

M.E DEGREE EXAMINATIONS: JANUARY 2011

First Semester

ENERGY ENGINEERING

MAT504: Applied Mathematics for Energy Engineering

Time: Three hours

Maximum Marks: 100

Answer all Questions:-

PART A (10 x 2 = 20 Marks)

1. If $F[f(x)] = F(s)$, Show that $F[e^{iax} f(x)] = F(s + a)$
2. Find the finite Fourier sine transform of 1 in $(0, \pi)$
3. Define the term functional.
4. Define the term extremal.
5. Define the term flux lines.
6. State the Schwarz- Christoffel transformation.
7. Write the order of convergence of the Newton- Raphson method.
8. State the condition for convergence of the Gauss – Seidal method.
9. Write down the standard five point formula in solving the Laplace equation.
10. Write down the Bender – Schmidt recurrence equation.

PART B (5 x 16 = 80 Marks)

11. a) Solve the diffusion equation : $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$: $-\infty < x < \infty$, $t > 0$ with conditions: $u(x, 0) = f(x)$
and $u, \frac{\partial u}{\partial x}$ tend to zero as x tend to $\pm \infty$.

(OR)

- b) Solve $\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}$, $0 < x < \infty$, $t > 0$ $u(0,t) = u_0$, $t > 0$; $u(x,0) = 0$, $0 < x < \infty$; $u, u_x \rightarrow 0$
as $x \rightarrow \infty$

12. a) (i) On what curve can the functional $I(y(x)) = \int_1^2 (y'^2 - 2xy) dx$, $y(1) = 0$ and $y(2) = -1$
attain an extremum? (8)
- (ii) Find the extremal of the functional $I = \int_0^1 (y'^2 + z'^2) dx$, $y(0) = 0$, $z(0) = 0$, $y(1) = 1$,
 $z(1) = 2$. (8)

(OR)

- b) Using Ritz method find an approximate solution to the problem of the minimum functional $J(y) = \int_0^1 (y'^2 + y^3) dx$, $y(0) = 4$, $y(1) = 1$.

13. a) (i) Find the complex potential due to a source at $z = -a$ and sink at $z = a$ of equal strengths k .
(ii) Determine the equipotential lines and stream – lines and represent graphically.
(iii) Find the speed of the fluid at any point.

(OR)

- b) Discuss the motion of a fluid having complex potential $\Omega(z) = V_0 \left(z + \frac{a^2}{z} \right) + \frac{i\gamma}{2\pi} \ln(z)$

14. a) (i) Find a positive root of $xe^x = 3$ by method of false position, root lies between 1 and 2. (8)

- (ii) Using power – method find the dominant eigen values and the corresponding eigen vector of

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(8)

(OR)

- b) (i) Find a positive root of $3x = \cos x + 1$ by Newton – Raphson method, root lies between 0 and 1. (8)

- (ii) Using Gauss – Seidel method solve

$$30x - 2y + 3z = 75$$

$$2x + 2y + 18z = 30$$

$$x + 17y - 2z = 48$$

(8)

15. a) Solve $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$, given $u(0, t) = u(4, t) = 0$, $u(x, 0) = x(4-x)$, assuming $h = k = 1$,

Compute u upto $t = 5$.

(OR)

- b) Solve $\nabla^2 u = 0$ in the square region bounded by $x = 0$, $x = 4$, $y = 0$, $y = 4$ with boundary conditions $u(0, y) = 0$, $u(4, y) = 8 + 2y$, $u(x, 0) = \frac{x^2}{2}$ and $u(x, 4) = x^2$ taking $h = k = 1$.
