Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐

D 4226

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Fourth Semester

Electronics and Communication Engineering

MA 1254 — RANDOM PROCESSES

(Regulation 2004)

Time : Three hours   Maximum : 100 marks

(Statistical Table permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. A box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good. What is the probability that the other one is also good?

2. Define moment generating function and write the formula to find mean and variance.

3. Define exponential density function and find mean and variance of the same.

4. If $Y = X^2$, where X is a Normal random variable with zero mean and variance σ^2, find the pdf of the random variable Y.

5. Let X and Y be any two random variables and a, b be constants. Prove that $\text{cov}(aX, bY) = ab\,\text{cov}(X, Y)$.

6. The joint probability density function of the Random variable (X, Y) is given by $f(x, y) = kxye^{-(x^2 + y^2)}$, $x > 0$, $y > 0$. Find the value of K and prove that X and Y are independent.

7. Define strict sense stationary process and wide sense stationary process.

8. Define Markov process.

9. Define autocorrelation of the process $\{X(t)\}$.

10. Given that the autocorrelation function for a stationary ergodic process with no periodic components is

$$R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}.$$

Find the mean value and variance of the process $\{X(t)\}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? (8)

(ii) A random variable X has the following probability distribution.

x :	0	1	2	3	4	5	6	7
$p(x)$:	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find :

(1) The value of k

(2) $P(1.5 < X < 4.5 / X > 2)$ and

(3) The smallest value of λ for which $p(X \le \lambda) > \frac{1}{2}$. (8)

Or

(b) (i) State and prove Baye's theorem. (8)

(ii) If the density function of a continuous RV X is given by

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ 3a - ax, & 2 \le x \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

(1) Find the value of a.

(2) Find the CDF of X.

(3) If x_1, x_2 and x_3 are 3 independent observations of X, what is the probability that exactly one of these 3 is greater that 1.5? (8)

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12. (a) (i) Prove that Poisson distribution is the limiting case of Binomial distribution. (8)

(ii) Define Gamma distribution and find mean and variance of the same. (8)

Or

(b) (i) In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students get distinction. Find the percentages of students who have got first class and second class. (Assume normal distribution of marks) (8)

(ii) Define Weibull distribution and find its mean and variance. (8)

13. (a) The joint probability mass function of (X, Y) is given by $p(x,y) = k(2x + 3y)$, $x = 0, 1, 2$; $y = 1, 2, 3$. Find all the marginal and conditional probability distributions. Also find the probability distribution of $(X + Y)$. (16)

Or

(b) (i) The lifetime of a certain brand of an electric bulb may be considered a RV with mean 1200h and standard deviation 250h. Find the probability, using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250h. (8)

(ii) The random variables X and Y are statistically independent having a gamma distribution with parameters $(m, \frac{1}{2})$ and $(n, \frac{1}{2})$, respectively. Derive the probability density function of a random variable $U = \dfrac{X}{X + Y}$. (8)

14. (a) (i) Define random process. Classify it with an example. (8)

(ii) If $X(t) = Y \cos wt + Z \sin wt$, where Y and Z are two independent normal RVs with $E(Y) = E(Z) = 0$, $E(Y^2) = E(Z^2) = \sigma^2$ and w is a constant, prove that $\{X(t)\}$ is a SSS process of order 2. (8)

Or

(b) (i) State the Postulates of a Poisson process and derive its probability law. (8)

(ii) Prove that the sum of two independent Poisson process is a Poisson process. (8)

15. (a) (i) Find the power spectral density of a WSS process with autocorrelation function . (8)

$$R(\tau) = e^{-\alpha\tau^2}$$

(ii) The power spectral density function of a zero mean WSS process $\{X(t)\}$ is given by

$$S(w) = \begin{cases} 1, & |w| < w_0 \\ 0, & \text{elesewhere} \end{cases}$$

Find $R(\tau)$ and show that $X(t)$ and $X\left(t + \dfrac{\tau}{w_0}\right)$ are uncorrelated. (8)

Or

(b) $X(t)$ is the input voltage to a circuit (system) and $Y(t)$ is the output voltage. $\{X(t)\}$ is a stationary random process with $\mu_x = 0$ and $R_{xx}(\tau) = e^{-\alpha|\tau|}$. Find μ_y, $S_{yy}(w)$ and $R_{yy}(\tau)$, if the power transfer function is

$$H(w) = \frac{R}{R + iLW}$$

(16)