_s

(iii) Find the average number of customers in the queue L_q

(iv) What is the probability that the server is idle?

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