

B.E/B.TECH. DEGREE EXAMINATIONS: NOV/DEC 2010

Third Semester

U07MA301: MATHEMATICS III

(Common to all Branches)

Time: Three Hours

Maximum Marks: 100

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. Lagrange's Linear equation is of the form

- a) $Pq+Qp=R$ b) $Pp+Qq=R$ c) $Pq+Qp=0$ d) $Pp+Qq=0$

2. The partial differential equation of $z = ax + by + ab$ by eliminating the arbitrary constants is

- a) $z = px+qy +pq$ b) $z=x+ y$ c) $z=qx + py + p$ d) $z=px + qy$

3. The value of b_n when $f(x) = x \sin x$ in $(-\pi, \pi)$ is

- a) 1 b) -1 c) 0 d) 2

4. The root mean square value of $f(x)=x$ in $0 < x < l$ is

- a) $\frac{l}{3}$ b) $\frac{l}{\sqrt{3}}$ c) $\frac{3}{l}$ d) $\frac{\sqrt{3}}{l}$

5. Classify the partial differential equation $U_{xx} = U_{yy}$

- a) elliptic b) parabolic c) hyperbolic d) neither elliptic nor hyperbolic

6. How many boundary conditions and initial conditions are required to solve one dimensional heat equation?

- a) 1 boundary condition & 2 initial conditions b) 2 boundary condition & 2 initial conditions
c) 3 boundary conditions d) 2 boundary condition & 1 initial conditions

7. The Fourier sine transform of e^{-bx} is

a) $\sqrt{\frac{2}{\pi}} \left[\frac{s}{s^2 + b^2} \right]$ b) $\sqrt{\frac{2}{\pi}} \left[\frac{s}{s^2 - b^2} \right]$ c) $\sqrt{\frac{2}{\pi}} \left[\frac{1}{s^2 + b^2} \right]$ d) $\sqrt{\frac{2}{\pi}} \left[\frac{1}{s^2 - b^2} \right]$

8. The Fourier transform of $f(x) = \begin{cases} 1 & \text{if } a < x < b \\ 0 & \text{otherwise} \end{cases}$ is

a) $\frac{1}{\sqrt{2\pi}} \left[\frac{e^{isa} - e^{isb}}{is} \right]$ b) $\frac{1}{\sqrt{2\pi}} \left[\frac{e^{-isa} - e^{-isb}}{is} \right]$ c) $\frac{1}{\sqrt{2\pi}} \left[\frac{e^{isb} - e^{isa}}{is} \right]$ d) $\frac{1}{\sqrt{\pi}} \left[\frac{e^{isb} - e^{isa}}{is} \right]$

9. $Z(3.4^n)$ is

a) $\frac{3z}{z-4}$ b) $\frac{z}{z-4}$ c) $\frac{3z}{z+4}$ d) $\frac{z}{z+4}$

10. $z^{-1} \left[\frac{az}{(z-a)^2} \right]$ is

a) a^n b) na^n c) n d) n^2

PART B (10 x 2 = 20 Marks)

11. Find the complete solution of the partial differential equation $p^2 + q^2 - 4pq = 0$.

12. Form the partial differential equation by eliminating 'f' from the relation

$$z = f(x^2 + y^2) + x + y.$$

13. Obtain the coefficient b_5 in the Fourier Cosine series of the function

$$f(x) = \sin 5x \text{ in } (0, 2\pi).$$

14. State Dirichlet's conditions for Fourier series.

15. State Fourier law of heat conduction.

16. Write the periodic solutions of Laplace's equation in Cartesian Co-ordinates.

17. Find the Fourier sine transform of $\frac{1}{x}$

18. If the Fourier transform of $f(x)$ is $F(s)$, what is the Fourier transform of $f(x-a)$?

19. Find $Z[f(n)]$ where $f(n) = n$ for $n = 0, 1, 2, 3, \dots$

20. State Convolution theorem on Z-transform.

PART C (5 x 14 = 70 Marks)

21. a) (i) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2zx$ (7)

(ii) Solve $(D^2 + 3DD' - 4(D')^2)z = x + \sin y$ (7)

(OR)

b) (i) Obtain a singular solution to the equation $z = px + qy + \sqrt{1 + p^2 + q^2}$. (7)

(ii) Solve $(D^3 - 2D^2D')z = e^{x+2y} + 3x^2y$ (7)

22. a) (i) Find the half range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$. (7)

(ii) Compute the first two harmonics of the Fourier series of $f(x)$ given by the following table. (7)

X	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

(OR)

b) (i) By finding the Fourier Cosine series for $f(x) = x$ in $(-\pi, \pi)$. Show that

$$\frac{\pi^4}{96} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^4}. \quad (7)$$

(ii) Determine the Fourier series for the function $f(x) = x^2$ of period 2π in $(-\pi, \pi)$. (7)

23. a) A tightly stretched string with fixed endpoints $x=0$ and $x=l$ is initially at rest in the equilibrium position. If it is set vibrating giving each point a initial velocity $3x(l-x)$.

Find the displacement.

(OR)

b) An infinitely long metal plate in the form of an area is enclosed between the lines

$y = 0$ and $y = \pi$ for $x > 0$. The temperature is zero along the edges $y = 0$ and $y = \pi$

and at infinity. If the edge $x=0$ is kept at a constant temperature $T^\circ\text{C}$, find the steady state temperature at any point of the plate.

24. a) (i) Find the Fourier transform of $f(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$.

Hence find $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$. (7)

(ii) Find the Fourier transform of $f(x) = \begin{cases} 1-x^2 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ (7)

Hence deduce that $\int_0^\infty \frac{\sin t - t \cos t}{t^3} dt = \frac{\pi}{4}$

(OR)

b) (i) By finding the Fourier cosine transform of $f(x) = e^{-ax}$, $a > 0$ and using Parseval's identity for cosine transform evaluate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$ (7)

(ii) Show that $e^{-\frac{x^2}{2}}$ is self reciprocal under Fourier transform. (7)

25. a) (i) Solve using Z transform, $y_{n+2} - 4y_n = 0$, given $y_0 = 2, y_1 = -1$. (7)

(ii) Find the Z-transform of $n(n+1)$ (7)

(OR)

b) (i) Find the inverse Z-transform of $\frac{z^2}{(z-a)^2}$ using convolution theorem. (7)

(ii) Find the Z-transform of $2^n(1-n)$, $n \geq 0$. (7)
