Reg. No. : | | | | | | | | | | | |

D 4224

B.E/B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Fourth Semester

(Regulation 2004)

Computer Science and Engineering

MA 1252 — PROBABILITY AND QUEUEING THEORY

(Common to B.E. Part-Time Third Semester Regulation 2005)

Time : Three hours Maximum : 100 marks

Use of statistical table is permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. When A and B are 2 mutually exclusive events, are the values $P(A) = 0.6$ and $P(A \cap \overline{B}) = 0.5$ consistent? Why?

2. A continuous random variable X has a density function given by $f(x) = k(1+x)$ $2<x<5$. Find $P(X<4)$.

3. The number of monthly breakdowns of a computer is a random variable having a Poisson distribution with mean equal to 1.8. Find the probability that this computer will function for a month (a) without a break down (b) with only one breakdown.

4. Find the distribution function of the random variable $Y = g(x)$, in terms of the distribution function of X, if it is given that

$$g(x) = \begin{cases} x - c & \text{for} \quad x > c \\ 0 & \text{for} \quad |x| \le c \\ x + c & \text{for} \quad x < -c \end{cases}$$

5. Define independence of two random variables X and Y, both in the discrete case and in the continuous case.

6. Comment on the following :

"The random variables X and Y are independent

iff $Cov(X, Y) = 0$ "

7. If $\{X(s, t)\}$ is a random process, what is the nature of $X(s, t)$ when (a) s is fixed (b) t is fixed?

8. What is a stochastic matrix? When is it said to be regular?

9. What do you mean by transient state and steady state queueing systems?

10. If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 mins before the picture starts and it its takes exactly 1.5 min to reach the correct seat after purchasing the ticket. Can he expect to be seated for the start of the picture?

PART B — (5 × 16 = 80 marks)

11. (a) (i) The first bag contains 3 white balls, 2 red balls and 4 black balls. Second bag contains 2 white, 3 red and 5 black balls and third bag contains 3 white, 4 red and 2 black balls. One bag is chosen at random and from it 3 balls are drawn. Out of 3 balls, 2 balls are white and 1 is red. What are the probabilities that they were taken from first bag, second bag, third bag.

(ii) A random variable X has the p.d.f.

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

find (1) $P\left(X < \frac{1}{2}\right)$ (2) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$ (3) $P\left(X > \frac{3}{4} \Big/ X > \frac{1}{2}\right)$.

Or

(b) (i) If the density function of a continuous random variable X is given by

$$f(x) = \begin{cases} ax & ; \quad 0 \le x \le 1 \\ a & ; \quad 1 \le x \le 2 \\ 3a - ax & ; \quad 2 \le x \le 3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

(1) Find 'a' (2) Find the cdf of X.

(ii) If the moments of a random variable 'X' are defined by $E(X^r) = 0.6; \quad r = 1, 2, 3, \ldots$

Show that $P(X = 0) = 0.4$, $P(X = 1) = 0.6$, $P(X \ge 2) = 0$.

12. (a) (i) A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are (1) exactly 3 defectives, (2) not more than 3 defectives.

(ii) A die is cast until 6 appears. What is the probability that it must be cast more than 5 times?

Or

(b) (i) If X is uniformly distributed over $(-\alpha, \alpha)$, $\alpha > 0$, find α so that

(1) $P(X > 1) = \frac{1}{3}$ (2) $P(|X| < 1) = P(|X| > 1)$.

(ii) Assume that mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall?

13. (a) (i) If the joint distribution function of X and Y is given by

$$F(x, y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}) & \text{for } x > 0, y > 0 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(1) Find the marginal densities of X and Y

(2) Are X and Y independent

(3) $P(1 < X < 3, \ 1 < Y < 2)$.

(ii) Find the coefficient of correlation between industrial production and export using the following data :

Production (X) :	55	56	58	59	60	60	62
Export (Y) :	35	38	37	39	44	43	44

Or

(b) (i) The lifetime of a certain brand of an electric bulb may be considered a RV with mean 1200 hrs and S.D. 250 hrs. Find the probability, using central limit theorem, that the average life time of 60 bulbs exceeds 1250 hrs.

(ii) The two lines of regression are

$$8x - 10y + 66 = 0$$
$$40x - 18y + 214 = 0$$

The variance of X is 9.

Find

(1) The mean values of X and Y

(2) Correlation coefficient between X and Y.

14. (a) (i) Given a random variable Ω with density $f(w)$ and another random variable ϕ uniformly distributed in $(-\pi, \pi)$ and independent of Ω and $X(t) = a\cos(\Omega t + \phi)$, prove that $\{X(t)\}$ is a WSS process.

(ii) The transition probability matrix of a Markov chain $\{X_n\}$, $n = 1, 2, 3, ...$ having 3 states 1, 2 and 3 is

$$P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

and the initial distribution is $(0.7,\ 0.2,\ 0.1)$

Find :

(1) $P\{X_2 = 3\}$.

(2) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

Or

(b) (i) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair die and drove to work iff a 6 appeared. Find

(1) The probability that he takes a train on the 3^{rd} day.

(2) The probability that he drives to work in the long run.

(ii) Write a short note on recurrent state, transient state, ergodic state.

15. (a) (i) A duplicating machine maintained for office use is operated by an office assistant who earns Rs. 5 per hour. The time to complete each jobs varies according to an exponential distribution with mean 6 mins. Assume a poisson input with an average arrival rate of 5 jobs per hour. If an 8-hrs day is used as a base, determine

(1) The percentage idle time of the machine.

(2) The average time a job is in the system.

(3) The average earning per day of the assistant.

(ii) A super market has two girls attending to sales at the counters. If the service time for each customer is exponential with mean 4 mins and if people arrive in Poisson fashion at the rate of 10 per hour,

(1) What is the probability that a customer has to wait for service?

(2) What is the expected percentage of idle time for each girl?

Or

(b) (i) Customers arrive at a one-man barber shop according to a Poisson process with a mean interarrival time of 12 min. Customers spend an average of 10 min in the barber's chair.

(1) What is the expected no. of customers in the barber shop and in the queue?

(2) What is the probability that more than 3 customers are in the system?

(ii) Derive the Pollaczek-Khinchine formula for M/G/1 queueing model.
