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B.E/B.TECH DEGREE EXAMINATIONS: APRIL/MAY 2012

Fourth Semester

MAT108: NUMERICAL METHODS

(Common to Civil, EEE, EIE, AERO, ME, MCE, TXT)

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions:-

PART A (10 x 1 = 10 Marks)

1. The order of convergence of Regula falsi method is

a) 2.618 b) 1.618 c) 1.518 d) – 1.618

2. In Gauss Jordan method the coefficient matrix is transformed into

a) an upper triangular matrix b) a lower triangular matrix

c) a diagonal matrix d) any square matrix

3. In the interpolation formula $y(x)=y_n+v\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_n+\ldots\ldots,$ v =

a) $\frac{x+x_0}{h}$ b) $\frac{x-x_0}{h}$ c) $\frac{x+x_n}{h}$ d) $\frac{x-x_n}{h}$

4. If $f(x)=\frac{1}{x}$, then $f(x_0,x_1,\ldots\ldots\ldots\ldots x_n)$ is

a) $\frac{(-1)^n}{x_0x_1x_2x_3\ldots\ldots x_n}$ b) $\frac{1}{x_0x_1x_2\ldots\ldots x_n}$ c) $\frac{1}{x_0x_1x_2\ldots\ldots x_{n-1}}$ d) $\frac{(-1)^n}{x_1x_2\ldots\ldots x_n}$

5. In Newton's forward difference formula the second derivative of y with respect to x at x = x_0 is

a. $\frac{1}{h^2}\left[\Delta^2 y_0+\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0+\ldots\ldots\right]$ b. $\frac{1}{h^2}\left[\Delta^2 y_0+\frac{11}{12}\Delta^4 y_0+\ldots\ldots\right]$

c. $\frac{1}{h^2}\left[\Delta^2 y_0-\Delta^3 y_0+\frac{11}{12}\Delta^4 y_0+\ldots\ldots\right]$ d. $\frac{1}{h^2}\left[\Delta^2 y_0+\Delta^3 y_0+\Delta^4 y_0+\ldots\ldots\right]$

6. The error in the Trapezoidal rule is of the order

a. h^2 b. h c. h^3 d. $\frac{h}{2}$

7. The Milne's predictor formula is

a) $y_{n+1}=y_{n-3}+\frac{4h}{3}\left(2y'_{n-2}-y'_{n-1}+2y'_n\right)$ b) $y_{n+1}=y_{n-3}-\frac{4h}{3}\left(2y'_{n-2}-y'_{n-1}+2y'_n\right)$

c) $y_{n+1}=y_{n-3}+\frac{4h}{3}\left(2y'_{n-2}-y'_{n-1}+2y'_n\right)$ d) $y_{n+1}=y_{n-3}+\frac{4h}{3}\left(2y'_{n-2}-y'_{n-1}-2y'_n\right)$

8. If $y'=x+y, y(0)=1, h=0.1$ then by fourth order Runge Kutta method k_2 is

a) 0.1 b) 0.1103 c) 0.11 d) 0.01

9. The equation $xu_{xx} + u_{yy} = 0, x > 0$ is

a) elliptic b) parabolic c) Hyperbolic d) None

10. The Crank – Nicholson formula reduces to $u_{i,j+1} = \frac{1}{4}\left[u_{i-1,j+1} + u_{i+1,j+1} + u_{i-1,j} + u_{i+1,j}\right]$ if

a) $k = ah^2$ b) $k = ah$ c) $k = ah^3$ d) $k = ah^4$

PART B (10 x 2 = 20 Marks)

11. Solve $3x - y = 2,\ x + 3y = 4$ using Gaussian elimination method.

12. What is the condition for the convergence to solve $\cos x = 3x - 2$ by iteration method?

13. Form the divided difference table for the following data.

x	5	15	22
y	7	36	160

14. State Stirling's formula for interpolation. When is it used?

15. Write down the expression for $\frac{dy}{dx} \& \frac{d^2y}{dx^2} at\ x = x_0$ by Newton's forward difference formula.

16. A curve passes through (0, 1), (0.25, 0.9412), (0.5, 0.8) (0.75, 0.64) and (1.0, 0.5). Find the area between the curve, x – axis and x = 0, x = 1 by Trapezoidal rule.

17. By using Euler's method for $y' = -y; y(0) = 1$ find $y(0.1)$.

18. Write R-K method of fourth order algorithm.

19. Name the two methods to solve one – dimensional heat equation?

20. Write down the Leibmann's iterative formula for solving the Laplace equation.

PART C (5 x 14 = 70 Marks)

21.a) (i) Find a positive root of $xe^x = 2$ by the method of False position.

(ii) Find an iterative formula to find the reciprocal of a given number N and hence find the value of $\frac{1}{19}$.

[OR]

b) (i) Solve by Gauss Jordan method

$10x + y + z = 12,\ 2x + 10y + z = 13,\ x + y + 5z = 7$

(ii) Solve the following system by Gauss-Jacobi method.

$10x - 5y - 2z = 3,\ 4x - 10y + 3z = -3, x + 6y + 10z = -3$

22. a) (i) Find the equation $y = f(x)$ of least degree and passing through the points

(-1, -21),(1, 15) (2, 12) (3, 3). Find also y at x = 0.

(ii) Use Lagrange's formula to fit a polynomial to the data

x:	-1	0	2	3
y:	-8	3	1	12

and hence find y(x =1)

[OR]

b) The population of a town is as follows:

Year :	1941	1951	1961	1971	1981	1991
Population in lakhs:	20	24	29	36	46	51

Estimate the population increase during the period 1946 to 1976.

23. a) Find the first two derivatives of $(x)^{1/3}$ at $x=50$ and $x=56$ given that table below

x:	50	51	52	53	54	55	56
$y=x^{1/3}$:	3.6840	3.7084	3.7325	3.7563	3.7798	3.8030	3.8259

[OR]

b) (i) Find $y'(5) \& y'(6)$ by using divided difference formula given:

x	0	2	3	4	7	9
y	4	26	58	112	466	922

(ii) A river is 80 meters wide. The depth'd' in meters at a distance x from the bank is given by the following table. Calculate the area of cross section of the river using Simpson's rule

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

24. a) (i) Using Taylor series method, find y at x = 1.1, 1.2, given $\frac{dy}{dx}=x+y,\ y(1)=0,$ correct to 4 decimal places.

(ii) Using Runge – Kutta method of fourth order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ given that $y(0)=1$ at $x=0.2,\ 0.4$

[OR]

b) Consider the initial value problem $\frac{dy}{dx}=1-y,\ y(0)=0$

(i) Using the Euler method, find $y(0.1)$.

(ii) Using Improved Euler method, find $y(0.2)$ and $y(0.3)$

(iii) Using Milne's Predictor – corrector method, find $y(0.4)$.

25. a) (i) Solve $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ given $u(0,t)=0,\ u(4,t)=0,\ u(x,0)=x(4-x),$ taking $h=1$.

Find the value of u upto t = 5 using Bender – Schmidt's explicit finite difference scheme.

(ii) Solve $\nabla^2 u = -10(x^2+y^2+10)$ over the squares mesh bounded by x = 0; y = 0; x = 3; y = 3 with u = 0 on the boundary and mesh length is 1 unit.

[OR]

b) (i) Solve $u_{xx} = 16u_t \quad 0<x<1, t>0$ given $u(0,t)=0\ u(l,t)=100t;\quad u(x,0)=0$ in $0<x<1$ compute u for one time step by using Crank – Nicholson method by assuming h = ¼.

(ii) Solve the elliptic equation $\nabla^2 u = 0$ over the square region of side 4, satisfying the boundary conditions: $u(0,y)=0$ for $0 \le y \le 4,\ u(4,y)=12+y$ for $0 \le y \le 4,\ u(x,0)=3x$ for $0 \le x \le 4,$ & $u(x,4)=x^2$ for $0 \le x \le 4$. Obtain solution correct to two decimal places. Take h = 1 = k.

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