Register Number: ………………..

**B.E. DEGREE EXAMINATIONS: NOV/DEC 2010**

First Semester

**U07MA101: MATHEMATICS – I**

(Common to All Branches)

**Time: Three Hours** **Maximum Marks: 100**

**Answer All Questions:-**

**PART A (10 x 1 = 10 Marks)**

1. The product of two eigen values of $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is 9. Then the third eigen value is

A. -5 B. 5 C. 15 D. 10

2. The signature of the Q.F. $x^2 + 2y^2 - 3z^2$ is

A. 1 B. 0 C. 2 D. -1

3. The general equation of a plane parallel to x-axis is

A. $ax + by + cz = 0$ B. $ax + by + cz = k$ C. $by + cz = 0$ D. $by + cz + d = 0$

4. The radius of the sphere passing through the origin and having its centre at (2, -2, 1) is

A. 1 B. 2 C. 3 D. $\sqrt{2}$

5. The radius of curvature of $y = \log \sin x$ at $x = \frac{\pi}{2}$ is

A. 1 B. $\infty$ C. 0 D. $1/\sqrt{2}$

6. The envelope of the family of straight lines $y = mx + \frac{1}{m}$, $m$ being the parameter is

A. $y^2 = 4x$ B. $x^2 = 4y$ C. $x^2 + y^2 = 1$ D. $xy = 1$

7. If $x = r\cos\theta, y = r\sin\theta$, then the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$ is

A. 1 B. -1 C. r D. $r^2$

8. Given $f_{xx} = 6x, f_{xy} = 0, f_{yy} = 6y$ the stationary point $(1,2)$ of $f(x,y)$ is

A. Minimum point B. Maximum point C. Saddle point D. Doubtful

9. The particular integral of $(D+1)^2 y = e^{-x}$ is

A. $xe^{-x}$ B. $\frac{x}{2}e^{-x}$ C. $\frac{-x^2}{2}e^{-x}$ D. $\frac{x^2}{2}e^{-x}$

10. Given $\frac{dx}{dt} - y = t$, $\frac{dy}{dt} + x = 1$, the solution for x is

A. $ae^t + be^{-t} + 2$ B. $(a + bt)e^t + 1$ C. $acost + bsint + 2$ D. $acost + bsint + 1$

**PART B (10 x 2 = 20 Marks)**

11. Two eigenvalues of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each. Find the eigenvalues of $A^{-1}$.

12. Determine the nature of the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$.

13. Find the equation of the plane passing through the intersection of $x + y + z = 5$ and $2x + 3y + 4z + 1 = 0$ and the point $(0,1,0)$.

14. Find the centre and radius of the sphere $2x^2 + 2y^2 + 2z^2 - 4x + 6y - 2z - 1 = 0$.

15. What is the curvature of the circle $x^2 + y^2 = 25$ at the point $(4,3)$ on it.

16. Find the radius of curvature of $r = \frac{a}{\theta}$ at $\theta = 1$.

17. If $u = \frac{y^2}{x}, v = \frac{x^2}{y}$ find $\frac{\partial(x,y)}{\partial(u,v)}$ .

18. Determine the stationary points of the function $x^4 + y^4 - 2x^2 + 4xy - 2y^2$.

19. Find the particular integral of $(D - 1)^2 y = xe^x \sin x$.

20. Transform the equation $x^2 y'' + xy' + y = 0$ into a linear equation with constant coefficients.

**PART C (5 x 14 = 70 Marks)**

21. a) (i) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (7)

(ii) Find the inverse of $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ using Cayley – Hamilton theorem. (7)

**(OR)**

b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ to canonical form by an Orthogonal transformation.

22. a) (i) Find the image of the point $P(1,3,4)$ in the plane $2x - y + z + 3 = 0$. (7)

(ii) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. (7)

**(OR)**

b) (i) Find the equation of shortest distance between the lines

$$\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1} \quad \text{and} \quad \frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}. \qquad (7)$$

(ii) Find the two tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane $2x+2y-z=0$. Find their points of contact. (7)

23. a) (i) Find the radius of curvature at any point $(r,\theta)$ of the curve $r^n = a^n \cos n\theta$. (6)

(ii) Find the evolute of the parabola $y^2 = 4ax$. (8)

**(OR)**

b) (i) Find the equation to the circle of curvature of the curve $\sqrt{x}+\sqrt{y}=\sqrt{a}$ at $(a/4, a/4)$ (7)

(ii) Find the envelope of $\frac{x}{l}+\frac{y}{m}=1$ where the parameters $l$ and $m$ are connected by the relation $\frac{l}{a}+\frac{m}{b}=1$ ($a$ and $b$ are constants). (7)

24. a) (i) Expand $f(x,y)=e^x\log(1+y)$ in powers of $x$ and $y$ upto terms of second degree. (6)

(ii) Find the minimum value of $x^2+y^2+z^2$ subject to $ax+by+cz=p$ (8)

**(OR)**

b) (i) If $u=2xy, v=x^2-y^2$ and $x=r\cos\theta, y=r\sin\theta$ evaluate $\frac{\partial(u,v)}{\partial(r,\theta)}$. (6)

(ii) Discuss the maxima and minima of the function $f(x,y)=3(x^2-y^2)-x^3+y^3$. (8)

25. a) (i) Solve $(D^2+2)\,y = x^2 + e^x\cos 2x$ where $D = \frac{d}{dx}$ (7)

(ii) Solve $\frac{dx}{dt}+2y=\sin 2t, \quad \frac{dy}{dt}-2x=\cos 2t$ (7)

**(OR)**

b) (i) Solve $(x^2D^2+xD+1)y=\log x\sin(\log x), \quad D=\frac{d}{dx}$ (7)

(ii) Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+a^2y=\tan ax.$ (7)

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