Register Number: ………………

B.E/B.TECH. DEGREE EXAMINATIONS: NOV/DEC 2010

First Semester

MAT 101: ENGINEERING MATHEMATICS I

(Common to all Branches)

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. Two eigen values of $A=\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ are 2 and 8 then the third eigen value is

 a) 2 b) -2 c) 3 d) 4

2. Nature of the quadratic form $x_1^2 - x_2^2 + 4x_3^2$ is

 a) Indefinite b) positive definite c) negative definite d) definite

3. The angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$ is

 a) 90^0 b) 30^0 c) 60^0 d) 45^0

4. If the direction ratios of a line are 3,4,12 then the direction cosines are

 a) $\frac{1}{13},\frac{2}{13},\frac{3}{13}$ b) $\frac{1}{12},\frac{4}{12},\frac{5}{12}$ c) $\frac{3}{13},\frac{4}{13},\frac{12}{13}$ d) $\frac{2}{13},\frac{3}{13},\frac{4}{13}$

5. The curvature of a circle $x^2 + y^2 = 25$ at any point on it is

 a) 5 b) $\frac{1}{5}$ c) 25 d) $\frac{1}{25}$

6. The family of curves which touches every member of the family is

 a) evolute b) envelope c) involute d) tangent

7. If $z = f(u,v)$ where u, v are functions of x and y, then the value of $\frac{\partial z}{\partial x} =$

 a) $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial u}.\frac{\partial v}{\partial x}$ b) $\frac{\partial z}{\partial u}.\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}.\frac{\partial v}{\partial x}$ c) $\frac{\partial z}{\partial u}.\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}.\frac{\partial v}{\partial y}$ d) $\frac{\partial z}{\partial u}.\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}.\frac{\partial v}{\partial x}$

8. If $x = r\cos\theta,\ y = r\sin\theta$ then the value of $\frac{\partial(x,y)}{\partial(r,\theta)}$ is

 a) $\frac{1}{r}$ b) r c) $-\frac{1}{r}$ d) – r

9. The solution of $\frac{d^2x}{dt^2} + n^2x = 0$ is

 a) $x = Ae^{nt} + Be^{-nt}$ b) $x = A\cos nt + B\sin nt$ c) $x = (At + B)e^{-nt}$ d) $x = (At + B)e^{nt}$

10. The particular integral of $(D-2)^2 y = e^{2x}$ is

 a) $-\frac{x^2}{4}e^{2x}$ b) $2xe^{2x}$ c) xe^{2x} d) $\frac{x^2}{2}e^{2x}$

PART B (10 x 2 = 20 Marks)

11. Find the eigen values of the matrix $\begin{pmatrix} 6 & 10 \\ 14 & 25 \end{pmatrix}$.

12. Using Cayley-Hamilton theorem find A^{-1} *if* $A=\begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$.

13. Find the direction cosines of the line joining the points (2, 3, - 6) and (3, - 4, 5).

14. Find the equation of a sphere having the points (- 4, 5, 1) and (4, 1, 7) as ends of a diameter.

15. Find the curvature at $x=0$ *on* $y=e^x$.

16. Find the envelope of $x\cos\alpha + y\sin\alpha = 1$ where α is the parameter.

17. Write the Maclaurin's series for a function of two variables x and y.

18. Find the stationary values of $x^2+y^2+6x+12$.

19. Transform $x^2y''-xy'-3y=0$ into a differential equation with constant coefficients.

20. Solve $\frac{dx}{dt}-y=0, \frac{dy}{dt}+x=0.$

PART C (5 x 14 = 70 Marks)

21. a) (i) Find the eigen values and eigen vectors of the matrix A = $A=\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. (7)

(ii) Verify Cayley Hamilton theorem for the matrix $A=\begin{pmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{pmatrix}$. (7)

(OR)

b) Reduce the quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to its canonical form through an orthogonal transformation.

22. a) (i) Find the equation of the plane through the line of intersection of the planes x + y + z = 1, 2x + 3y + 4z = 5 which is perpendicular to the plane x + y + z =0. (7)

(ii) Show the lines $\frac{x-4}{2}=\frac{y-5}{3}=\frac{z-6}{4}$ & $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar and find the equation of the plane in which they lie. (7)

(OR)

b) (i) Find the equation of the line of shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ *and* $\frac{x-3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. (7)

(ii) Find the equation of the sphere having the circle $x^2+y^2+z^2+10y-4z-8=0$, $x+y+z=3$ as a great circle. (7)

23. a) (i) Find the radius of curvature of $y^2 = \frac{a^3 - x^3}{x}$ at the point (a,0) . (7)

(ii) Find the evolute of the parabola $y^2 = 4ax$. (7)

(OR)

b) (i) Find the envelope of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where a and b are connected by the relation $a + b = c$. (7)

(ii) Find the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$. (7)

24. a) (i) Given the transformation $u = e^x \cos y \,\&\, v = e^x \sin y$ and that f is function of u and v and also of x and y, prove that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$. (7)

(ii) Expand $e^x \cos y$ in power of x and y as far as the terms of the 3rd degree. (7)

(OR)

b) (i) A rectangular box open at the top is to have a volume of 32cc. Find the dimensions of the box, that requires the least material for its construction. (7)

(ii) If $x = r\cos\theta, y = r\sin\theta$, prove that $\frac{\partial(x, y)}{\partial(r,\theta)} \times \frac{\partial(r,\theta)}{\partial(x, y)} = 1$. (7)

25.a) (i) Solve $\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 4y = e^{-x}\sin 2x$. (7)

(ii) Solve $x^2 \frac{d^{2y}}{dx^2} + x\frac{dy}{dx} + y = 4\sin(\log x)$. (7)

(OR)

b)(i) Solve $y'' + 4y = \tan 2x$ by the method of variation of parameters. (7)

(ii) Solve the simultaneous differential equations $\frac{dx}{dt} + 2y = \sin 2t; \frac{dy}{dt} - 2x = \cos 2t$. (7)

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