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B.E. DEGREE EXAMINATIONS: APRIL /MAY 2009

Fourth Semester

ELECTRONICS AND COMMUNICATION ENGINEERING

U07MA403 Random Processes

Time: Three Hours **Maximum Marks: 100**

(Statistical table are permitted)

Answer ALL the Questions:-

PART A (20 × 1 = 20 Marks)

1. Let S be the sample space and A be the event such that P (A) lies between

   (A) -1 to1 (B) 0 to 1 (C) $-\infty$ to ∞ (D) -1 to 0

2. If the events A and B are independent then P ($\bar{A} \cap \bar{B}$) is

   (A) P ($\bar{A}$) + P ($\bar{B}$) (B) P (A) +P (B) (C) P ($\bar{A}$).P ($\bar{B}$) (D) 1

3. A random variable X has the following probability distribution

x :	-2	-1	0	1	2	3
P(x):	0.1	k	0.2	2k	0.3	3k

   then the value of k is

   (A) $\frac{1}{12}$ (B) $\frac{1}{10}$ (C) $\frac{1}{15}$ (D) $\frac{1}{20}$

4. The function f(x) = kx, 0<x<1 is a pdf of a continuous random variable x. Then the value of k is

   (A) 1 (B) 4 (C) $\frac{1}{2}$ (D) 2

5. The mean and variance of the binomial distribution is

   (A) np, $\sqrt{npq}$ (B) np, npq (C) nq, np (D) $\sqrt{npq}$, np

6. The Poisson distribution is a limiting case of

   (A) Geometric distribution (B) Normal distribution
   (C) Binomial distribution (D) Exponential distribution

7. The mean and variance of the exponential distribution is

   (A) $\frac{1}{\lambda}, \frac{1}{\sqrt{\lambda}}$ (B) $\frac{1}{\lambda}, \frac{1}{\lambda^2}$ (C) $\frac{1}{\lambda}, \frac{1}{\lambda}$ (D) $\frac{1}{\lambda^2}, \frac{1}{\lambda}$

8. For a normal distribution

   (A) mean ≠median ≠ mode (B) mean ≠ median = mode
   (C) mean =median = mode (D) mean = median ≠mode

9. The correlation coefficient ρ_{xy} lies between

   (A) $-\infty$ to ∞ (B) 0 to ∞ (C) $-\infty$ to 0 (D) -1 to 1

10. If X and Y are orthogonal then E (XY) =

(A) 1 (B) 0 (C) -1 (D) 2 21

11. Cov (ax, by) is

(A) a^2b^2 cov (X,Y) (B) a^2b cov (X,Y) (C) ab cov (X,Y) (D) cov (X,Y)

12. The line of regression of y on x is given by

(A) $y - \bar{y} = b_{yx}(x - \bar{x})$ (B)) $y + \bar{y} = b_{yx}(x - \bar{x})$

(C)) $y - \bar{y} = b_{xy}(x - \bar{x})$ (D) $y - \bar{y} = b_{yx}(x + \bar{x})$ 22

13. Every strict sense stationary process is a

(A) Ergodic process (B) WSS process (C) not WSS process (D) Random process

14. A random process X (t) is WSS if

(A) $E(X(t)) \neq$ constant $R_{xx}(t, t+\tau) = R_{xx}(\tau)$ (B) $E(X(t)) =$ constant $R_{xx}(t, t+\tau) \neq R_{xx}(\tau)$ 22
(C) $E(X(t)) =$constant $R_{xx}(t, t+\tau) = R_{xx}(\tau)$ (D) $E(X(t)) \neq$ constant $R_{xx}(t, t+\tau) \neq R_{xx}(\tau)$

15. The sum of two independent Poisson process is
(A) Normal process (B) sine wave process (C) Poisson process (D) Not a Poisson process

16. Normal process is
(A) WSS and SSS (B) not WSS and SSS (C) WSS and not SSS (D) not WSS and Not SSS 23

17. Auto correlation function of a random process X (t) is an
(A) Odd function (B) even function
(C) neither odd nor even function (D) not an even function

18. Two independent random process will have their cross correlation as ---------- of individual means
(A) Sum (B) subtraction (C) product (D) division

19. If $R_{xy}(\tau) = 0$ then the process X (t) and Y (t) are

(A) Independent (B) not orthogonal (C) orthogonal (D) not independent

20. The autocorrelation of a random process X (t) is $R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$ then the mean of X (t) is
(A) 4 (B) 10 (C) 2 (D) 5

PART B (5 × 16 = 80 Marks)

21. a (i) An urn contains 10 white and 3 black balls. Another urn contains 3 white and 5 black balls. Two balls are drawn at random from the first urn and placed in the second urn and then one ball is taken at random from the latter. What is the probability that it is a white ball. (10)

(ii) 'A' can hit the target in 4 out of 5 shots and 'B' can hit the target in 3 out of 4 shots. Find the probability that the target being hit when both try. (6) 23

(OR)

21. b (i) A continuous random variable X has a pdf $f(x) = k x^2 e^{-x}$, $x > 0$. Find k, mean and variance (8)

(ii) A random variable X has the probability function $P(X = x) = \frac{1}{2^x}$, $x = 1, 2, \ldots$ Find

(1) M.G.F (2) mean (8)

22 . a (i) Find the mean ,variance , M.G.F of the binomial distribution. (10)

(ii) Find the M.G.F of the exponential distribution and hence find mean. (6)

(OR)

22.b (i) Find the mean ,variance , M.G.F of Poisson distribution. (8)

(ii) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, find the probability that at least one of them would have scored above 75. (8)

23. a (i) The joint pdf of the random variable (X,Y) is given by

$$f(x, y) = \begin{cases} C x (x-y), & 0 < x < 2, \ -x < y < x \\ 0 & \text{otherwise} \end{cases}$$

Find (i) C (ii) $f_X(x)$ (iii) $f(y/x)$. (8)

(ii) The following table gives the joint probability distribution of the random variable (X, Y). Find all the marginal and conditional distributions. (8)

y \ x	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

(OR)

23. b (i) The random variable X and Y has the pdf

$$f(x, y) = \begin{cases} x + y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Obtain the correlation coefficient between x and y. (10)

(ii) If $X_1, X_2 \ldots.X_n$ are Poisson variates with parameter $\lambda = 2$ .Use central limit theorem to estimate P $(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + \ldots.. + X_n$ and n = 75 (6)

24 .a (i) Show that the random process X (t) = A cos (ω_0 t +θ) is WSS , if A and ω_0 are constants and θ is uniformly distributed variable in $(0,2\pi)$. (8)

(ii) The process X(t) whose probability distribution under certain conditions is given by

$$P(X(t)=n) = \begin{cases} \dfrac{(at)^{n-1}}{(1+at)^{n+1}}, & n = 1, 2, 3, \ldots\ldots \\ \dfrac{at}{1+at}, & n = 0 \end{cases}$$

Show that it is not stationary. (8)

(OR)

24.b (i) A man either drives a car or catches a train to go to office each day. He never goes 2 days in row by train but if he drives one day , then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work if and only if a 6 appeared. Find (1) the probability that he takes a train on the third day and (2) the probability that he drives to work in the long run. (10)

(ii) Prove that the sum of two independent Poisson process is a Poisson process. (6)

25. a (i) A random process X (t) = 10 cos (100t +θ) where $\theta \in (-\pi, \pi)$ followed uniform distribution. Find the autocorrelation function of the process. (8)

(ii) If the autocorrelation of X(t) is

$$R_{xx}(\tau) = \begin{cases} 1-|\tau|, & |\tau| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Obtain the spectral density of X. (8)

(OR)

25 .b. If Y(t) = X (t + a) – X(t - a), prove that $R_{yy}(\tau) = 2 R_{xx}(\tau) - R_{xx}(\tau + 2a) - R_{XX}(\tau - 2a)$.

Hence prove that $S_{yy}(\omega) = 4 \sin^2 a\omega \, S_{xx}(\omega)$ (16)

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