Reg. No. :

B 2256

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Fourth Semester

Electronics and Communication Engineering

MA 034 — RANDOM PROCESSES

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Suppose that 25% of the population of a country are unemployed women and a total of 35% are unemployed. What percent of the unemployed are women?

2. A factory produces its entire output with three machines. Machines I, II and III produce 50%, 30% and 20% of the output, but 4%, 2% and 4% of their outputs are defective respectively. What fraction of the total output is defective?

3. The joint probability density function of a random variable (X, Y) is given by $f_{XY}(X,Y) = Cxy$, $0 < x < 2$, $0 < y < 2$, where C is a constant. Find the value of C.

4. Let X and Y be continuous random variables with joint probability density function $f_{XY}(x,y) = \frac{3}{2}(x^2 + y^2)$, $0 < x < 1, 0 < y < 1$. Find $f_{X/Y}(x/y)$.

5. Show that a Binomial process is a Markov process.

6. For a Poisson process with parameter λ and for $s < t$ show that $P\{N(s) = k \mid N(t) = n\} = \binom{n}{k}\left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}, k = 0,1,2,\ldots,n$.

7. Show that the autocorrelation function $R_{xx}(\tau)$ is maximum at $\tau = 0$.

8. Statistically independent zero-mean random processes $x(t)$ and $y(t)$ have autocorrelation functions $R_{xx}(\tau) = e^{-|\tau|}$ and $R_{yy}(\tau) = \cos 2\pi\tau$ respectively. Find the autocorrelation function of the sum $Z(t) = x(t) + y(t)$

9. A wide sense stationary noise process $N(t)$ has an autocorrelation function $R_{NN}(\tau) = Pe^{-3|\tau|}$, $-\infty < \tau < \infty$ with P as a constant. Find its power density spectrum.

10. The power spectral density of a stationary random process is a constant in a symmetrical interval about zero and zero outside the interval. Compute the autocorrelation function.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Suppose that, on an average, a post office handles 10,000 letters a day with a variance of 2000. What can be said about the probability that this post office will handle between 8000 and 12000 letters tomorrow? (6)

(ii) The moment generating function of a random variable X is $(.6e^{t} + .4)^2$, what is the moment generating function of $Y = 3X + 2$. Also find the mean and variance of the random variable X. (6)

(iii) Let X be a continuous random variable with probability density function $f_X(x) = \frac{2}{x^2}, 1 < x < 2$, Find $E[\log X]$. (4)

Or

(b) (i) Let X be a Gamma random variable with parameters n and λ. Find the moment generating function of X and use it to find $E[X]$ and var $[X]$. (6)

(ii) A stack of cards consists of 6 red and 5 blue cards. A second stack of cards consists of 9 red cards. A stack is selected at random and 3 of its cards are drawn. If all of them are red, what is the probability that the first stack was selected? (6)

(iii) The cumulative distribution function for a random variable X is given by $F_X(x) = 1 - e^{-5x}, x \geq 0$. Find var$(5X + 4)$. (4)

12. (a) Let the joint probability density function of (X,Y) be $f_{XY}(x,y)=e^{-y}$, $0<x<y<\infty$. Find the correlation coefficient $r(X,Y)$. (16)

Or

(b) (i) If 20 random numbers are selected independently from the interval $(0,1)$ with the probability density function of each number is $f_X(x)=1,\ 0<x<1$, what is the approximate probability that the sum of these numbers is at least 8? (8)

(ii) Let X and Y be independent random variables with common probability density function $f_X(x)=e^{-2x}, x>0$. Find the joint pdf of $U=X+Y$ and $V=e^{x}$. (8)

13. (a) Show that the random process $x(t)=\cos(t+\phi)$ where ϕ is uniformly distributed in $(0,\ 2\pi)$ with the probability density function $f_\phi(x)=\dfrac{1}{2\pi}, 0<x<2\pi$ is

(i) First order stationary

(ii) Stationary in wide sense

(iii) Ergodic. (16)

Or

(b) (i) Prove that the process $x(t)=9+A$ with $f_A(\alpha)=1, 0<\alpha<1$ is

(1) First order stationary

(2) Second-order stationary

(3) Strictly stationary

(4) Not Ergodic. (8)

(ii) A stochastic process is described by $x(t)=U\sin 2t+V\cos 2t$ where U and V are independent random variables with zero means and equal variances. Find the variance and covariance of the given process. (8)

14. (a) Given the random process $x(t)=\sum_{n=-\infty}^{\infty}A_n g(t-nb+\phi)$ where

$g(t)=\begin{cases}1 & 0<t<b\\ 0 & \text{otherwise}\end{cases}$, $A_i' s$ and ϕ are independent random variables with density functions $f_A(\alpha)=\frac{1}{2}\delta(\alpha-1)+\frac{1}{2}\delta(\alpha+1)$ and $f_\phi(\alpha)=\frac{1}{b}, 0<\alpha<b$. The joint probability mass function of A_i is given by

A_i \ A_{i+1}	−1	+1
−1	0.4	0.1
+1	0.1	0.4

Find the autocorrelation function $R_{xx}(\tau)$ in $0 < \tau < b$ for the above process. (16)

Or

(b) Given $x(n) = \sum_k X_k \delta(n-k)$ where the joint mass function for the random variables X_k is given below :

X_k \ X_{k+1}	0	1
0	0.25	0.25
1	0.25	0.25

Find the autocorrelation function $R_{xx}(k)$ and covariance function $L_{xx}(k)$ for k = 0, 1 2, 3.

15. (a) (i) For a linear system with random input $x(t)$, the impulse response $h(t)$ and output $y(t)$, obtain the cross correlation function and cross power spectral density functions. (8)

(ii) Given the power spectral density $\dfrac{1+\omega^2}{\omega^4 + 4\omega^2 + 4}$. Use residue theory to find the average power in the process $x(t)$. (8)

Or

(b) (i) Show that in an input-output system the energy of a signal is equal to the energy of its spectrum. (8)

(ii) Find the cross-correlation function corresponding to the cross-power spectrum $S_{xy}(\omega) = \dfrac{6}{(9+\omega^2)(3+j\omega)^2}$. (8)