Register Number: ………………

B.E/B.TECH. DEGREE EXAMINATIONS: NOV/DEC 2010

Second Semester

MAT102: ENGINEERING MATHEMATICS II

(Common to Aeronautical, Civil , Electronics and Communication Engineering, Electronics and Instrumentation Engineering, Electrical & Electronics Engineering, Mechtronics & Mechanical Engineering)

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. The value of $\int_0^1\int_0^2 y\,dx\,dy$ is ………………

(a) ∞ (b) $\frac{1}{6}$ (c) $\frac{1}{2}$ (d) 1

2. The value of $\int_0^1\int_0^2\int_0^3 dx\,dy\,dz$ ………………….

(a) 0 (b) 3.5 (c) 6 (d) ∞

3. If $\overline{F}$ is solenoidal then div $\overline{F}$ is

(a) 0 (b) 1 (c) -2 (d) 10

4. div(curl $\overline{F}$) is

(a) 1 (b) 2 (c)0 (d) ∞

5. If $f(z)=u(x,y)+iv(x,y)$ is analytic then $f'(z)=$

(a) $\frac{\partial u}{\partial x}-i\frac{\partial v}{\partial x}$ (b) $\frac{\partial u}{\partial x}-i\frac{\partial u}{\partial y}$ (c) $\frac{\partial v}{\partial x}+i\frac{\partial v}{\partial y}$ (d) $\frac{\partial v}{\partial y}-i\frac{\partial u}{\partial y}$

6. If $u_{xx}+u_{yy}=0$, where $f(z)=u+iv$ is analytic, then u is

(a) harmonic (b)conjugate harmonic (c) analytic (d) non-harmonic

7. If $f(z)$ is analytic inside and on a simple closed curve C that encloses a simply connected region R and if 'a' is any point in R then $\frac{1}{2\pi i}\int_c \frac{f(z)}{(z-a)}dz$ is

(a) $f(a)$ (b) $f'(a)$ (c) $f\left(\frac{1}{a}\right)$ (d) $f^n(a)$

8. If principal part of the Laurent's expansion of $f(z)$ about an isolated singularity z = a contains **infinite** number of terms then singularity z = a is called

(a) removable (b) pole of order m (c) essential (d) zero of $f(z)$

9. If $L(f(t)) = \phi(s)$,then $L(e^{at} f(t))$ is

(a) $\phi(s/a)$ (b) $\phi(s+a)$ (c) $\phi(s-a)$ (d) $\phi(sa)$

10. If $L(t^2)$ is

(a) $\frac{\Gamma(3)}{s^3}$ (b) $\frac{\Gamma(2)}{s^{2+1}}$ (c) $\frac{1}{s^3}$ (d) 0

PART B (10 x 2= 20 Marks)

11. Evaluate $\int_1^3\int_1^2 \frac{dxdy}{xy}$

12. Change the order of integration in $\int_0^1\int_y^1 f(x,y)dydx$.

13. If $\overline{F} = 3xyz^2\overline{i} + 2xy^3\overline{j} - x^2yz\overline{k}$ then find $\nabla.\overline{F}$ at $(1,-1,1)$

14. State Gauss Divergence theorem.

15. State C.R equations in cartesian form.

16. Find the image of the circle $|z| = 2$ under the transformation $w = z + 3 + 2i$.

17. Evaluate $\int_C \frac{z^2}{z-1}$, $C:|z| = \frac{1}{2}$.

18. Find by Cauchy residue theorem, $\int_C \frac{\cos \pi z^2 + \sin \pi z^2}{(z+2)}dz, C:|z| = 3$.

19. Find the Laplace transform of $(at+b)^3$.

20. Find L^{-1} $(\tan^{-1} s)$

PART C (5 x 14 = 70 Marks)

21. a) (i) Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dxdydz$. (7)

(ii) Find the area bounded by the parabolas $y^2 = 4 - x$ and $y^2 = x$ by double integration. (7)

(OR)

b) (i) Change the order of integration in $\int_0^a\int_y^a \frac{x}{\sqrt{x^2+y^2}}\,dxdy$ and evaluate. Sketch the region of integration. (7)

(ii) By changing into polar coordinates, evaluate $\int_0^a\int_y^a \frac{x^2}{(x^2+y^2)^{\frac{3}{2}}}\,dxdy$. (7)

22. a) Verify Stoke's theorem for $\overline{F} = y^2 z\overline{i} + z^2 x\overline{j} + x^2 y\overline{k}$ where S is the open surface of the cube formed by the planes $x = \pm a, y = \pm a, z = \pm a$, in which the plane $z = -a$ is cut.

(OR)

b) Verify Gauss divergence theorem for $\overline{F} = x^2\overline{i} + z\overline{j} + yz\overline{k}$, over the cube formed by $x = \pm 1 y = \pm 1, z = \pm 1$. . (7)

23. a) (i) Determine the analytic function f(z)= u + i v, given (7)

$u = x^3 - 3xy^2+3x^2-3y^2+1$ also find v

(ii) Show that the transformation $w = \frac{1}{z}$ maps the circle $|z-3| = 5$ onto the circle $\left|w + \frac{3}{16}\right| = \frac{5}{16}$. (7)

(OR)

b) (i) If $f(z)$ is an analytic function of z in a region R, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (7)

(ii) Find the bilinear transformation that maps $0, 1, \infty$ into the points $i, 1, -i$. (7)

24. a) (i) Evaluate $\int_C \frac{z}{(z-1)(z-2)^2}dz$, where C is the circle $|z-2|=\frac{1}{2}$, using Cauchy's integral formula. (7)

(ii) Evaluate $\int_0^\infty \left(\frac{dx}{(x^2+a^2)^2}\right)^2, a>0$ using contour integration. (7)

(OR)

b) (i) Find the Laurent's series of $f(z)=\frac{z^2-1}{z^2+5z+6}$ valid in the region (7)

(1) $|z|<2$, (2) $2<|z|<3$,(3) $|z|>3$

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+\sin\theta}$, using contour integration. (7)

25. a) (i) Find the Laplace transform of $\frac{\sin^2 t}{t}$. (6)

(ii)Use convolution theorem to find $L^{-1}\left(\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right)$. (8)

(OR)

b) (i) Find the Laplace transform of the "half-sine wave rectifier" function $f(t)$ given by

$$f(t)=\begin{cases} a\sin\omega t, 0\le t\le \frac{\pi}{\omega} \\ 0, \frac{\pi}{\omega}\le t\le \frac{2\pi}{\omega} \end{cases}.$$

(ii) Solve the differential equation $(D^2 + 5D+6)\, y = e^{-t}$ given that y(0) = 0 and y'(0) = 0. (7)

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