Reg. No. : [ | | | | | | | | | | ]

**D 4227**

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2008.

Fourth Semester

Biotechnology

MA 1255 — PROBABILITY AND STATISTICS

(Regulation 2004)

Time : Three hours | Maximum : 100 marks

Use of Statistical and Control Chart is permitted

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. The probability that a student will get an A in Biology is .32 and the probability that he or she will get either A or a B is .27. Explain why there must be a mistake in the above statement.

2. Out of 800 families with 4 children each how many families would be expected to have atmost 2 girls? (Assume equal probabilities for boys and girls)

3. State central limit theorem.

4. If the joint pdf of a two dimensional r.v $(x,y)$ is given by $f(x,y) = k(6-x-y)$, $0 < x < 2$, $2 < y < 4$. Find the value of $k$ and $p\ (x < 1, y < 3)$.

5. Prove that the Poisson process is a Markov process.

6. Cars arrive at a petrol pump having one petrol unit in Poisson fashion with an average of 10 cars per hour. The service time is exponentially distributed with mean of 3 minutes. Find the average queue length.

7. A density function of the time to failure in years of a component is given by $f(t) = \dfrac{200}{(t+10)^3}$, $t \geq 0$ compute the MTTF.

8. Show that if a system has constant failure rate preventive maintenance does not improve the reliability of the system.

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9. What is the main advantage of LSD over RBD?

10. Find the lower and upper control limits for $\overline{X}$ chart and $R$-chart when each sample is of size 4 and $\overline{\overline{X}} = 10.80$, $\overline{R} = .46$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the MGF of the binomial distribution and hence find its mean and variance. (8)

(ii) If the life of certain type of car has a Weibull distribution with parameter $\beta = 2$ find the value of parameter $\alpha$ given that probability that the life of car exceeds 5 years is $e^{-2.5}$. (8)

Or

(b) (i) The probability that a student passes a certain exam is .9 given that he studied. The probability that he passes the exam without studying is 2. Assume that the probability that the student studies for exam is .75. Give that the student passed the exam what is the probability that he studied. (8)

(ii) State and prove the memoryless property of an exponential distribution. (8)

12. (a) (i) For the bivariate probability distribution of RV $(X,Y)$ given by $f(x,y) = \frac{x+y}{21}$ $x = 1, 2, 3$ ; $y = 1, 2$. Find marginal distribution of $X$ and $Y$ and $P(X/Y = 1)$. (8)

(ii) Let $X_1$ and $X_2$ be two continuous random variables with joint probability distribution $f(x_1,x_2) = \begin{cases} 4x_1x_2 & 0 < x_1 < 1 \quad 0 < x_2 < 1 \\ 0 & \text{elsewhere} \end{cases}$.

Find the joint probability distribution of $Y_1 = X_1^2$ and $Y_2 = X_1X_2$.

Or

(b) (i) Find the estimated regression line for height-weight data for seven boys given below : (8)

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Height $x$ : | 36 | 38 | 40 | 42 | 44 | 46 | 48 |
| Weight $y$ : | 30 | 35 | 35 | 42 | 51 | 48 | 53 |

(ii) If $X$ is a discrete random variable with probability distribution.

| | | | |
|---|---|---|---|
| $x$ : | $-1$ | 0 | 1 |
| $p(X = x)$ : | .3 | .4 | .3 |

If $Y = 1 + 2X$ find joint probability distribution of $X$ and $Y$ and the correlation coefficient. (8)

13. (a) (i) Define a random process and explain the classifications of random process with example. (8)

(ii) Three boys A, B and C are throwing ball to each other. A always throws the ball to B and B always throws to C, but C is just as likely to show the ball to B as to A. Show that process is Markovian. Find the transition probability matrix and classify the states.

Or

(b) (i) A telephone exchange has two long distance operators. The telephone company finds that during peak time long distance calls arrive in Poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length 5 minutes. What is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day? If the subscribers will wait and are serviced in turn what is the expected waiting time? (8)

(ii) Derive the differential equation for Birth and Death process. (8)

14. (a) (i) Thermocouples of a particular design have a failure rate 0.008 per hour. How many thermocouples must be placed in parallel if the system is to run for 100 hours with a system failure probability of no more than 0.05? Assume that all the failures are independent. (5)

(ii) Obtain the MTTF of reliability under preventive maintenance. (11)

Or

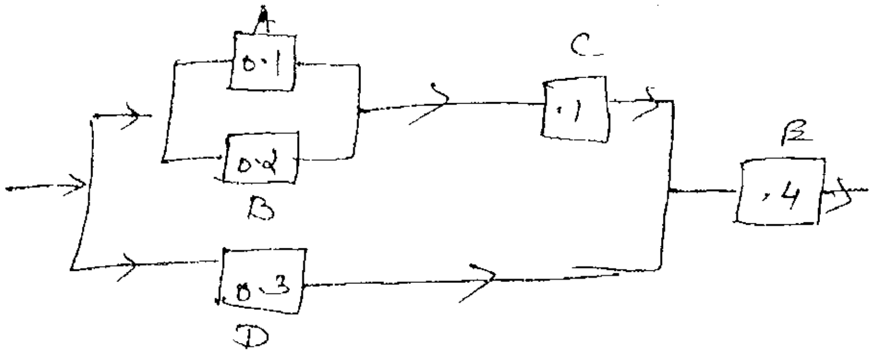
(b) (i) The density function of time to failure of an appliance is $f(t) = \frac{32}{(t+4)^3}$, $t > 0$.

(1) Find the reliability function

(2) Find failure rate

(3) Find the MTTF. (8)

(ii) Calculate the system reliability for five elements A, B, C, D, E connected as shown below : (8)




15. (a) Four doctors each test four treatments for a certain disease and observe the number of days each patient takes to recover. The results are as follows :

| | Treatment | | | |
|---|---|---|---|---|
| Doctor | 1 | 2 | 3 | 4 |
| A | 10 | 14 | 19 | 20 |
| B | 11 | 15 | 17 | 21 |
| C | 9 | 12 | 16 | 19 |
| D | 8 | 13 | 17 | 20 |

Discuss the difference between (i) doctors (ii) treatments. (16)

Or

(b) (i) The following data give the average life in hours and range in hours of 12 samples each of 5 lamps. Construct control chart for $\overline{X}$ and $R$ and comment on the state of control. (8)

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\overline{X}$ : | 120 | 127 | 152 | 157 | 160 | 134 | 137 | 123 | 140 | 144 | 120 | 127 |
| $R$ : | 30 | 44 | 60 | 34 | 38 | 35 | 45 | 62 | 39 | 50 | 35 | 41 |

(ii) On inspection of 10 samples each of size 400 the numbers of defective articles were :

19, 4, 9, 12, 9, 15, 26, 14, 15, 17

Draw nP chart and P chart and comment on the state of control. (8)

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