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B.E./B.TECH. DEGREE EXAMINATION - DECEMBER 2008

2007 Batch - First Semester

U07MA101 MATHEMATICS - I

Time: Three Hours **Maximum Marks: 100**

Answer ALL Questions

PART A (20 x 2 = 40 Marks)

1. The eigen values of the matrix $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ is

a) (3,1,4) b) (3,2,5) c) (3,0,0) d) (3,2,6)

2. The sum and product of the eigen values of $A = \begin{bmatrix} 1 & 6 & 1 \\ 2 & 1 & 0 \\ 0 & 5 & 3 \end{bmatrix}$ is

a) 5, -23 b) 8, 12 c) 10, 15 d) 5, 4

3. Nature of the quadratic form $2x^2 + 3y^2 + 2xy$ is

a) positive definite b) positive semi definite

c) negative definite d) negative semi definite

4. If a square matrix A satisfies $AA^T = A^TA = I$ then A is called

a) orthogonal matrix b) column matrix

c) symmetric matrix d) triangular matrix.

5. The point where the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ meets the plane $x + y + z = 15$ is

a) (3,5,7) b) (5,3,7) c) (3,7,5) d) (5,7,3)

6. The angle subtended at the origin by points P (2,3-6) and Q(3, -4,5) is

a) $\theta = \cos^{-1}\left(\frac{18\sqrt{2}}{35}\right)$ b) $\theta = \cos^{-1}\left(\frac{9\sqrt{2}}{35}\right)$

c) $\theta = \cos^{-1}\left(\frac{30\sqrt{2}}{47}\right)$ d) $\theta = \sin^{-1}\left(\frac{18\sqrt{2}}{35}\right)$

7. The tangent plane at (1,2,0) to the sphere $3(x^2+y^2+z^2) + 8x + 12y + 16z - 47 = 0$ is

a) $7x + 12y + 8z - 31 = 0$ b) $12x + 7y + 8z = 0$

c) $7x + 8z = 0$ d) $12x + 13y = 31$.

8. The equation to the sphere, having the points (-4, 5 ,1), (4, 1 , 7) as ends of a diameter is

a) $x^2 + y^2 + z^2 = 0$ b) $x^2 + y^2 + z^2 - 6y + 8z + 4$

c) $x^2 + y^2 + z^2 - 3x + 3y$ d) $x^2 + y^2 + z^2 - 3x + 12$

9. The radius of curvature for a circle $x^2 + y^2 = r^2$ is

a) r b) $\frac{1}{r}$ c) r^2 d) $\frac{1}{r^2}$.

10. Curvature of a straight line y = mx + c is

a) 0 b) 1 c) m d) c

11. The envelope of the family of straight lines $y = mx + \frac{a}{m}$ is

a) $y^2 = 4ax$ b) $x^2 = 4ay$ c) $x^2 + y^2 = a^2$ d) xy = a.

12. Locus of the centre of curvature is

a) involute b) evolute c) envelope d) radius of curvature.

13. If x = uv, $y = \frac{u}{v}$ then $\frac{\partial(x,y)}{\partial(u,v)} =$

a) $\frac{-2u}{v}$ b) $\frac{2u}{v}$ c) $\frac{2v}{u}$ d) $\frac{-2v}{u}$.

14. If $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$ then $\frac{dz}{dt}$

a) $10a^3t^4 + 16a^3t^3$ b) $10a^3t^2 + 7a^3t^3$

c) $16a^3t^4 + 10a^3t^2$ d) $7a^2t^2 + 14a^3t^2$.

15. The stationery point of the function $x^2 + y^2 + 6x + 12$ is

a) (−3, 0) b) (0, −3) c) (1, 3) d) (2, 3)

16. If $x = e^r \sec\theta$, $y = e^r \tan\theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)}$ is

a) 1 b) 0 c) 2 d) −1

17. The solution of $(D^2 - 2D + 1)y = e^{-x}$ is

a) $e^x(Ax + B) + \frac{1}{4}e^{-x}$ b) $e^{-x}(Ax + B) + \frac{1}{4}e^{x}$

c) $e^x(Ax + B) - \frac{1}{4}e^{-x}$ d) $e^x(Ax + B) - \frac{1}{4}e^{x}$.

18. The Particular integral $(D^2 + 1)y = \sin 2x$

a) $\frac{\sin 2x}{-3}$ b) $\frac{\cos 2x}{3}$ c) $\frac{\sin 2x}{4}$ d) $\frac{\cos 2x}{4}$.

19. The solution of $(xD^2 + D)y = 0$ is

a) $A e^x + B$ b) A logx + B c) $A e^{-x}$ d) $A\log x^2 + B$.

20. If $\frac{dx}{dt} + y = 0$, $\frac{dy}{dt} + x = 0$, then x is

a) $A e^t + B e^{-t}$ b) $A e^{2t} + B e^{-t}$ c) $A e^{3t} + B e^t$ d) $A e^{3t} + B e^{-t}$

PART B (5 x 12 = 60 Marks)

21. (a) (i) Prove that the eigen values of a real symmetric matrix are real (4 marks)

(ii) Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & 1 \end{pmatrix}$ and hence find A^{-1}. (8 marks)

(OR)

21. (b) Reduce the quadratic form $8x_1^2 + 7x_2^2 + 3x_3^2 - 12x_1x_2 - 8x_2x_3 + 4x_1x_3$ to the canonical form through an orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.

22 (a) (i) Find the foot of the perpendicular drawn from the point (1, 0, -3) to the line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ (4 marks)

(ii) Find the equation of the sphere that passes through the circle $x^2 + y^2 + z^2 + 2x + 3y + z - 2 = 0$, $2x - y - 3z - 1 = 0$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 3x + y - 2 = 0$ (8 marks)

(OR)

22 b) (i) Find the equation of the plane which passes through the point (1,2,-1) and contains the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z+2}{-1}$ (4 marks)

(ii) Find the length and equation of the shortest distance between the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ (8 marks)

23. (a) (i) Find the equation of circle of curvature at the point $\left(\frac{a}{4}, \frac{a}{4}\right)$ of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ (7 marks)

(ii) Find the evolute of the parabola $x^2 = 4ay$ (5 marks)

(OR)

23. (b) (i) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where the parameters a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant. (7 marks)

(ii) Find the radius of curvature of the curve $r = a(1+\cos\theta)$ at the point $\theta = \frac{\pi}{2}$. (5 marks)

24. (a) (i) Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$$ (7 marks)

(ii) Discuss the maxima and minima of the function $f(x, y) = x^3 + y^3 - 12x^2 - 9y^2 + 256$ (5 marks)

(OR)

24.(b)(i) Find the Taylor's series expansion of the function $f(x, y) = e^x \log(1+y)$ upto the third degree terms.

(5 marks)

(ii) Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ (7 marks)

25 (a) (i) Solve the simultaneous differential equations:

$\frac{dx}{dt} + 2y = \sin 2t$ and $\frac{dy}{dt} - 2x = \cos 2t$ (6 marks)

(ii) Solve $\frac{d^2 y}{dx^2} + \frac{1}{x}\frac{dy}{dx} = \frac{12\log x}{x^2}$ (6 marks)

(OR)

25. (b) (i) Solve the equation $\frac{d^2 y}{dx^2} + y = \sec^2 x$ using the method of variation of parameters.

(8 marks)

(ii) Solve $(D^2 - 2D + 2)y = e^x \cos x$ (4 marks)

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