Reg. No. : | | | | | | | | | | | |

A 1424

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Second Semester

Information Technology

MA 039 — PROBABILITY AND STATISTICS

(Common to Fourth Semester B.Tech. Industrial Bio-Technology, B.E. Metallurgical Engineering)

(Common to Fifth Semester Civil Engineering)

Time : Three hours — Maximum : 100 marks

Use of Statistical Table and Control Chart is permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. The probabilities of A, B and C solving a problem are 1/3,2/5 and 3/8 respectively. If all three try to solve the problem simultaneously, find the probability that exactly one of them will solve it.

2. For a binomial distribution with mean 8 and standard deviation $\sqrt{2}$, find the first two terms of the distribution.

3. The joint density function of the two continuous random variable X and Y is

$$f(x,y) = \begin{cases} \dfrac{xy}{96}, & 0 < x < 4,\ 1 < y < 5 \\ 0, & \text{otherwise} \end{cases}$$

Find the value of $P(1 < X < 2,\ 2 < Y < 3)$.

4. Let X and Y be any two random variables and a, b be constants. Prove that $Cov(aX, bY) = abCov(X,Y)$.

5. A fair dice is tossed repeatedly. If X_n denotes the maximum of the number occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$.

6. Find the variance of the stationary process $\{X(t)\}$, whose auto correlation function is given by $R_{xx}(\tau) = 16 + \dfrac{9}{1 + 6\tau^2}$.

7. Four units are connected with reliabilities 0.96, 0.93, 0.94 and 0.95. Determine the system reliability when they are connected

(a) in series (b) in parallel.

8. It is known that the reliability function for a system is $R(t) = \frac{10}{10+t}, t \geq 0$. How many units must be placed in parallel in order to achieve a reliability of 0.98 for 5 year operation?

9. What are the basic principles of experimental design?

10. Depict the ANOVA table for two way classification.

PART B — (5 × 16 = 80 marks)

11. (a) (i) The probability function of an infinite distribution is given by $P(X = j) = \frac{1}{2^j}$ for $j = 1, 2, ...\infty$. Verify if it is a legitimate probability mass function and also find P(X is even), $P(X \geq 5)$ and P(X is divisible by 3). (8)

(ii) The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1000 taxi drivers, find approximately the number of drivers with

(1) no accidents in a year

(2) more than 3 accidents in a year. (8)

Or

(b) (i) Find the moment generating function of a exponential random variable and hence find its mean and variance. (8)

(ii) The time required to repair a machine is exponentially distributed with parameter 1/2.

(1) What is the probability that the repair time exceeds 3 hours?

(2) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours? (8)

12. (a) Let X and Y be random variables having joint density function

$$f(x,y) = \begin{cases} x + y, & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find the correlation coefficient r_{XY}. (16)

Or

(b) The joint pdf of two random variables X and Y is given by

$$f(x,y) = \begin{cases} k[(x+y) - (x^2 + y^2)], & 0 < (x,y) < 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that X and Y are uncorrelated but not independent. (16)

13. (a) (i) Let $X(t) = A\cos\omega t + B\sin\omega t$, $Y(t) = B\cos\omega t - A\sin\omega t$ where A and B are random variables and ω is a constant. Show that X(t) and Y(t) are jointly wide sense stationary if A and B are uncorrelated random variables with mean zero and same variance and ω is a constant. (8)

(ii) Consider a Markov chain with transition probability matrix

$$P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.4 & 0.5 & 0.1 \\ 0.6 & 0 & 0.4 \end{pmatrix}$$

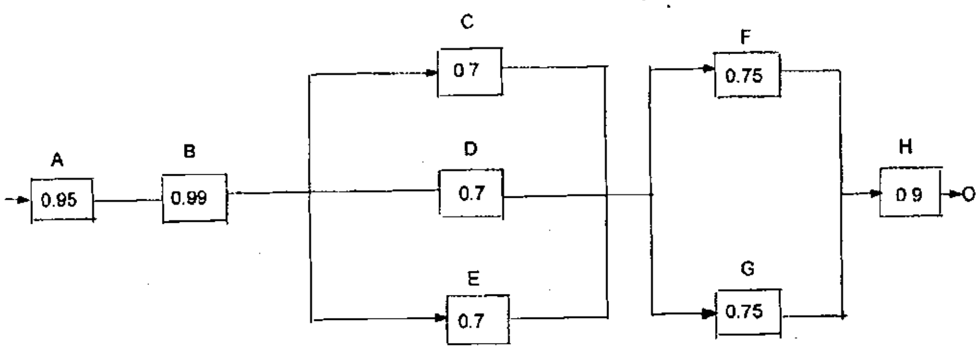
Find the steady state probabilities of the system. (8)

Or

(b) (i) Find the autocorrelation co-efficient between $X(t)$ and $X(t + s)$ if $\{X(t), t \geq 0\}$ is a Poisson process. (8)

(ii) Arrivals at the telephone booth are considered to be Poisson with an average time of 12 min. between one arrival and the other. The length of the phone call is assumed to be distributed exponentially with mean 4 min. Find the average number of persons waiting in the system. What is the probability that a person arriving at the booth will have to wait in the queue? (8)

14. (a) (i) Find the reliability of the system diagrammed below : (8)



(ii) The density function of time to failure of an appliance is $f(t) = \dfrac{32}{(t+4)^3}$, $t > 0$. Find the reliability function R(t), the failure rate $\lambda(t)$ and the MTTF. (8)

Or

(b) (i) Given that $R(t) = e^{-\sqrt{0.001t}}$, $t \geq 0$,

(1) Compute the reliability for a 50 hour mission.

(2) Find the hazard rate function.

(3) Given a 10 hour warranty period, compute the reliability for a 60 hour mission.

(4) What is the average design life for a reliability of 0.95, given a 10 hour warranty period? (8)

(ii) A critical communication relay has a constant failure rate of 0.1 per day. Once it has failed the mean time to repair is 2.5 days. What are the point availability at the end of 2 days, the interval availability over a 2 day period and the steady state availability. (8)

15. (a) Three machines A, B, C gave the production of pieces in four days as below :

A	17	16	14	13
B	15	12	19	18
C	20	8	11	17

Is there a significant difference between machines? (16)

Or

(b) Fifteen samples of 4 readings each gave the following values of $\overline{X}$ and R:

Sample No.	1	2	3	4	5	6	7	8	9	10
$\overline{X}$	16.1	15.2	14.2	13.9	15.4	15.7	15.2	15.0	16.5	14.9
R	3.0	2.1	5.6	2.4	4.1	2.7	2.3	3.8	5.0	2.9

Sample No.	11	12	13	14	15
$\overline{X}$	15.3	17.8	15.9	14.6	15.2
R	13.8	14.2	4.8	5.0	2.2

Compute the control limits for $\overline{X}$ and R charts using the above data for all the samples Hence examine if the process is in control. (16)