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B.TECH DEGREE EXAMINATIONS: NOV/DEC 2010

Seventh Semester

INFORMATION TECHNOLOGY

U07MA503: Discrete Mathematics

Time: Three Hours **Maximum Marks: 100**

Answer all the Questions:-

PART A (10 x 1 = 10 Marks)

1. Let p: You drive over 75 kms per hour; q: You set a speeding ticket. Then the logical equivalent of the statement "Whenever you get a speeding ticket, you are driving over 75 kms per hour" is

   A. $p \to q$ B. $\sim p \to \sim q$ C. $p \to \sim q$ D. $q \to p$.

2. The contrapositive of the proposition $p \to q$ is

   A. $q \to p$ B. $\sim q \to \sim p$ C. $\sim q \to p$ D. $\sim p \to \sim q$

3. Let B(x): x is a bird; F(x): x can fly. Then the logical equivalent of the statement "Not all birds can fly" is

   A. $\sim[(x)(B(x) \to F(x))]$ B. $\sim (x)(B(x) \to F(x))$ C. $\sim(x)B(x) \to F(x)$ D. $(x)B(x) \to \sim F(x)$.

4. Let Q(x, y) denote the statement " x + y= x-y" . If the universe of discourse for both variables consists of all integers then which of the following statements has truth value true?

   A. Q(1, 1) B. Q(2, 0) C. (y) Q(1, y) D. (y)Q(2, y)

5. Let A and B be any two sets. Then $A-(A \cap B)$ is the same as

   A. B B. A-B C. B-A D. $A \cup B$.

6. Let A, B, C be any three sets with n(A) = 4; n(B) = 3; and n(C) = 2 and B and C are disjoint.

   Then $n(A \times (B \cup C)$ is

   A. 6 B.12 C.18 D. 20

7. The condition for the existence of inverse of a function is

   A. Injective but not surjective B. both injective and surjective
   C. surjective and not injective D. Neither surjective nor injective

8. The function which describes the membership of an element in a set is

   A. Identity function B. Constant function C. Idempotent function D. Characteristic function

9. Let N be the set of all natural numbers. Then which of the following is not true about (N, +).

   A. Algebraic system B. Semi-group C. Monoid D. Group.

10. Which of the following is not true for a group (G, *).

    A. The order of G is finite.
    B. Identity element is unique
    C. Identity element is the only idempotent element.
    D. The inverse of every element is unique.

PART B (10 x 2 = 20 Marks)

11. Verfify whether the proposition $(p \wedge q) \wedge \sim(p \vee q)$ is a contradiction without using truth table.

12. If $H_1, \ldots, H_n$ and P imply Q then prove that $H_1, \ldots, H_n \Rightarrow P \to Q$.

13. Translate the statement $(x)(y)\ ((x > 0) \wedge (y<0) \to (xy<0)$ into English where the Universe of discourse for both variables consists of real numbers.

14. Let Q(x, y) denote "x +y =0". What are the truth values of the quantifications

    (i) $(\exists y)\ (x)\ Q(x, y)$ (ii) $(x)(y)\ Q(x, y)$

    Where the universe discourse consists of real numbers.

15. Prove that for any two sets A and B , if $A \subseteq B$ then $B^c \subseteq A^c$.

16. Draw the Hasse diagram of the partially ordered set (P(S), $\subseteq$) where P(S) is the power set of a set S and S = {a, b, c}.

17. Show that the function f: $R \to R$ given by f(x) = cosx for all $x \in R$ is neither one-one nor onto.

18. Distinguish between a relation and a function.

19. Find all the right cosets of H= {1, -1} in the multiplicative group G= { 1, -1, i, -i}.

20. Define Hamming distance.

PART C (5 × 14 = 70 Marks)

21. (a) (i) Establish the equivalence $\sim(P \wedge Q) \to (\sim P \vee (\sim P \vee Q)) \Leftrightarrow \sim P \vee Q$ without using truth table. (7)

    (ii) If there was a meeting then catching the bus was difficult. If they arrived on time then catching the bus was not difficult. They arrived on time. Therefore there was no meeting. Prove that the above statements constitute a valid argument. (7)

(OR)

(b) (i) Obtain the pdnf and pcnf of pv (Tp .$\to$(qv (Tq $\to$ r)))

(ii) Check the validity of the following arguments:

If the band could not play rock music or the refreshments were not delivered on time, then the new years party would have been canceled and Alice would have been angry. If the party were cancelled, then the refunds would have to be made. No refunds were made. Therefore the band could play rock music. (7)

22. (a) (i) Rewrite the following using the quantifiers, Variables and predicate symbols

    (1) Some men are genius.

    (2) Some numbers are not rational

    (3) There is a student who likes Mathematics but not geography. (7)

    (ii) Prove that $(x)p(x) \vee (x)q(x) \Rightarrow (x)(p(x) \vee q(x))$. (7)

(OR)

(b) Show the following implication by indirect method of proof

$(x)\,(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$.

23. (a) (i) Prove that $(A\times B) \cap (R\times S) = (A\cap R) \times (B\cap S)$. (7)

(ii) Let A = {1, 2,3 4, 5, 6,7 } and R = { (x,y) } x-y is divisible by 3} show that R is equivalence relation on A and draw the graph of R. (7)

(OR)

(b) (i) Prove that the set of all natural numbers(N) is a partially ordered set under the relation$\leq$. (7)

(ii) If R and S are relations on a set A. Prove that

(1) If R and S are reflexive then $R\cup S$ and $R\cap S$ are reflexive.

(2) If R and S are symmetric then $R\cup S$ and $R\cap S$ are symmetric.

(3) If R and S are transitive then $R\cap S$ is transitive. (7)

24. (a) Let A = R- {3) and B= R-{1} and a function $f: A\to B$ be defined as f(x) = ((x-2)/(x-3)) for $x \in A$, Find (i) if f is one-one. (ii) if f is onto. (iii) find f^{-1} if it exists. In case if f^{-1} does not exist, give reasons why is it so?

(OR)

(b) (i) Show that $A\cap(B\cup C)= (A\cap B)\cup(A\cap C)$ using Characteristic function. (7)

(ii) Let $f: A \to B$ be a bijection. Then prove that $f^{-1}: B\to A$ is also a bijection. (7)

25. (a) (i) State and prove Lagrange's theorem (7)

(ii) Show that (2,5) encoding function define by

e(00) = 00000

e(01) = 01110

e(10) = 10101

e(11) = 11011 is a group code

(OR)

(b) (i) Prove that kernel of homomorphism is a normal subgroup (7)

(ii) Let $H = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ be a parity check matrix. Find the hamming code generated by H.

(7)

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