B.E. DEGREE EXAMINATIONS: NOV/DEC 2010

Third Semester

MAT 104: Engineering Mathematics III

Common to

(Aeronautical Engineering, Civil, Electrical & Electronics, Electronics Communications,

Electronics & Instrumentation, Mechanical, Mechatronics)

Time: Three Hours   **Maximum Marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. A Solution obtained by giving particular values to the arbitrary constants in a complete integral is called.

   (a) General integral   (b) Singular integral   (c) Particular Solution   (d) None of these.

2. Particular integral of $(D^2 - D'^2) = e^x$ is

   (a) e^x (b) $\dfrac{e^x}{2}$ (c) ½   (d) 1

3. The value of b_n in the expression of x^2 as a Fourier Series in $(-\pi, \pi)$ is

   (a) 0   (b) 1   (c) -1   (d) $1/\pi$

4. The Value of a_0 in Harmonic analysis is given by,

   (a) $\dfrac{1}{n}\sum f(x)$ (b) $\dfrac{2}{n}\sum f(x)$ (c) $\sum f(x)$ (d) $2\sum f(x)$

5. The equation $f_{xx} + 2f_{xy} + f_{yy} = 0$ is

   (a) parabolic   (b) a hyperbolic   (c) elliptic   (d) none of these

6. The Constant a^2 in the wave equation $U_{tt} = a^2 U_{xx}$ Stands for

   (a) $\dfrac{T}{m}$ (b) $\dfrac{m}{T}$ (c) $\dfrac{T^2}{m^2}$ (d) $\dfrac{m^2}{T^2}$

7. IF F $[f(x)] = F(s)$ then $F[(-x)]$ is

   (a) $F(-s)$ (b) $F(s)$ (c) $-F(s)$ (d) $-F[(-s)]$

8. If $f(x)$ is defined in $(-\infty, \infty)$ and $F[f(x)] = F(s)$ then $\displaystyle\int_{-\infty}^{\infty} |f(x)|^2 dx$ is

(a) $\int_0^\infty |F(s)|^2 ds$　　(b) $\int_{-\alpha}^\infty |F(s)|^2 ds$　　(c) $\int_{-\alpha}^\infty |F(s)| ds$　　(d) $\int_0^\infty |F(s)| ds$

9. $Z(a^{n-1}) =$

(a) $\dfrac{1}{Z-a}$ if $|z| > |a|$　　(b) $\dfrac{1}{Z-a}$ if $|z| \leq |a|$　　(c) $\dfrac{1}{(Z-a)^2}$　　(d) $Z - a$

10. If $z[f(x)] = f(z)$ then $\underset{Z \to \infty}{Lt} f(z) =$

(a) $f(0)$　　　(b) $f(z)$　　　(c) $f(x)$　　　(d) 0

PART B (10x2=20 Marks)

11. Form the PDE by eliminating the arbitrary constants a and b from $Z = (x+a)(y+b)$

12. Solve $(D^4 - D'^4)z = 0$

13. If $f(x) = 2x$ in $(0,4)$, then find the value of a_n in Fourier Series expansion.

14. What do you mean by Harmonic Analysis?

15. Write two solutions of Laplace equation $U_{xx} + U_{yy} = 0$ involing exponential terms in x or y

16. An insulated rod of length 60cm has its ends at A and B maintained at 200C and 80^0C respectively, Find the steady state solution of the rod.

17. Write down the FCT pair of formulae.

18. State the change of scale property of Fourier transform.

19. Find the z – transform of $\dfrac{1}{n!}$

20. State the Convolution theorem for Z-transform.

PART C (5x14=70 Marks)

21. a) (i) Form the partial differential equation by eliminating the arbitrary functions f and
　　　g from $Z = xf(y/x) + yg(x)$.

　　(ii) Solve $(mz - ny)p + (nx - lz)q = ly - mx$

(OR)

　　b) (i) Find the complete integral of $p(1+q) = q^z$

(ii) Solve $(D^2 - 2DD')Z = e^{2x} + x^3 y$

22. a) (i) Find the Fourier series expansion for $f(x) = x + x^2$ in $-\pi < x < \pi$ and hence

deduce that $\sum_{1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

(ii) Find the half range sine series for $f(x) = x(\pi - x)$ in $0 < x < \pi$ and hence deduce

that $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - .... = \frac{\pi^3}{32}$

(OR)

b) (i) Obtain the Fourier series for $f(x) = /\cos x/ in (-\pi, \pi)$

(ii) Find the first fundamental harmonic of the fourier series of $f(x)$ given by the

following table:

x :	0	1	2	3	4	5
$f(x):$	9	18	24	28	26	20

23. a) A string of length $2l$ is tightly stretched and fixed at it ends at the points $(0,0)$ and

$(2l,0)$ of the xy-plane. It is made to vibrate transversely in the xy plane by giving to

each of its points a transverse velocity v in the $xy-plane$, where v is given by

$$v = \begin{cases} kx & 0 \le x \le l \\ k(2l-x) & l < x \le 2l \end{cases}$$

Fine the expression for the transverse displacement of the string at any time t.

(OR)

b) An insulated rod of length 30 cm has its ends A and B kept at 20^0 C and 80^0C

respectively until steady state conditions prevail. The temperature at each end is then

suddenly reduced to 0^0C and kept so, Find the resulting temperature distribution

$U(x,t)$ taking origin at A.

24. a) (i) Find the Fourier Transform of $f(x)$, if

$$f(x) = \begin{cases} 1 - |x| & if \ |x| < 1 \\ 0 & if \ |x| > 1 \end{cases}$$ Hence deduce that $\int_{0}^{\infty} \left(\frac{S \, int}{t} \right)^4 dt = \pi/3$ (8)

(ii) Evaluate $\int_{0}^{\alpha} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$ using Fourier cosine transform. (6)

(OR)

b) (i) Find the Fourier sine transform of $f(x) = \dfrac{e^{-ax}}{x}$

(ii) State and prove convolution theorem on Fourier transform.

25. a) (i) Find the z-transform of $\dfrac{1}{2}(n+1)(n+2)$

(ii) Find the inverse z- transform of $\dfrac{4z^2 - 2z}{z^3 - 5z^2 + 8z - 4}$

(OR)

b) (i) Using Convolution theorem evaluate

$$Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$$

(ii) Solve $U_{n+2} + 4U_{n+1} + 3U_n = 3^n$ with $u_0 = 0,\ u_1 = 1$

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