

B.E. DEGREE EXAMINATIONS: NOV/DEC 2010

Fourth Semester

ELECTRONICS COMMUNICATION ENGINEERING

U07MA403: Random Processes

(Use of Statistical Tables permitted)

Time: Three Hours

Maximum Marks: 100

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. The probability of a number that is divisible by 5 in the interval (21, 60) is
(A) 1/3 (B) 1/4 (C) 1/5 (D) 1/6
2. Given the pdf of a RV X: $f(x) = 12(x^2 - x^3)$, $0 < x < 1$, its mean is:
(A) 1/5 (B) 2/5 (C) 3/5 (D) 4/5
3. For binomial distribution:
(A) Variance = Mean (B) Variance > Mean (C) Variance < Mean (D) mean = S.D.
4. The first three central moments of a Poisson distribution are:
(A) Equal (B) Distinct (C) Complex (D) Equal and Distinct
5. Given the regression lines: $4x - 5y + 33 = 0$ and $20x - 9y - 107 = 0$, the coefficient of correlation r between x and y is:
(A) 0 (B) 0.4 (C) 0.6 (D) 1
6. The marginal pdf $f(x)$ of a 2-D RV (X, Y) with joint pdf $f(x, y) = \begin{cases} e^{-(x+y)}, & 0 < x, y < \infty \\ 0, & \text{otherwise} \end{cases}$ is
(A) e^{-x} (B) e^{-y} (C) e^x (D) e^y
7. A random process is said to be a continuous RP if the state space S and parameter set T are
(A) both continuous (B) both discrete (C) one continuous (D) one discrete
8. A Poisson random process is:
(A) Markovian (B) Stationary (C) Continuous (D) Markovian and Continuous
9. The Wiener relationship of a RP relates:
(A) Time (B) Frequency (C) Time and frequency (D) Time and tide

10. The cross spectral density of two process is:

- (A) 0 (B) 1 (C) 2 (D) 100

PART B (10 x 2 = 20 Marks)

11. Find the mean of the RV X, given that $M_X(t) = (2e^t/5) + (e^{2t}/5) + (2e^{3t}/5)$.

12. State probability mass function and probability density function.

13. State any four properties of normal distribution.

14. If X is uniformly distributed with mean 1 and variance 4/3, find $P(X < 0)$.

15. Find the value of k if $f(x, y) = k(1-x)(1-y)$, $0 < x, y < 1$ is to be a joint density function.

16. If X and Y are independent RV with variance 2 and 3, find the $V(3X+4Y)$.

17. Define Markov process and Markov chain.

18. Classify the different types of random processes.

19. Define a linear system with a random input.

20. Define cross spectral density.

PART C (5 x 14 = 70 Marks)

21. a) (i) The probability function of an infinite discrete distribution is given by

$$P(x) = 1/2^x, x = 1, 2, \dots \text{ Find the mean and variance.} \quad (7)$$

(ii) If $E(X) = 1$ and $E(X^2 - X) = 4$, find $V(X)$ and $V(2 - 3X)$. (7)

(OR)

b) (i) Find the mean and variance of a continuous RV X with pdf $f(x) = 0.5x^2 e^{-x}$, $x \geq 0$. (7)

(ii) Using the MGF of a continuous RV X with pdf $f(x) = \begin{cases} x/2, & 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}$, find the

first two moments about origin. (7)

22. a) (i) Fit a binomial distribution to the following data:

x:	0	1	2	3	4
f:	5	29	36	25	5

(ii) If X is a Poisson RV with $P(X = 2) = 9 P(X = 4) + 90 P(X = 6)$, find $P(X \geq 2)$. (7)

(OR)

- b) (i) If X is a normal RV with $\mu = 30$ and $\sigma = 5$, find $P(26 \leq X \leq 40)$ and $P(X \geq 45)$. (7)
(ii) Find the MGF and hence the mean and variance of the uniform distribution. (7)

23. a) Given the joint probability function $p(x, y) = (x + y)/21$, $x = 1, 2, 3$; $y = 1, 2$. Find the coefficient of correlation between X and Y.

(OR)

b) Given the joint pdf of a 2 – D RV (X, Y): $f(x, y) = \begin{cases} k(4 - x - y), & 0 \leq x, y \leq 2 \\ 0, & \text{Otherwise.} \end{cases}$. . Find the

coefficient of correlation between X and Y.

24. a) If $U(t) = X \cos t + Y \sin t$, and $V(t) = Y \cos t + X \sin t$, where X and Y are independent RVs such that $E(X) = 0 = E(Y)$, $E(X^2) = 1 = E(Y^2)$. Show that U(t) and V(t) are individually stationary in Wide Sense, but they are not jointly WSS.

(OR)

b) Show that the Sum of two independent Poisson Process is again a Poisson Process, but their difference is not.

25. a) (i) The autocorrelation function for a Stationary Process X(t) is given by $R_{xx}(\tau) = 9 + 2e^{-|\tau|}$

Find the mean value of the random variable $Y = \int_0^2 X(t) dt$ and variance of X(t). (7)

(ii) The Cross Power Spectrum of a real RP X(t) and Y(t) is given by

$S_{XY}(\omega) = \begin{cases} a + jb\omega, & |\omega| \leq 1 \\ 0, & \text{otherwise} \end{cases}$. Find the Cross Correlation function. (7)

(OR)

b) (i) Show that $S_{YY}(\omega) = |H(\omega)|^2 S_{XX}(\omega)$ where $S_{XX}(\omega)$ and $S_{YY}(\omega)$ are the Power Spectral Density function of the input X(t) and the output Y(t) and H(ω) is the system transfer function. (7)

(ii) Given that a RP X(t) has the autocorrelation function $R_{xx}(\tau) = Ae^{-\alpha|\tau|} \cos(\omega\tau)$,

where $A > 0$, $\alpha > 0$ and ω are real constants, find the Power Spectrum. (7)
