# D 4225

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Fourth Semester

(Regulation 2004)

Mechanical Engineering

MA 1253/MA 1259 — PROBABILITY AND STATISTICS

(Common to Automobile Engineering, Production Engineering,
Information Technology, Textile Technology (Textile Chemistry), Textile Technology
Mechanical Engineering and Textile Technology (Fashion Technology))

(Common to B.E. (Part-Time) Third Semester–Regulation 2005)

Time : Three hours                                    Maximum : 100 marks

Statistical tables are permitted.

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1.  A fair die is tossed. The event $A$ is defined as 'the number obtained is a multiple of 3'. Find $P(A)$.

2.  The random variable $X$ can only take the values 2 and 5. Given that the value 5 is twice as likely the value 2, determine the expectation of $X$.

3.  The random variable $X$ has a binomial distribution with parameters $n = 20$ and $p = 0.4$. Determine $P\{X = 3\}$.

4.  The random variable $X$ has a Poisson distribution with mean 1.7. Determine $P\{X \geq 3\}$.

5.  The joint density function of $X$ and $Y$ is given by

$$f(x,y) = \begin{cases} 2e^{-x} e^{-2y}, & 0 < x < \infty, \quad 0 < y < \infty \\ 0 & , \text{ otherwise} \end{cases}$$

Compute $P\{X > 1, Y < 1\}$.

6. If $X$ and $Y$ are independent random variables, then find the covariance between $X$ and $Y$.

7. The specifications for a certain kind of ribbon call for a mean breaking strength of 180 pounds. If five pieces of the ribbon have a mean breaking strength of 169.5 pounds with standard deviation of 5.7 pounds, test the null hypothesis $\mu = 180$ pounds against the alternative hypothesis $\mu < 180$ pounds at the 0.01 level of significance. Assume that the population distribution is normal.

8. To test the claim that the resistance of electric wire can be reduced by more than 0.050 ohm by alloying, 32 values obtained for standard wire yielded $\bar{x}_1 = 0.136$ ohm and $S_1 = 0.004$ ohm, and 32 values obtained for alloyed wire yielded. $\bar{x}_2 = 0.083$ ohm and $S_2 = 0.005$ ohm. At the 0.05 level of significance, does this support the claim?

9. Write a note on completely randomized design.

10. Briefly explain the term Latin - square design.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Two events $A$ and $B$ are such that $P(A) = 0.4$ $P(B') = 0.3$ and $P(A \cap B) = 0.2$. Determine $P(A \cup B)$ and $P(A' \cap B')$. (4)

(ii) Two events $A$ and $B$ are such that $P(A) = 0.5$ $P(B) = 0.4$ and $P(A/B) = 0.3$. State whether the events are independent and find the value of $P(A \cap B)$. (4)

(iii) Urn I has 2 white and 3 black balls, Urn II has 4 white and 1 black balls and Urn III has 3 white and 4 black balls. An Urn is selected at random and a ball drawn at random is found to be white. Find the probability that Urn I was selected. (8)

Or

(b) (i) The diameter of an electric cable $X$ is a continuous r.v. with pdf $f(x) = Kx(1-x)$, $0 \leq x \leq 1$. Find the value of $k$, $F(x)$ and $P\left(X \leq \frac{1}{2}\middle|\frac{1}{3} < X < \frac{2}{3}\right)$. (8)

(ii) Find the first four moments about the origin for a random variable $X$ having the pdf. (8)

$$f(x) = \begin{cases} \dfrac{4x(9-x^2)}{81}, & 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

12. (a) (i) Find the mean and variance of a geometric distribution. (4)

(ii) If $X$ is a normal random variable with mean $\mu = 3$ and variance $\sigma^2 = 16$, find $P\{X<1\}$, $P\{X>-1\}$ and $P\{2<X<7\}$ (6)

(iii) If $X$ is uniformly distributed over the interval $[0,10]$, compute $P\{2<X<9\}$, $P\{1<X<4\}$ and $P\{X<5\}$ . (6)

Or

(b) (i) Find the moment generating function of a Poisson random variable and hence find its mean variance. (8)

(ii) The time required to repair a machine is exponentially distributed with parameter 1/2. What is the probability that the repair exceeds 2 hours? What is the conditional probability that the repair takes atleast 10 hour given that the duration exceeds 9 hours? (8)

13. (a) (i) The joint probability of $X$ and $Y$ is

$$f(x, y) = \begin{cases} 2/3(x + 2y) & : 0 < x < 1, \ 0 < y < 1 \\ 0 & : \text{otherwise} \end{cases}$$

Find the conditional density of $X$ given the value of $y$. (6)

(ii) Find the correlation coefficient between the marks obtained by ten students in Maths and Physics. (10)

Maths : 65 45 40 55 60 50 80 30 70 65

Physics : 60 60 55 70 80 40 85 50 70 80

Or

(b) (i) The joint probability distribution $f(x, y)$ of $X$ and $Y$ is

$$X$$

| | | 0 | 1 | 2 |
|---|---|---|---|---|
| $Y$ | 0 | .1 | .4 | .1 |
| | 1 | .2 | .2 | 0 |

Find $P(X+Y>1)$ and $P(X=x)$. (6)

(ii) Find the most likely price in City A corresponding to the price of Rs. 70 at City B from the following

| | City B | City A |
|---|---|---|
| Average price | 65 | 67 |
| S.D. of price | 2.5 | 3.5 |

Correlation coefficient is 0.8. (10)

14. (a) (i) The following random samples are measurements of the heat - producing capacity (in millions of calories per ton) of specimen's of coals from two mines :

Mine 1 :   8,260   8,130   8,350   8,070   8,340
Mine 2 :   7,950   7,890   7,900   8,140   7,920   7,840

Use the 0.01 level of significance to test whether the difference between the means of these two samples is significant. (8)

(ii) The lapping process which is used to grind certain silicon wafers to the proper thickness is acceptable only if $\sigma$, the population S.D. of the thickness of dice cut from the wafers, is at most 0.5 mil. Use the 0.05 level of significance to test the null hypothesis $\sigma = 0.5$ against the alternative hypothesis $\sigma > 0.5$, if the thickness of 15 dice cut from such wafers have S.D. of 0.64 mil. (8)

Or

(b) (i) The following are the average weekly losses of working hours due to accidents in 10 industrial plants before and after an introduction of a safety program was put into operation.

Before :   45   73   46   124   33   57   83   34   26   17
After :   36   60   44   119   35   51   77   29   24   11

Use to 0.05 level of significance to test whether the safety program is effective. (8)

(ii) A study shows that 16 out of 200 tractors produced on one assembly line required extensive adjustments before they could be shipped, while the same was true for 14 out of 400 tractors produced on another assembly line. At the 0.01 level of significance, does this support the claim that the second production line does support work? (8)

15. (a) Perform an analysis of variance at 0.01 level of significance for the following data : (16)

|   | I | II | III |
|---|---|----|-----|
| A | 45 | 43 | 51 |
| B | 47 | 46 | 52 |
| C | 48 | 50 | 55 |
| D | 42 | 37 | 49 |

Or

(b) On a feeding experiment a farmer has four types of hogs denoted by I, II, III, IV. These types are each divided into three groups, which are fed variental relations A, B and C. The following results are obtained. The number being the gains in weight in pounds in various groups. (16)

|   | I | II | III | IV |
|---|---|----|-----|----|
| A | 7 | 16 | 10.5 | 13.5 |
| B | 14 | 15.5 | 15 | 21 |
| C | 8.5 | 16.5 | 9.5 | 13.5 |

Perform an analysis of variance test at 5% level of significance to the difference between the variental relation A, B and C and hogs I, II, III and IV.

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