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B.E/B.TECH. DEGREE EXAMINATIONS: NOV/DEC 2010

Second Semester

MAT103 ENGINEERING MATHEMATICS II

(Common To CSE, IT, TXT, FT & BT Branches)

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. $\int_0^1 \int_0^{1-x} dx dy$ represents

a) Area of triangle b) Area of circle c) Area of rectangle d) Area of square

2. The value of $\int_0^1 \int_0^2 \int_0^3 xyz \, dx \, dy \, dz$ is ___________

a) 9/2 b) 9 c) 36 / 2 d) 18

3. If $\overline{F}$ is irrotational then it can be expressed as

a) $\overline{F} = \nabla\phi$ b) $\nabla \times \overline{F}x = \nabla\phi$ c) $\overline{F} = \nabla \times \phi$ d) $\nabla . F = \nabla\phi$

4. If R is a region bounded by a closed Curve C then the area of R is

a) $\int_c xdy + ydx$ b) $\int xdy - ydx$ c) $\int_c xdx + ydy$ d) $\frac{1}{2}\int_c xdy - ydx$

5. The function $f(z) = \frac{1}{z^2 + 1}$ is analytic everywhere except

a) $z = 1$ b) $z = \pm 1$ c) $z = \pm i$ d) $z = i$ only

6. An analytic function with constant real part is

a) imaginary b) constant c) unknown d) not defined

7. The transformation w = c z where 'c' is complex constant consists of

a) Magnification b) rotation

c) Magnification and rotation d) Translation

8. The transformation w = sin z is not conformal at

a) 0 b) $\frac{\pi}{3}$ c) $\frac{3\pi}{2}$ d) $\frac{2\pi}{3}$

9. The value of $\int_C \frac{dz}{z-2}$ where C is $|z|=1$

a) 1 b) 0 c) 2 d) 3

10. If the Laurent's expansion of f(z) contains only a finite number of negative powers of (z-a) then z = a is called a

a) pole b) residue c) removable singularity d) essential singularity

PART B (10 × 2 = 20 Marks)

11. Evaluate $\int_0^{\pi}\int_0^{5} r^4 \sin\theta \, dr \, d\theta$.

12. Find the area of the circle, $x^2+y^2=a^2$ using double integral.

13.If $\nabla\varphi$ is solenoidal then find $\nabla^2\varphi$.

14. Evaluate $\iint \vec{r} \cdot \vec{ds}$ $where$ S is the surface of the tetrahedron whose vertices are (0,0,0), (1,0,0), (0,1,0), (0,0,1).

15. State any two properties of Analytic function.

16. Show that the function $x^4-6x^2y^2+y^4$ is harmonic.

17. Find the invariant point of the transformation $w = \frac{6z-9}{z}$.

18. Find the image of the infinite strip $0 \le x \le 1$ under the transformation $w = e^z$.

19. State Cauchy's integral formula.

20. Determine the poles of $f(z)=\frac{z^2}{(z-1)(z-2)^2}$ and write their orders.

PART C (5 × 14 = 70 Marks)

21. a) (i) Evaluate $\iint_R xy \, dx \, dy$ where R is the domain bounded by X axis, ordinate x = 2a and and the curve $x^2=4ay$ (7)

(ii) Find the volume of the tetrahedron formed by the planes x = 0, y = 0, z = 0 & 2x + 3y + 4z = 12. (7)

(OR)

b) (i) Express $\int_0^a \int_y^a \frac{x}{x^2+y^2}\,dx\,dy$ in polar co-ordinates and then evaluate it. (6)

(ii) Change of order of integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$. (8)

22. a) (i) Prove that $div\,grad\,r^n = n(n+1)r^{n-2}$. (7)

(ii) Prove that $\vec{F} = (y^2\cos x + z^3)\vec{i} + (2y\sin x - 4)\vec{j} + 3xz^2\vec{k}$ is irrotational and find its scalar potential. (7)

(OR)

b) Verify Gauss divergence theorem for $\vec{F} = x^2\boldsymbol{i} + y^2\boldsymbol{j} + z^2\mathbf{k}$ **,** where S is the surface of the cuboid formed by the planes x =0 , x= a , y = 0, y = b, z = 0, and z = c .

23. a) (i) Show that an analytic function with constant modulus is constant. (6)

(ii) If f(z) is analytic prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$$ (8)

(OR)

b) (i) Prove that $x^2 - y^2 + e^{-2x}\cos 2y$ is harmonic and find its harmonic conjugate. (7)

(ii) Show that f(z) = z^3 is analytic and find its derivative. (7)

24. a) (i) Show that transformation $w = \frac{1}{z}$ maps the circle $|z-3| = 5$ onto the circle

$$\left|w+\frac{3}{16}\right| = \frac{5}{16}.$$ (7)

(ii) Find the bilinear transformation which maps the points $z = 0, 1, \infty$ into $w = i, 1, -i$ respectively. (7)

(OR)

b) (i) Show that the image of the circle $|z-1|$=1under the transformation w = z^2 is the cardioid R = 2(1+ $\cos\varphi$). (6)

(ii) Discuss the transformation w = $\cos z$. (8)

25. a) (i) Evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz, C\, is\, |z| = 3.$ using Cauchy's integral formula. (6)

(ii) Using residue calculus prove that $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta} = \frac{\pi}{2}$. (8)

(OR)

b) (i) Expand the function $f(z) = \frac{z}{(z-1)(z-3)}$ as Laurent series, valid in the region

$3 < |z+2| < 5$. (6)

(ii) Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)}$, using contour integration. (8)

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