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B 2262

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Fourth Semester

Computer Science and Engineering

MA 040 — PROBABILITY AND QUEUEING THEORY

Time : Three hours | Maximum : 100 marks

Statistical tables are permitted

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If A and B are independent events with $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{5}$, find $P(\overline{A} \cap \overline{B})$ and $P(\overline{A} \cap B)$.

2. If a random variable X has the probability density function $f(x) = \begin{cases} \frac{1}{7}; -3 < x < 4, \text{ find } P(X < 1) \\ 0, \text{ otherwise} \end{cases}$.

3. State the central limit theorem.

4. If the joint probability density function of (X, Y) is $f(x,y) = \begin{cases} \frac{1}{6}, 0 \le x \le 3, 0 \le y \le 2, \text{ find } P(X+Y \le 1) \\ 0, \text{ otherwise} \end{cases}$

5. If $X(t)$ is a stationary process, $R(t_1, t_2) = E\{X(t_1)X(t_2)\}$ is function of $(t_1 - t_2)$, what are $E\{X^2(t)\}$ and $Var\{X(t)\}$.

6. Obtain the mean of the binomial process.

7. Consider a Markov chain $\{X_n; n = 0,1,2,...\}$ with state space $S = \{1,2\}$ and one-step transition probability matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Is state 1 periodic?

8. A component has MTBF = 100 hours and MTTR = 20 hours with both failure and repair distributions exponential. Find the steady state availability and non-availability of the component.

9. In a given M|M|1|FCFS queue, $\rho = 0.7$. Find the expected number of customers in the queue.

10. In an M|G|1|FCFS with infinite capacity queue, the arrival rate $\lambda = 5$ and the mean service time $E(S) = \frac{1}{8}$ hour and $Var(S) = 0$. Compute the mean waiting time Wq in the queue.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) A continuous random variable X has probability density function $f(x) = \begin{cases} Cxe^{-x}, x > 0 \\ 0, \quad x \leq 0 \end{cases}$. Find the value of constant C. Obtain the moment generating function of the random variable X and hence obtain its mean and variance. (8)

(ii) A bag contains 3 black and 4 white balls. 2 balls are drawn at random one at a time without replacement.

(1) What is the probability that a second ball drawn is white if the first ball is black?

(2) What is the conditional probability that first ball drawn is white if the second ball is known to be white? (8)

Or

(b) (i) Three machines A, B and C produce identical items of their respective output 5%, 4% and 3% of the items are faulty. On a certain day A has produced 35%, B has produced 40% and C has produced 25% of the total output. An item selected at random is found to be faulty. What is the chance that it was produced by A? (8)

(ii) A test engineer discovered that the cumulative distribution function of the lifetime of an equipment in years is given by

$$F(x)=\begin{cases}1-e^{-\frac{1}{5}x}, & x \geq 0\\ 0, & x<0\end{cases}.$$

(1) What is the expected lifetime of the equipment?

(2) What is the variance of the lifetime of the equipment? (8)

12. (a) (i) Let X and Y have the joint probability mass funtion

y \ x	0	1	2
0	0.2	0.3	0.15
1	0.1	0.1	0.25

(1) Find $P(X+Y>1)$.

(2) Are X and Y independent random variables? Justify your answer. (8)

(ii) The joint probability density function of random variables X and Y is given by $f(x,y)=\begin{cases}Cxy^2, & 0 \leq x \leq y \leq 1\\ 0, & \text{otherwise}\end{cases}$.

(1) Determine the value of C.

(2) Find the marginal probability density functions of X and Y.

(3) Find the conditional density functions $f_{Y/X}(y/x)$ and $f_{X/Y}(x/y)$. (8)

Or

(b) Let X and Y be positive independent random variable with the identical probability density function $f(x) = e^{-x}, x > 0$. Find the joint probability density function of $U = X + Y$ and $V = X/Y$. Are U and V independent?

13. (a) (i) Define :

(1) Markov process.

(2) Independent increment random process.

(3) Strict-sense stationary process.

(4) Second order stationary process. (8)

(ii) Let $X(t)$ be a Poisson process with rate λ. Find $E\{[X(t) - X(s)^2]\}$ for $t > s$ (8)

Or

(b) (i) Define renewal process and renewal density function. Establish the integral equation for the renewal function. (8)

(ii) Define Poisson process and obtain its probability distribution. (8)

14. (a) (i) Let $\{X_n; n \geq 0\}$ be a Markov chain with three states 0, 1, 2 and one-step transition probability matrix

$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

and the initial distribution $P(X_0 = i) = 1/3$, $i = 0,1,2$. Find

(1) $P(X_2 = 1, X_1 = 1/X_0 = 2)$.

(2) $P(X_2 = 2, X_1 = 1, X_0 = 1)$.

(3) $P(X_3 = 1, X_2 = 2, X_1 = 0, X_0 = 0)$.

(4) Is the chain irreducible? Explain. (8)

(ii) Consider a Markov chain $\{X_n; n \geq 0\}$ with state space $S = \{0,1\}$ and one-step transition probability matrix $P = \begin{vmatrix} 1 & 0 \\ 1/2 & 1/2 \end{vmatrix}$.

(1) Draw the state transition diagram.

(2) Is the chain irreducible? Explain.

(3) Show that state 0 is ergodic.

(4) Show that state 1 is transient. (8)

Or

(b) (i) Discuss the reliability analysis for 2 – unit parallel system with repair. (8)

(ii) The life length of a device is exponentially distributed. It is found that the reliability of the device for 100 hour period of operation is 0.90. How many hours of operation are necessary to get a reliability of 0.95? (8)

15. (a) (i) A service station expects a customer every 4 minutes on the average. Service takes, on the average. 3 minutes. Assume Poisson input and exponential service.

(1) What is the average number of customers waiting for service?

(2) How long can a customer expect to wait for service?

(3) What is the probability that a customer will spend less than 15 minutes waiting for and getting service?

(4) What is the probability that a customer will spend longer than 10 minutes waiting for and getting service? (8)

(ii) A one-man barber shop has a total of 10 seats. Inter-arrival times are exponentially distributed, and an average of 20 prospective customers arrives each hour at the shop. Those customers who find the shop full do not enter. The barber takes an average of 12 minutes to cut each customer's hair. Haircut times are exponentially distributed.

(1) On the average how many haircuts per hour will the barber complete?

(2) On the average, how much time will be spent in the shop by a customer who enters? (8)

Or

(b) (i) For an M|M|2 queuing system with a waiting room of capacity 5, find the average number of customers in the system, assuming that arrival rate as 4 per hour and mean service time 30 minutes. (8)

(ii) A car wash facility operates with only one bay. Cars arrive according to a Poisson distribution with a mean of 4 cars per hour and may wait in the facility's parking lot if the bay is busy. The parking lot is large enough to accommodate any number of cars. Find the average number of cars waiting in the parking lot, if the time for washing and cleaning a car as follows :

(1) uniform distribution between 8 and 12 minutes?

(2) a normal distribution with mean 12 minutes and standard deviation 3 minutes? (8)
