Reg. No. : | | | | | | | | | | | |

A 1425

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Third Semester

Civil Engineering

MA 231 — MATHEMATICS – III

(Common to all branches of B.E/B.Tech except fashion Technology Industrial Bio Technology, Bio Medical, Textile chemistry)

Time : Three hours · Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the partial differential equation from $z=x+y+f(xy)$ by eliminating the arbitrary function f.

2. Solve $(D-D')(D+2D'+1)z=0$.

3. State the Dirichlet's conditions for the existence of Fourier series of $f(x)$.

4. Define the root mean square value of a function $f(x)$ in $(0,2l)$.

5. Classify the partial differential equation

$$(x+1)z_{xx}+\sqrt{2}(x+y+1)z_{xy}+(y+1)z_{yy}+yz_y-xz_y+2\sin x=0.$$

6. A taut string of length L cm fastened at both ends, is disturbed from its position of equilibrium by imparting to each of its points an initial velocity of magnitude $kx(L-x)$ for $0<x<L$. Formulate the problem mathematically.

7. Find the Laplace transform of $f(t)=e^{-t}\cos t\, t$.

8. Find the inverse Laplace transform of $F(s)=\frac{s}{(s+a)^2+b^2}$.

9. State the Fourier integral theorem.

10. Show that $\mathcal{F}c\{f(t)\cos at\}=\frac{1}{2}[F_c(s+a)+F_c(s-a)]$, where $F_c(s)$ is the Fourier cosine transform of $f(x)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the singular integral of the partial differential equation $z=px+qy+p^2+pq+q^2$. (8)

(ii) Solve the equation $(D^2-4DD'+D'^2)z=e^{2x-y}+2x$. (8)

Or

(b) (i) Solve $z^2(p^2+q^2)=x+y$. (8)

(ii) Solve the Lagrange's equation $x^2(y-z)+y^2(z-x)=z^2(x-y)$. (8)

12. (a) (i) Find the Fourier series of $f(x)=e^x$ in $(-\pi,\pi)$. (8)

(ii) Find the half range cosine series of $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x & 1<x<2 \end{cases}$. (8)

Or

(b) (i) Find the Fourier series of periodicity 2π for $f(x)=x^2$, in $-\pi<x<\pi$. Hence show that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots=\frac{\pi^4}{90}$. (8)

(ii) Find the constant term and the first two harmonics of the Fourier cosine series of $y=f(x)$ using the following table. (8)

x	0	$\pi/6$	$\pi/3$	$\pi/2$	$2\pi/3$	$5\pi/6$
$f(x)$	10	12	15	20	17	11

13. (a) A tightly stretched string with fixed end points $x=0$ and $x=50$ is initially at rest in its equilibrium position. If it is set to vibrate by giving each point a velocity $v=v_0 \sin^3 \frac{\pi x}{50}$, find the displacement of the string at any subsequent time. (16)

Or

(b) A rod of length 30cm has its ends A and B kept at 20°C and 80°C respectively until steady state conditions prevail. If the temperature of A is suddenly raised to 40°C while that the other end B is reduced to 60°C, find the temperature distribution at any point in the rod. (16)

14. (a) (i) Find the Laplace transform of the function $f(t)=\begin{cases} t^2 & 0<t<3 \\ 6, & t>3 \end{cases}$. (8)

(ii) Using convolution theorem, find $L^{-1}\left\{\frac{16}{(s-2)(s+4)}\right\}$. (8)

Or

(b) (i) Find the Laplace transform of the periodic function

$$f(t)=\begin{cases} 1, & 0\le t<2 \\ -1, & 2\le x\le 4 \end{cases}$$

and $f(t+4)=f(t)$. (8)

(ii) Solve the initial value problem $y''-6y'+9y=t^2e^{2t}, y(0)=2$ and $y'(0)=6$, using Laplace transform. (8)

15. (a) (i) Find the Fourier integral representation of $f(x)=\begin{cases} 0, & x<0 \\ e^{-x}, & x>0 \end{cases}$. (8)

(ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+4)(x^2+9)}$ using transform methods. (8)

Or

(b) Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & |x|<a \\ 0, & |x|>a>0 \end{cases}$. Hence evaluate

(i) $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}dx$ and

(ii) $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)^2 dx$. (16)
