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B.E DEGREE EXAMINATIONS: NOV/DEC 2010

Seventh Semester

AERONAUTICAL ENGINEERING

U07ARE06: Finite Element Method

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. The sum of shape function is equal to

a) Two b) Zero c) One d) four

2. Stiffness matrix (K) for a one dimensional bar element.

a) $\frac{AE}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ b) $\frac{AE}{L}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ c) $\frac{A}{EL}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ d) $\frac{A}{EL}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

3. Give the general expression for element stiffness matrix.

a) $[K] = \int B^T[D][B]\,dA$ b) $[K] = \int B^T[D][B]\,dV$ c) $[K] = \int [B][D]^T[B]\,dV$ d) $[K] = \int [B][D]^T[B]\,dA$

4. Which one is variational method.

a) Galerkin's method b) Least square method

c) Rayleigh-Ritz method d) Subdomain or Point collocation method

5. For plane strain problem, the stresses satisfy the condition.

a. $\tau_{xz} = \tau_{yz} = \sigma_z = 0$ b. $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \gamma(\sigma_x + \sigma_y)$

c. $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \gamma\tau_{xy}$ d. $\tau_{xz} = \tau_{yz} = 0, \sigma_z = \gamma(\sigma_x + \sigma_y) + (1-\gamma)\tau_{xy}$

6. Which one of the plane strain condition is correct for a long body of uniform cross section subjected to transverse loading along the length?

a) $\varepsilon_y, \gamma_{xz}$, and γ_{yz} are zero b) $\varepsilon_x, \gamma_{xz}$, and γ_{yz} are zero

c) $\varepsilon_z, \gamma_{xz}$, and γ_{yz} are zero d) $\varepsilon_z, \gamma_{xy}$, and γ_{yz} are zero.

7. Jacobian of transformation matrix [J] is

a) $\begin{bmatrix}\frac{\partial y}{\partial \xi} & \frac{\partial x}{\partial \xi}\\ \frac{\partial y}{\partial \eta} & \frac{\partial x}{\partial \eta}\end{bmatrix}$ b) $\begin{bmatrix}\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}\\ \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi}\end{bmatrix}$ c) $\begin{bmatrix}\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi}\\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta}\end{bmatrix}$ d) $\begin{bmatrix}\frac{\partial y}{\partial \eta} & \frac{\partial y}{\partial \xi}\\ \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \xi}\end{bmatrix}$

8. The stiffness matrix equation for four noded iso-parametric quadrilateral elements.

a) $\int_{-1}^{1}\int_{-1}^{1}[B]^T[D]X|J|\,X\,\partial\varepsilon\,X\,\partial\eta$ b) $t\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]X\,\partial\varepsilon\,X\,\partial\eta$

c) $t\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]X|J|\,X\,\partial\varepsilon\,X\,\partial\eta$ d) $\int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]X|J|\,X\,\partial\varepsilon\,X\,\partial\eta$

9. Stiffness matrix equation for one dimensional heat conduction element is

a) $\frac{AK}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ b) $\frac{AK}{L}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ c) $\frac{A}{KL}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ d) $\frac{A}{KL}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

10. Stiffness matrix for the torsional bar is

a) $\frac{G}{JL}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ b) $\frac{GJ}{L}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}$ c) $\frac{J}{GL}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ d) $\frac{GJ}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$

PART B (10 x 2 = 20 Marks)

11. State the principle of minimum potential energy
12. State the properties of stiffness matrix.
13. Define shape function.
14. Write down the stress-strain relationship matrix for plane stress condition.
15. Explain the important properties of CST element.
16. Write down the strain displacement matrix for two dimensional CST elements.
17. What are serendipity elements?
18. Define the sub-parametric, iso-parametric and super parametric elements.
19. Write down the stiffness matrix equation for four noded iso-parametric quadrilateral element.
20. Write down the finite element equation for one dimensional heat conduction with free end convection

PART C (5 x 14 = 70 Marks)

21. a) (i) Derive the interpolation function for one dimensional three noded quadratic bar element in local coordinate system.

(ii) Derive the strain displacement matrix for one dimensional three noded quadratic bar element.

(OR)

b) For a tapered bar of uniform thickness t = 10 mm as shown in fig1. Find the displacement at the nodes by forming into two element model. The bar has mass density ρ= 7800kg/m^3, Young's modulus E= 2x10^5 MN/m^2. In addition to self-weight, the plate is subjected to a point load P=10kN at its centre. Also determine the reaction forces at the support.

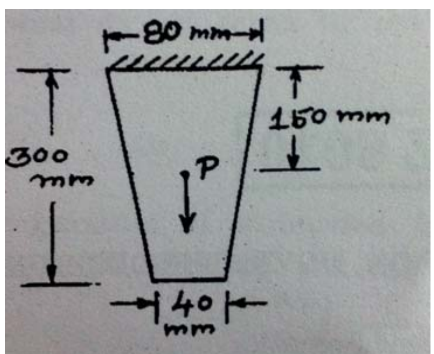

Fig. (1)

22. a) Consider the bar with an axial load P= 200KN is applied as shown in Fig 2.

(i) Determine the nodal displacements.

(ii) Determine the stress in each material.

(iii) Determine the reaction forces.

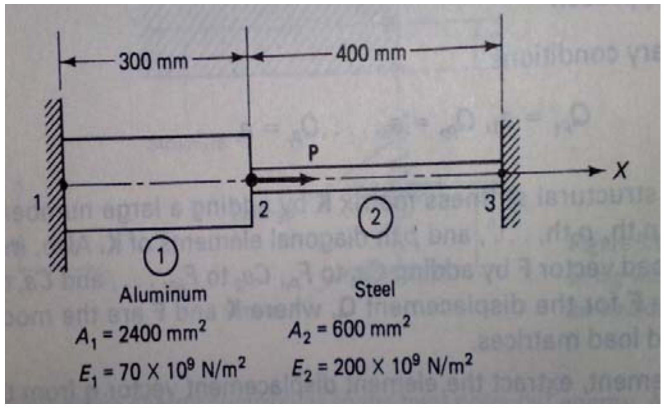

Fig. 2

(OR)

b) Consider the four- bar truss shown in Fig 3. It is given that E= 200Gpa and A_e=1cm^2. For all element.

(i) Determine the elemental stiffness matrix for each element.

(ii) Assemble the structural stiffness matrix K for the entire truss.

(iii) Using the elimination approach, solve for the nodal displacement.

(iv) Recover the stress in each element.

(v) Calculate the reaction forces.

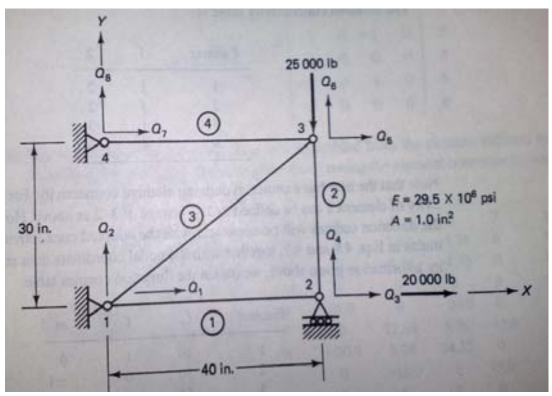

Fig 3.

23. a) Evaluate the stiffness matrix for the element shown in Fig. 4 Thickness t = 1 cm cross sectional area A = 1 cm^2, E=200Gpa, and Poisson's ratio γ=0.25.Assume the plane stress condition and also find the elemental stresses.

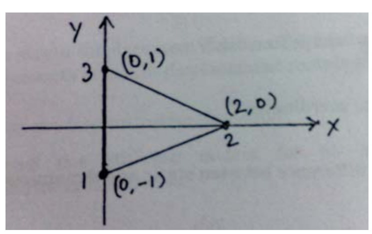

Fig. 4

(OR)

b) For the plane stress element shown in Fig. 5 the nodal displacement are.

u_1 = 0.005mm v_1= 0.002mm

u_2= 0.0mm v_2= 0.0mm

u_3 = 0.005 mm v_3=0.0mm

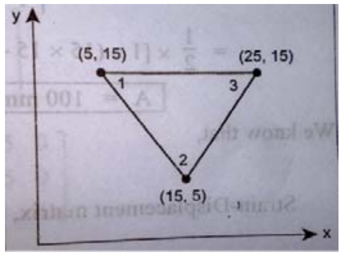

Fig. 5

Determine the elemental stress σ_x ,σ_y, τ_{xy},σ_1 ,σ_2 and the principle angle θp, let E= 70Gpa, and Poisson's ratio=0.3 and use unit thickness for plain strain. All co-ordinates are in millimetre.

24. a) Evaluate the jacobian matrix for the isoparametric quadrilateral element shown in Fig 6.

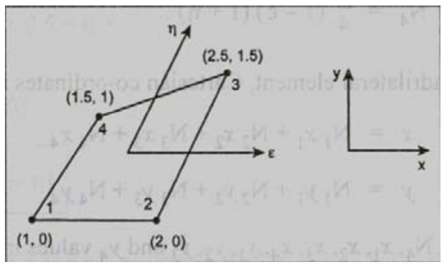

Fig. 6

(OR)

b) Derive the element stiffness matrix for a linear isoparametric quadrilateral element.

25. a) Calculate the temperature distribution in a one dimensional fin with physical properties given in Fig 7. the fin is rectangular in shape and is 120mm long , 40mm wide and 10 mm thick. Assume that convection heat loss occurs from end of the fin. Use two elements. Take k=0.3W/mm℃ , h= $1x10^{-3}$ W/mm^2℃ , Tα = 20℃ .

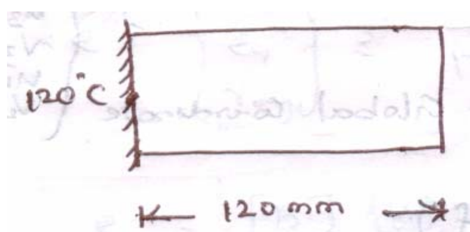

Fig. 7

(OR)

b) A cantilever beam is subjected to uniformly distributed load. Using Rayleigh Ritz method, find the maximum deflection of the beam by considering the mode function as

$$V = C_1 \left[1 - \cos\frac{\pi x}{2l}\right]$$

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