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B.E. DEGREE EXAMINATIONS: NOV / DEC 2010

Fifth Semester

COMPUTER SCIENCE AND ENGINEERING

U07MA404: Probability and Queuing Theory

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. If the probability of John solving a problem is 3/7 and that for Ravi is 5/7. What is the probability that the problem is solved when both of them try?

   (a) $\frac{\sigma_1}{\sigma_7}$ (b) $\frac{2}{9}$ (c) 0 (d) 1 .

2. The probability of an impossible event is _____

   (a) 0 (b) 1 (c) S (d) 5

3. The moment generating function of uniform distribution is

   (a) $(q + pe^t)^n$ (b) $\frac{1}{b+a}$ (c) $\frac{e^{bt} - e^{at}}{t(b-a)}$ (d) $(q + pe^t)^{-n}$

4. If X has a geometric distribution, then for any two positive integers 'm' and 'n' P [X > m+n /X>m] = _____

   (a) P[x = n] (b) P[x>n] (c) P [x<n] (d) P [x= 0]

5. If X and Y are independent Random Variables then Cov (X,Y) = _____

   (a) 0 (b) E (xy) – E (x) E (y) (c) E (xy) (d) E (x) E (y)

6. Regression Co-effecient of y on x is

   (a) $\frac{r\sigma_y}{\sigma_x}$ (b) $\frac{r\sigma_x}{\sigma_y}$ (c) $\frac{\sigma_y}{\sigma_x}$ (d) $\frac{\sigma_x}{\sigma_y}$

7. If T is Continuous and S is discrete then the random process is

   (a) Continuous random process (b) continuous random sequence

   (c) discrete random process (d) discrete random sequence

8. The sum of two independent Poisson process is a _____ Process

   (a) Binomial (b) Poisson (c) Normal (d) Markov.

9. What is the probability that a customer has to unit more than 15 min to get his service completed in a M/M/I queuing system , if λ = 6 per hour and μ = 10 per hour ?

   (a) e^2 (b) e^1 (c) e^{-1} (d) e^0

10. At a public telephone booth, arrivals are considered to be poisson with an average inter-arrival time of 12 minutes. The length of phone call is 4 minutes. The average length of Queues formed from time to time is

(a) 4 (b) 6 (c) 10 (d) 1.5

PART B (10 x 2 = 20 Marks)

11. State any two properties of probability density function.
12. A box contains 4 bad and 6 good tubes, Two are drawn out from the box at a time. One of them is tested and found to be good .What is the probability that the other one is also good?
13. If X ~ B (10, 1/2). Find the mean of the Binomial R.V.
14. If X is uniformly distributed R.V with mean 1 and Variance 4/3, find P (X< 0).
15. The joint pdf of two random variables X and Y is

$$f(x,y) = \begin{cases} cx(x-y), & 0 < x < 2, \quad -x < y < x \\ 0, & \text{otherwise} \end{cases}$$ Find C.

16. State the central limit Theorem.
17. Define Birth and death process.
18. Distinguish between wide sense stationary and strict sense stationary random processes.
19. Write any two basic characteristics of a Queuing System.
20. State Little's Formula.

PART C (5 x 14 = 70 Marks)

21. (a) (i) Find the constant C so that the function $f(x) = \begin{cases} c\,x^2 & 0 < x < 3 \\ 0, & \text{otherwise} \end{cases}$ Is a pdf . Find the distribution function. (7)

(ii) An Urn contains 10 white and 3 black balls. Another Urn contains 3 white and 5 black balls. Two balls are drawn at random, from the first urn and placed in the second urn and then 1 ball is taken at random from the latter. What is the probability that it is a white ball? (7)

(OR)

(b) (i) Given the probability distribution

X	:	0	1	2	3	4
P(x)	:	k	3k	5k	7k	9k

a. Find the value of k.

b. Find P(x< 3); P(x≥3) ; P(0<x<4)

c. Find the distribution function of x. (7)

(ii) Find the mgf of a R.V x having density function $f(x) = \begin{cases} \frac{x}{2}, & 0 \le x \le 2 \\ 0, & otherwise \end{cases}$

Find the first two moments about the origin. (7)

22. (a) (i) Find the MGF of Poisson distribution. Also find its mean and variance. (7)

(ii) State and prove the memory less property of Exponential distribution. (7)

(OR)

(b) (i) In a normal distribution , 31 % of the items are under 45 and 8% are over 64, Find the mean and variance. (7)

(ii) Six dice are thrown 729 times. How many times do you expect atleast 3 dice show a 5 or 6? (7)

23. (a) Obtain the equations of the lines of regression for the following data.

X:	65	66	67	67	68	69	70	72
Y:	67	68	65	68	72	72	69	71

(OR)

(b) (i) The lifetime of a certain kind of electric bulb may be considered as a random variable with mean 1200 hours and S.D 250 hours. Find the probability that the average lifetime of 60 bulbs exceeds 1250 hours using Central limit Theorem. (7)

(ii) If X and Y are two random variables having joint pdf $f(x,y) = \begin{cases} \frac{1}{8}(6-x-y), & 0 \le x \le 2, \ 2 \le y \le 4 \\ 0, & otherwise \end{cases}$

Find a) P[x<1, y<3] b) P[x+y<3] and c) P[x<1/y<3]. (7)

24. (a) (i) The process {X(t)} whose probability distribution under certain conditions is given by $P[X(t) = n] = \begin{cases} \frac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1,2,3 \ldots . \\ \frac{at}{1+at}, & n = 0 \end{cases}$ Show that [x(t)] is not stationary. (7)

(ii) Show that the random process X(t) = A cos (ωt+θ) is wide sense stationary if A and ω are constants and θ is uniformly distributed random variable on (0, 2π) (7)

(OR)

(b) (i) The one step tpm of a markov chain $\{ X_n : n=0,1,2,3,\ldots..\}$ having state space S={1,2,3}

is $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ and the initial distribution is $\pi_0 = (0.7,0.2,0.1)$.

Find a) $P(X_2 = 3/X_0=1)$ b) $P(X_2 = 3)$ c) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 1)$ (7)

(ii) Prove that the difference of two independent Poisson process is not a Poisson process. (7)

25. (a) (i) In a M| M | 1 queue model, the average arrival is 4 customers per minute and $\rho = 0.7$. What are

(a) Mean number of customers in the system

(b) Mean number of customers in the queue

(c) Probability that the server is idle

(ii) An automatic car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars/hr. and may wait in the facility's parking lot if the bay is busy. Find Ls, Lq, Ws, Wq if the service time is constant and equal to 10 minutes.

(OR)

(b) There are 3 typists in an office. Each typist can type an average of 6 letters per hour. If letters arrive for being typed at a rate of 15 letters per hour.

(i) What fraction of the time all the typists will be busy?

(ii) What is the average number of letters waiting to be typed?

(iii) What is the average time a letter has to spend for waiting and for being typed?

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