Register Number.................................

B.E. / B. TECH. DEGREE EXAMINATIONS: APRIL / MAY 2009

Third Semester

U07MA301 MATHEMATICS III

(Common to All Branches)

Time: Three Hours **Maximum Marks: 100**

Answer ALL the Questions:-

PART A (20 x 1 = 20 Marks)

1. The partial differential equation by eliminating the arbitrary constants a and b from $z=(x+a)^2+(y-b)^2$ is ________.

   (A) $z=p^2+q^2$ (B) $z=p^2-q^2$ (C) $4z=p^2+q^2$ (D) $4z=p^2-q^2$

2. The Complete Integral of $z=px+qy-\sqrt{pq}$ is

   (A) $z=ax+by+\sqrt{ab}$ (B) $z=ax+by-\sqrt{ab}$ (C) $z=ax-by-\sqrt{ab}$ (D) $z=ax-by+\sqrt{ab}$

3. The solution of $(D^3-3DD'^2+2D'^3)z=0$

   (A) $z=f_1(y+x)+xf_2(y+x)+f_3(y-2x)$ (B) $z=f_1(y+x)+xf_2(y+x)-f_3(y-2x)$

   (C) $z=f_1(y-x)+xf_2(y-x)-f_3(y-2x)$ (D) $z=f_1(y-x)+xf_2(y-x)+f_3(y-2x)$

4. The partial differential equation by eliminating the arbitrary function from $z=f(x^2-y^2)$ is

   (A) $px+qy=0$ (B) $px-qy=0$ (C) $py+qx=0$ (D) $py+qx=0$

5. The value of (a_0, a_n) in the Fourier series expansion $f(x)=x+x^3$, $(-\pi,\pi)$ is

   (A) (1 , 5) (B) (1 , 3) (C) (0 , 0) (D) (0 , 1)

6. If $f(x)=x+x^2$ is expressed as a Fourier series in the interval (-2, 2) to which value this series converges at x = 2

   (A) 5 (B) 3 (C) 8 (D) 4

7. If f (x) = x, $(0,2\pi)$ the value for $\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$ is

   (A) $\frac{8}{2}\pi^2$ (B) $\frac{2}{8}\pi^2$ (C) $\frac{8}{3}\pi^2$ (D) $\frac{3}{8}\pi$

8. If f (x) = $\begin{cases} 1+\dfrac{2x}{\pi} & -\pi < x < 0 \\ 1-\dfrac{2x}{\pi} & 0 < x < \pi \end{cases}$, the value of b_n is

(A) 1 (B) 0 (C) 8 (D) 10 cos nx

9. The constant a^2 in the wave equation $u_{tt} = a^2 u_{xx}$

(A) $\dfrac{\text{tension}}{\text{mass per unit length of the string}}$ (B) $\dfrac{\text{mass per unit length of the string}}{\text{tension}}$

(C) mass per unit length X tension (D) $\dfrac{\text{velocity}}{\text{tension}}$

10. A rod of length $20\ cm$ whose one end is kept at 30^0C and the other end is kept at 70^0C is maintained so until steady state previles, the steady state temperature is

(A) 20 x + 30 (B) 2 x + 30 (C) 3 x + 20 (D) 30 x + 20

11. The suitable solution of the one dimensional heat equation $\dfrac{\partial u}{\partial t} = a^2 \dfrac{\partial^2 u}{\partial x^2}$

(A) (A cos px +B sin px) $e^{-c^2p^2t}$ (B) (A cos px +B sin px) $e^{c^2p^2t}$

(C) (A cos px - B sin px) $e^{-c^2p^2t}$ (D) (A cos px -B sin px) $e^{c^2p^2t}$

12. The nature of the partial differential equation $4u_{xx} + 4u_{xy} + u_{yy} + 2u_x - u_y = 0$ is

(A) Hyperbolic equation (B) Elliptic equation (C) Parabolic equation (D) Circle

13. The Fourier transform of f(x) if $f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a > 0 \end{cases}$

(A) $\sqrt{\dfrac{2}{\pi}}\dfrac{\cos as}{s}$ (B) $\sqrt{\dfrac{2}{\pi}}\dfrac{\sin as}{a}$ (C) $\sqrt{\dfrac{2}{\pi}}\dfrac{\cos as}{a}$ (D) $\sqrt{\dfrac{2}{\pi}}\dfrac{\sin as}{s}$

14. If F (f (x)) = F (s) then $F_s(f(ax))$

(A) $\dfrac{1}{s}F_s\left(\dfrac{s}{a}\right)$ (B) $\dfrac{1}{a}F_s\left(\dfrac{a}{s}\right)$ (C) $\dfrac{1}{s}F_s\left(\dfrac{a}{s}\right)$ (D) $\dfrac{1}{a}F_s\left(\dfrac{s}{a}\right)$

15. The Fourier transform of $e^{-\frac{x^2}{2}}$

(A) $e^{-\frac{x^2}{2}}$ (B) e^{-x^2} (C) $e^{\frac{x^2}{2}}$ (D) $e^{-\frac{x^2}{s}}$

16. The Fourier cosine transform of e^{-ax} , $a > 0$.

(A) $\sqrt{\frac{2}{\pi}}\frac{s}{s^2+a^2}$ (B) $\sqrt{\frac{\pi}{2}}\frac{s}{s^2+a^2}$ (C) $\sqrt{\frac{\pi}{2}}\frac{a}{s^2+a^2}$ (D) $\sqrt{\frac{2}{\pi}}\frac{a}{s^2+a^2}$

17. Definition of Z – transform (two-sided), $Z(x(n))$ is

(A) $\sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$ (B) $\sum_{n=-\infty}^{\infty} x(n)\, z^{n}$ (C) $\sum_{n=0}^{\infty} x(n)\, z^{-n}$ (D) $\sum_{n=0}^{\infty} x(n)\, z^{n}$

18. The value of $Z(a^n)$ if $|z| > |a|$

(A) $\frac{z}{z+a}$ (B) $\frac{z}{z-a}$ (C) $\frac{z^2}{z+a}$ (D) $\frac{z^2}{z-a}$

19. The value of $Z\left(a^{at+b}\right)$

(A) $\frac{z}{z+z^{aT}}$ (B) $\frac{aT}{z-z^{aT}}$ (C) $e^b\frac{z}{z-z^{aT}}$ (D) $e^b\frac{z}{z+z^{aT}}$

20. If $Z(f(n)) = F(Z)$, then $Z(n f(n))$ is

(A) $z\frac{d}{dz}F(Z)$ (B) $-\frac{d}{dz}F(Z)$ (C) $-z^2\frac{d}{dz}F(Z)$ (D) $-z\frac{d}{dz}F(Z)$

PART B (5 x 16 = 80 Marks)

21. (a) . (i) . Obtain the complete integral and singular integral of

$$z = px + qy + \sqrt{p^2 + q^2 + 16} \quad (8)$$

(ii) . Solve $(D^3 - 7DD'^2 - 6D'^3)z = x + e^{2x+3y}$ (8)

(OR)

(b) . (i) . Solve $x[y^2 - z^2]p + y[z^2 - x^2]q - z[x^2 - y^2] = 0$ (8)

(ii). Solve $(D^3 + D^2D' - DD'^2 - D'^3)z = e^{2x+y} + \cos(x+y)$ (8)

22. (a) . (i) . Obtain Fourier series for f(x) of period 2l for the function

$$f(x) = \begin{cases} l - x & 0 < x \le l \\ 0 & l \le x < 2l \end{cases}$$

Hence deduce that

(1) $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots\dots\dots = \frac{\pi}{4}$ (2) $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots\dots\dots = \frac{\pi^2}{8}$ (16)

(OR)

(b) . (i). Find the half range cosine series of

$$f(x) = x, \quad 0 < x < \pi$$

(8)

(ii). Obtain Fourier series up to the second harmonic for the following data: (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	0.8	0.6	0.4	0.7	0.9	1.1

23. (a). A string is stretched and fastened to two points $x = 0$ and $x = l$ apart. Motion is started by displacing the string into the form $y = k\left(lx - x^2\right)$ from which it is released at time $t = 0$. Find the displacement of any point on the string at a distance of x from one end at time t. (16)

(OR)

(b). A square plate is bounded by $x = 0, x = l, y = 0, y = l$ The edge $x = 0$ is maintained at 100^0C and the other three edges are kept at 0^0C. Find steady state temperature at any point within the plate. (16)

24. (a) . Find the Fourier transforms of

$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| \geq a \end{cases}$. Hence deduce that (i) $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$ (ii) $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$

(16)

(OR)

(b) . Find the Fourier sine and cosine transform of e^{-x} and hence find the Fourier sine transform of $\frac{x}{1+x^2}$ and Fourier cosine transform of $\frac{1}{1+x^2}$ (16)

25. (a). (i) Find $Z\left(r^n \cos n\theta\right)$ and hence find $Z\left(a^n r^n \cos n\theta\right)$ (8)

(ii) Using Z – transform, solve $u_{n+2} + 3u_{n+1} + 2u_n = 0$ *given that* $u_0 = 1$, $u_1 = 2$. (8)

(OR)

(b). (i). Find $Z^{-1}\left(\frac{z\left(z^2 - z + 2\right)}{(z+1)(z-1)^2}\right)$ using partial fraction method. (8)

(ii) Using Z – transform, solve $u_{n+2} - 5u_{n+1} + 6u_n = 4^n$ *given that* $u_0 = 0$, $u_1 = 1$. (8)

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