

B.E. DEGREE EXAMINATIONS: NOV/DEC 2010

Second Semester

U07MA201: MATHEMATICS II

(Common to all Branches)

Time: Three Hours

Maximum Marks: 100

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. $\int_0^2 \int_1^3 \int_1^2 xy^2 z dz dy dx =$

- A) 0 B) $\frac{26}{3}$ C) $\frac{3}{2}$ D) 26

2. Formula to find the area using double integral is

- A) $\iint dx dy$ B) $\iint xy dx dy$ C) $\iint x dx dy$ D) $\iint y dx dy$

3. If F is a vector such that $\text{div } F = 0$ for all points in a given region, then F is said to be

- A) Irrotational B) Solenoidal C) Lamellar D) Directional derivative

4. If ϕ is scalar, then $\text{grad } \phi$ is

- A) Scalar B) Solenoidal C) Vector D) Irrotational

5. The equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$ is known as

- A) Cauchy- Riemann equation B) Laplace's equation
C) Harmonic conjugate D) Poisson's equation

6. A mapping $w = f(z)$ is conformal if $f(z)$ is analytic and if

- A) $f'(z) \neq 0$ B) $f'(z) = \infty$ C) $f(z) = \infty$ D) $f'(z) = 0$

7. The singular points for the function $f(z) = \frac{z^2 + 1}{(z-1)(z-2)}$ are
- A) 0 and 1 B) 0 and 2 C) 1 and 2 D) -1 and -2
8. The poles which lie inside $|z| = \frac{3}{2}$ for the function $f(z) = \frac{z^2}{(z-1)(z-3)}$ are
- A) 1,3 B) 1 C) 3 D) 0
9. $L[2]$ is
- A) $\frac{2}{s^2}$ B) 2 C) $\frac{1}{s}$ D) $\frac{2}{s}$
10. If $L[f(t)] = F(s)$ and $L[g(t)] = G(s)$ then $L[f(t) * g(t)]$ is
- A) $F(s) * G(s)$ B) $F(s) + G(s)$ C) $F(s) - G(s)$ D) $F(s) \cdot G(s)$

PART B (10 x 2 = 20 Marks)

11. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin(\theta + \phi) d\theta d\phi$
12. Change the order of integration in $\int_0^1 \int_x^1 \frac{x}{x^2 + y^2} dx dy$.
13. State Gauss divergence theorem.
14. Find the function ϕ , if $\text{grad } \phi = (y^2 - 2xyz^3)\vec{i} + (3 + 2xy - x^2z^3)\vec{j} + (6z^3 - 3x^2yz^2)\vec{k}$.
15. Show that an analytic function with constant real part is a constant.
16. Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic.
17. Expand the function $f(z) = \cos z$ in Taylor's series about the point $z = -\frac{\pi}{2}$.
18. Evaluate $\int_C \frac{z+1}{z^3 - 2z^2} dz$, where C is the circle $|z - 2 - i| = 2$.

19. Find $L[\cos at]$

20. P.T $L[f(at)] = 1/a F(s/a)$ if $L[f(t)] = F(s)$.

PART C (5 x 14 = 70 Marks)

21. a) Evaluate $\iiint \sqrt{1-x^2-y^2-z^2} dx dy dz$, taken throughout the volume of the sphere

$$x^2 + y^2 + z^2 = 1, \text{ by transforming to spherical polar co-ordinates.}$$

(OR)

b) Transform $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dx dy}{\sqrt{a^2-x^2-y^2}}$ into polar co-ordinates and then evaluate.

22. a) Verify Stoke's theorem for $\vec{F} = y^2 z \vec{i} + z^2 x \vec{j} + x^2 y \vec{k}$, where S is the open surface of the cube formed by the plane $x = \pm a, y = \pm a, z = \pm a$, in which the plane $z = -a$ is cut.

(OR)

b) Verify Gauss divergence theorem for $\vec{F} = x^2 \vec{i} + z \vec{j} + yz \vec{k}$ over the cube formed by

$$x = \pm 1, y = \pm 1, z = \pm 1.$$

23. a) Determine the analytic function $f(z) = u + iv$ given that $u - v = \frac{\cos x + \sin x - e^{-y}}{2 \cos x - e^y - e^{-y}}$ and

$$f\left(\frac{\pi}{2}\right) = 0.$$

(OR)

b) (i) If $f(z) = u + iv$ is analytic, prove that $\nabla^2 |f(z)|^2 = 4 |f'(z)|^2 = 0$ (7)

(ii) Find the bilinear transformation which maps the points $z = 0, z = 1$ and $z = \infty$ into

$$\text{the points } w = i, w = 1, w = -i. \quad (4)$$

24. a) Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region

(i) $|z + 1| < 1$

(ii) $1 < |z + 1| < 2$.

(OR)

b) Use Contour integration to prove that $\int_0^{\infty} \frac{x^2}{x^4 + a^4} dx = \frac{\pi}{a2\sqrt{2}}$, where $a > 0$.

25. a) (i) Solve $y'' - 4y' + 8y = e^{2t}$, $y(0) = 2$ and $y'(0) = -2$. (10)

(ii) Evaluate $\int_0^{\infty} \frac{e^{-2t} - e^{-4t}}{t} dt$. by using Laplace transform. (4)

(Or)

b) (i) Find the Laplace transform of the "square wave" function $f(t)$ defined by

$$f(t) = \begin{cases} k, & 0 \leq t \leq a \\ -k, & a \leq t \leq 2a \end{cases} \text{ and } f(t + 2a) = f(t), \text{ for all } t. \quad (7)$$

(ii) Use convolution theorem to find $L^{-1} \left[\frac{s^2}{(s^2 + a^2)(s^2 + b^2)} \right]$. (7)
