Register Number.......................

M.E.DEGREE EXAMINATIONS: DECEMBER - 2008

First Semester

ENERGY ENGINEERING

P07MA103 Applied Mathematics for Thermal Engineering

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions:-

PART – A (20 × 1 = 20 Marks)

1. If F (f(x)] = F(S), then F (f(x-a)) is

   A) e^{isa} F (S) B) e^{-isa} F (S) C) e^{is} F (S) D) $\frac{1}{e^{isa}}$ F (S)

2. If F (f(x)) = F(s) then F (f (x) $\cos ax$) is

   A) F(s-a) B) F (s-a) + F (s +a) C) $\frac{1}{2}$ [F (s-a) + F (s +a)] D) $\frac{1}{2}$ [F (s-a) - F (s +a)]

3. If F [f(x)] = F(S) then F [f (ax)] is

   A) F($\frac{s}{a}$) B) $\frac{1}{a}$ F ($\frac{s}{a}$) C) $\frac{1}{a}$ F(s) D) $\frac{1}{a}$ F (as)

4. The one dimensional heat equation is

   A) $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$ B) $\frac{\partial u}{\partial x} = K\frac{\partial^2 u}{\partial t^2}$ C) k $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ D) k $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}$

5. The Euler's equation is

   A) $\frac{\partial f}{\partial y} - \frac{d}{dx}[\frac{\partial f}{\partial y'}] = 0$ B) $\frac{\partial f}{\partial x} - \frac{d}{dy}[\frac{\partial f}{\partial y'}] = 0$

   C) $\frac{\partial f}{\partial y} + \frac{d}{dx}[\frac{\partial f}{\partial y'}] = 0$ D) $\frac{\partial f}{\partial y} - \frac{d}{dy'}[\frac{\partial f}{\partial x}] = 0$

6. The short distance between two points in a plane is

   A) Curve B) Straight line C) Tangent D) Chord

7. The necessary condition for $I = \int_{x_1}^{x_2} f(x, y_1, y_2, \ldots\ldots\ldots y_n, y_1', y_2', \ldots\ldots y_n')\,dx$ to be an extremum is

   A) $\frac{\partial f}{\partial y_i} - \frac{d}{dx}(\frac{\partial f}{\partial y_i'}) = 0$ B) $\frac{\partial f}{\partial y_i'} = (\frac{\partial f}{\partial y_i'})$ C) $\frac{\partial f}{\partial y_i} - \frac{d}{dx}(\frac{\partial f}{\partial y_i'}) \neq 0$ D) $\frac{d}{dx}(\frac{\partial f}{\partial y_i})-(\frac{\partial f}{\partial y_i'}) = 0$

8. A geodesics on a surface is a curve along which the distance between any two points of the surface is

   A) Minimum B) Maximum C) neither maximum nor minimum D) Saddle point

9. The Laplace equation is

A) $\nabla^2\phi \neq 0$ B) $\nabla^2\phi = 1$ C) $\nabla^2\phi = 0$ D) $\nabla^2\phi \neq 1$

10. The point at which velocity is zero is called

A) Saddle point B) stagnation point C) critical point D) invariant point

11. The function ϕ = sinx coshy is

A) Analytic B) not analytic C) harmonic D) non harmonic

12. The Cauchy Riemann equations for $\phi + i\psi$ is

A) $\phi_x=\psi_y$, $\phi_y=i\psi_x$ B) $\phi_x=i\psi_y$, $\phi_y=\psi_x$ C) $\phi_x=-\psi_y$, $\phi_y=\psi_x$ D) $\phi_x=\psi_y$, $\phi_y=-\psi$

13. The sufficient condition for Gauss Jacobi method to converge is

A) Diagonal matrix B) Unit matrix C) Diagonal dominant D) Upper triangular matrix

14. Solving the equations 5x+4y = 15, 3x+7y = 12 by Gauss Jordan then the value of x and y are

A) 4.478 , 1.652 B) 1.478, 0.652 C) 2.478, 0.652 D) 2.478 ,1.652.

15. The rate of convergence in Gauss - Seidal method is .... times than that of Gauss Jacobi method.

A) 1 B) 2 C) 3 D) 4

16. In Gauss elimination method the augmented matrix is converted to

A) Lower triangular B) unit matrix C) upper triangular D) diagonal matrix

17. The Liebmann's principle is

A) $u_{ij}^{(n+1)} = \frac{1}{4}\left(u_{i+1,j}^{(n)} + u_{i-1,j}^{(n)} + u_{i,j+1}^{(n)} + u_{i,j-1}^{(n)}\right)$

B) $u_{ij}^{(n+1)} = \frac{1}{4}\left(u_{i,j}^{(n)} + u_{i-1,j}^{(n)} + u_{i,j+1}^{(n)} + u_{i,j-1}^{(n)}\right)$

C) $u_{ij}^{(n+1)} = \frac{1}{4}\left(u_{i+1,j}^{(n)} + u_{i-1,j-1}^{(n)} + u_{i,j+1}^{(n)} + u_{i,j-1}^{(n)}\right)$

D) $u_{ij}^{(n+1)} = \frac{1}{4}\left(u_{i+1,j}^{(n)} + u_{i-1,j}^{(n)} + u_{i,j+1}^{(n)} + u_{i+1,j-1}^{(n)}\right)$

18. In finite difference explicit formula the limit for λ is

A) $-1<\lambda<0$, B) $-1 < \lambda <1$, C) $0 < \lambda <1$, D) $0 \leq \lambda \leq ½$

19. The Schmidt's formula for solving the heat flow equation is

A) $u_{i,j+1} = \lambda u_{i+1,j} + (1-2\lambda) u_{i,j} + \lambda u_{i-1,j}$ B) $u_{i,j+1} = u_{i+1,j} + (1-2\lambda) u_{i,j} + u_{i-1,j}$

C) $u_{i,j+1} = \lambda u_{i+1,j} + (1+2\lambda) u_{i,j} + \lambda u_{i-1,j}$ D) $u_{i,j+1} = \lambda u_{i+1,j} + (1+2\lambda) u_{i,j} - \lambda u_{i-1,j}$

20. The standard five point formula to solve Laplace equation $u_{xx} + u_{yy} = 0$ is

A) $u_{i,j} = \frac{1}{4} [u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}]$ B) $u_{i,j} = \frac{1}{4} [u_{i-1,j} - u_{i+1,j} - u_{i,j-1} + u_{i,j+1}]$

C) $u_{i,j} = \frac{1}{4} [u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}]$ D) $u_{i,j} = [u_{i-1,j} + u_{i+1,j} + u_{i,j-1} + u_{i,j+1}]$

PART B (5× 16 = 80 Marks)

21. a. Solve the heat conduction problem by using Fourier transform

$$u_t = k\, u_{xx},\quad 0 < x < \infty,\ t > 0$$
$$u_{(0,t)} = u_0,\quad t \geq 0$$
$$u_{(x,0)} = 0,\quad 0 < x < \infty,$$
$$u,\ \frac{\partial u}{\partial x} \text{ tends to zero as } x \to \infty$$

(16)

(OR)

21. b. (i) Find the Fourier transform of f(x) defined by

$$f(x) = \begin{cases} 1 & |x| < a \\ 0 & |x| > a \end{cases}$$

and hence evaluate $\int_{-\infty}^{\infty} \frac{\sin as \cos sx \; ds}{s}$ (8)

(ii) Find the Fourier sine transform of f(x) defined by

$$f(x) = \begin{cases} 0 & 0 < x < a \\ x & a \leq x \leq b \\ 0 & x > b \end{cases}$$

(8)

22. a. (i) Using Ritz method solve the boundary value problem

$$y'' - y + x = 0,\ (0 \leq x \leq 1),\ y(0) = y(1) = 0.$$

(8)

(ii) Find the curves on which the functional

$\int_0^1 (y'^2 + 12xy)\, dx$ with y (0) =0 and y(1) =1 can be extremised. (8)

(OR)

22. b. Show that the functional $\int_0^{\frac{\pi}{2}} \left(2xy + \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right) dt$ such that

x (0) = 0, x ($\frac{\pi}{2}$) = -1, y(0) = 0 , y ($\frac{\pi}{2}$) = 1 is stationary for x = - sint, y = sint. (16)

23. a. Find the complex potential due to a source at z =-a and a sink at z = a of strength k,

(i) Determine the equipotential lines and stream lines graphically

(ii) Find the speed of the fluid at any point. (16)

(OR)

23. b. The complex potential of a fluid flow is given by Ω (z) = V_0 (z +$\frac{a^2}{z}$) where V_0 and 'a' are positive constants.

(i) Obtain equations for the stream lines and equipotential lines and represent them graphically

(ii) find the velocity point and its value far from the obstacle.

(iii) Find the stagnation point. (16)

24. a. (i) Solve the following equations by Gauss – Seidel method

$20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$ (8)

(ii) Solve the following equation by Gauss Elimination method

$3x + y - z = 3$; $2x - 8y + z = 5$; $x - 2y + 9z = 8$ (8)

(OR)

24. b. Solve the following system of equations by Factorisation method

$3x_1+2x_2+7x_3 = 4$; $2x_1+3x_2+x_3 = 5$; $3x_1+4x_2 + x_3 = 7$ (16)

25. a. Solve the Poisson equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides x = 0, y = 0, x =3 and y =3 with u = 0, on the boundary and mesh length = 1. (16)

(OR)

b. Solve $u_t = u_{xx}$, by Crank – Nicholson method subject to

u(x,0)= 0, u (0,t)=0, and u(1,t)= t for two time steps. (16)

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