Register Number ....

B.E./B.TECH. DEGREE EXAMINATIONS - DECEMBER 2008

First Semester

U07MA101 MATHEMATICS I

(Common to ALL Branches)

Time: Three Hours **Maximum marks: 100**

Answer ALL Questions:-

PART A (20 x 1 = 20 Marks)

1. If the vectors (1,1,2), (1,2,5) and $(5,3,\lambda)$ are linearly dependent then the value of λ is
   A. 4 B. -4 C. 0 D. 6

2. The product of the two eigenvalues of the matrix $A=\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16, then the third eigenvalue is
   A. 1 B. 2 C. 3 D. 4

3. The eigenvectors corresponding to distinct eigenvalues of a real symmetric matrix are
   A. zero B. equal C. linearly independent D. orthogonal

4. The nature of the quadratic form $f(x_1,x_2,x_3) = x_1 + 2x_2^2$ is
   A. Positive semi definite B. Negative semi definite
   C. Positive definite D. Indefinite

5. The length of the line whose projections on the co-ordinate axes are 2,3,6 is
   A. 11 B. 36 C. 7 D. 49

6. If the lines $\frac{x-4}{5} = \frac{y-3}{-2} = \frac{z-2}{k}$ and $\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z-1}{-7}$ are coplanar then the value of k is
   A. 2 B. -6 C. 4 D. 6

7. The equation the plane through the intersection the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and passing through the origin is
   A. $x - 3y = 0$ B. $x + 3y = 0$
   C. $5x + y + 4z - 8 = 0$ D. $3x - y = 0$

8. The radius of the sphere $7x^2 + 7y^2 + 7z^2 + 28x - 42y + 56z + 3 = 0$ is
   A. $10\sqrt{\frac{2}{7}}$ B. $\frac{200}{7}$ C. $\frac{206}{7}$ D. $\sqrt{\frac{206}{7}}$

9. The radius of curvature of $xy = 4$ at the point (2,2) is
   A. $\sqrt{2}$ B. 0 C. 2 D. $2\sqrt{2}$

4

10. The radius of curvature of $r = a\theta$ at the point $\theta = 1$ is

A. $\sqrt{2}\,a$ B. $\frac{2\sqrt{2}}{3}a$ C. $\frac{\sqrt{2}}{3}a$ D. $\frac{2a}{3}$

11. The curvature of a straight line is

A. 1 B. -1 C. ∞ D. 0

12. The envelope of the family of straight lines $\frac{x}{t} + yt = 2c$, t being the parameter is

A. $y^2 = 4x$ B. $xy = c^2$ C. $x^2 = 4y$ D. $y^2 = x$

13. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$ then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$ is equal to

A. $\frac{x}{y}$ B. 0 C. $\frac{y}{x}$ D. $\frac{z}{y}$

14. The minimum point of $f(x,y) = x^2 + y^2 + 6x + 12$ is

A. (0, 3) B. (0, -3) C. (-3, 0) D. (3, 0)

15. The nature of the stationary point (1, 2) of the function $f(x,y)$, if $f_{xx} = 6x$, $f_{xy} = 0$, $f_{yy} = 6y$ is

A. Minimum point B. Saddle point
C. Maximum point D. Fixed point

16. If $u = \frac{y^2}{x}$ and $v = \frac{x^2}{y}$ then the value of the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ is

A. 3 B. -3 C. $\frac{1}{3}$ D. $-\frac{1}{3}$

17. The solution of the equation $(D^2 + 4)y = 0$ is

A. $y = (Ax + B)e^{-2x}$ B. $y = (Ax + B)e^{2x}$
C. $y = Ae^{2x} + Be^{-2x}$ D. $y = A\cos 2x + B\sin 2x$

18. The particular integral of $(D+1)^2 y = e^{-x}\cos x$ is

A. $e^{-x}\cos x$ B. $-e^{-x}\cos x$
C. $-e^{-x}\sin x$ D. $e^{-x}\sin x$

19. The solution of the equation $(x^2 D^2 + 4xD + 2)y = 0$ is

A. $y = Ae^{-x} + Be^{-4x}$ B. $y = Ae^{x} + Be^{4x}$
C. $y = \frac{A}{x} + \frac{B}{x^2}$ D. $y = A\log x + B\log 4x$

20. If $Dx + y = \sin t$ and $x + Dy = \cos t$ then the value of x is

A. $x = (At + B)e^{-t}$ B. $x = Ae^{t} + Be^{-t}$
C. $x = A\cos t + B\sin t$ D. $x = (At + B)e^{t}$

is

PART- B (5 x 16 = 80 Marks)

21 a. Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$ to a canonical form by an orthogonal reduction. (16)

(OR)

b. i) Find the eigen value and eigen vector of the matrix $A = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ (8)

ii) Verify Cayley-Hamilton theorem, and hence find A^{-1} if

$$A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$$ (8)

22 a. i) Find the equation of the plane through the line of intersection of x + y + z = 1 and 2x + 3y + 4z = 5 and perpendicular to x – y + z = 0. (8)

ii) Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{1} \text{ and } \frac{x-5}{3} = \frac{y-5}{2} = \frac{z+1}{-5}$$ (8)

(OR)

b. i) Show that the lines $\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5}$ and $\frac{x-8}{7} = y-4 = \frac{z-5}{3}$ are coplanar. Find their common point and the equation of the plane in which they lie. (8)

ii) Find the equation of the sphere that posses through the circle $x^2 + y^2 + z^2 + x - 3y + 2z = 0$, $2x + 5y - z + 7 = 0$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 3x + 5y - 7z - 6 = 0$. (8)

23 a. i) Find the radius of curvature of the curve $xy = c^2$ at the point $x = c$. (8)

ii) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ subject to the condition $a^2 + b^2 = c^2$, where c is a known constant. (8)

(OR)

b. i) Find the centre of curvature for the curve $\sqrt{x} + \sqrt{y} = 1$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$. (8)

ii) Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another cycloid $x = a(\theta + \sin\theta)$, $y = -a(1 - \cos\theta)$. (8)

24 a. i) If $w = f(x-y, y-z, z-x)$, show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (8)

ii) Obtain maximum and minimum values of $f(x,y) = x^3 + y^3 - 3xy$ (8)

(OR)

b. i) Expand $e^x \cos y$ in powers of x and y upto the terms of the second degree using Taylor's series. (8)

ii) A rectangular box, open at the top is to have a volume of 32cc. Find the dimensions of the box which requires least material for its construction. (8)

25 a. i) Solve $(D^2 - 5D + 6)y = e^x \cos 2x$. (8)

ii) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos(\log(1+x))$. (8)

(OR)

b. i) Solve $(x^2D^2 + xD - 9)y = \frac{5}{x^2}$. (8)

ii) Using the method of variation of parameters solve $y'' + a^2 y = \tan ax$ (8)

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