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B.E. / B.TECH DEGREE EXAMINATIONS: OCTOBER/NOVEMBER–2008

Third Semester

U07MA301 MATHEMATICS - III

Time: Three Hours **Maximum Marks: 100**

Answer All Questions:

PART – A (20 x 1 = 20 Marks)

1. The PDE obtained by eliminating 'a' and 'b' from $z = (x+a)^2 + (y-b)^2$

   a. $4z = p^2 + q^2$ b. $z = p^2q^2$ c. $4z = p + q$ d. $z^2 = p^2q^2$

2. The complete solution of $p^2 + q^2 = 4$ is

   a. $z = -ax \pm \sqrt{4 - a^2}\, y + c$ b. $z = -ax \pm (4 - a^2)y + c$

   c. $z = ax \pm \sqrt{4 - a^2}\, y + c$ d. $z = ax \pm \sqrt{4 - a^2}\, y + c$

3. The solution of $(D^3 - 2D^2D' - 4DD'^2 + 8D'^3)\ z=0$ is

   a. $z = f_1(y+2x) - xf_2(y+2x) + f_3(y-2x)$

   b. $z = f_1(y+2x) + xf_2(y+2x) + f_3(y-2x)$

   c. $z = f_1(y+2x) + f_2(y+2x) + f_3(y-2x)$

   d. $z = f_1(y+2x) + f_2(y+2x) - f_3(y+2x)$

4. The P.I. of $(D^2 - 3DD' + 2D'^2)z = \cos(x+2y)$ is

   a. $\frac{\cos(x+2y)}{3}$ b. $\frac{1}{2}\cos(x+2y)$ c. $-\frac{\cos(x+2y)}{3}$ d. $\frac{\cos(x-2y)}{3}$

5. If f(x) = x^2 in $(-\pi, \pi)$ then the value of a_0 is

   a. $\frac{\pi^2}{3}$ b. $\frac{2\pi^2}{3}$ c. $2\pi^2$ d. $\frac{4\pi^2}{3}$

6. The R.M.S. value of f(x) = x in $(-\pi, \pi)$ is

   a. $\frac{\pi}{3}$ b. $\frac{2\pi}{3}$ c. $\frac{\pi}{\sqrt{3}}$ d. $\frac{\pi}{\sqrt{2}}$

7. The half range sine series of unity in $(0, \pi)$ is

   a. $\frac{2}{n\pi}[1-(-1)^n]$ b. $\frac{2}{n\pi}[1+(-1)^n]$ c. $\frac{1}{n\pi}[1+(-1)^n]$ d. $2n[1+(-1)^n]$

8. If f(x) = e^x in $(0, 2\pi)$, then the value of a_0 is

   a. $\frac{1-e^{2\pi}}{\pi}$ b. $\frac{e^{2\pi}-1}{\pi}$ c. $\frac{e^{\pi}-1}{2\pi}$ d. $\frac{1-e^{\pi}}{2}$

9. Classification of the PDE $u_{xx} + 4u_{xy} + 4u_{yy} - 12u_x + u_y + 7u = x^2 + y^2$ is

a. Elliptic b. Hyperbolic c. Parabolic d. Cycloid

10. The one dimensional wave equation is

a. $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ b. $\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial^2 y}{\partial t^2}$ c. $\frac{\partial^2 y}{\partial x^2} = c^2 \frac{\partial y}{\partial t}$ d. $\frac{\partial y}{\partial x} = c^2 \frac{\partial^2 y}{\partial t^2}$

11. The correct solution for one dimensional heat equation

a. $u(x,t) = (c_1 \cos px + c_2 \sin px)e^{\alpha^2 p^2 t}$ b. $u(x,t) = (c_1 e^{px} + c_2 e^{-px})e^{\alpha^2 p^2 t}$

c. $u(x,t) = (c_1 \cos px + c_2 \sin px)e^{-\alpha^2 p^2 t}$ d. $u(x,t) = (c_1 e^{px} + c_2 e^{-px})e^{-\alpha^2 p^2 t}$

12. A rod of 30cm long has its ends A and B kept at 20 c and 80° c respectively until steady state prevails. The steady state temperature of the rod is

a. u=20x+2 b. u=x+10 c. u = 2x+20 d. u=x+2

13. Fc $\left[2e^{-3x}\right]$ is

a. $\sqrt{\frac{2}{\pi}} \frac{6}{s^2+9}$ b. $\sqrt{\frac{2}{\pi}} \frac{3}{(s^2+9)^2}$ c. $\sqrt{\frac{2}{\pi}} \frac{6}{(s^2+9)^2}$ d. $\sqrt{\frac{\pi}{2}} \frac{6}{(s^2+9)^2}$

14. Fs $\left[xe^{-ax}\right]$ is

a. $\sqrt{\frac{2}{\pi}} \frac{2as}{(s^2+a^2)}$ b. $\sqrt{\frac{2}{\pi}} \frac{2as}{(s^2+a^2)^2}$ c. $\sqrt{\frac{2}{\pi}} \frac{2a}{(s^2+a^2)^2}$ d. $\frac{2}{\pi} \frac{as}{(s^2+a^2)^2}$

15. The Fourier sine transform of $\frac{1}{x}$ is

a. $\frac{\pi}{2}$ b. $\frac{2}{\pi}$ c. $\sqrt{\frac{\pi}{2}}$ d. $\frac{\pi}{\sqrt{2}}$

16. The Fourier cosine transform f e^{-x} is

a. $\sqrt{\frac{2}{\pi}} \frac{1}{s^2+1}$ b. $\sqrt{\frac{2}{\pi}} \frac{s}{s^2+1}$ c. $\sqrt{\frac{\pi}{2}} \frac{1}{s^2+1}$ d. $\frac{2}{\pi} \frac{s}{s^2+1}$

17. Z [$\delta_{(n)}$] is equal to

a. 0 b. 1 c. $\frac{z}{(z-1)^2}$ d. $\frac{z}{z-1}$

18. $z\left[\cos\frac{n\pi}{2}\right]$ is equal to

a. $\frac{z}{z^2-1}$ b. $\frac{z^2-1}{z}$ c. $\frac{z^2}{z^2+1}$ d. $\frac{z^2}{z^2-1}$

19. $z\left[(-3)^n\right]$ is equal to

a. $\frac{z+3}{z}$ b. $\frac{(z-a)^2}{z}$ c. $\frac{z^2}{z-a}$ d. $\frac{z}{z+3}$

20. $z\left[\frac{1}{n!}\right]$ is

a. $e^{1/z}$ b. e^{z} c. $\frac{1}{e^{1/z}}$ d. e^{-z}

PART-B(5 X 16 = 80 Marks)

21.a (i). Find the Fourier series for f(x) = | cos x | in the interval $(-\pi, \pi)$ (8)

(ii). Find Fourier series upto second harmonic from the following data: (8)

X	0	1	2	3	4	5
Y	9	18	24	28	26	20

(OR)

b (i). Find the Fourier series of the function $f(x) = \begin{cases} kx & 0 < x < \ell \\ 0 & 0 < x < 2\ell \end{cases}$, k-constant. (8)

(ii). Find the half range cosine series for the function $f(x) = (x-1)^2$ in $0 < x < 1$. (8)

22. a (i) Form the PDE by eliminating the function from $z = xf\left(\frac{y}{x}\right) + y\phi(x)$. (8)

(ii) Solve $(D^2 - 2DD' + D'^2 - 3D + 3D' + 2)z = e^{2x+y}$ (8)

(OR)

b (i). Solve $(mz - ny)\frac{\partial z}{\partial x} + (nx - lz)\frac{\partial z}{\partial y} = (ly - mx)$ (8)

(ii). Solve $(D^2 + 4DD' - 5D'^2)z = xy + \sin(2x + 3y)$ (8)

23.a). A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $v_0 \sin^3 \frac{\pi x}{l}$, find the displacement y(x, t) of the string at any time t and at a distance x from the zero end. (16)

(OR)

b) A bar, 10cm. long, with insulated sides, has its ends A and B kept at 20^0C and 40^0C respectively, until steady-state conditions prevail, that is, until the temperature at an interior point no longer changes with time. The temperature at A is then suddenly raised to 50^0C and at the same instant that at B is lowered to 10^0C. Find the subsequent temperature function u(x,t) at any time. (16)

24. a (i) Find the Fourier transform of $f(x) = \begin{cases} (1-x^2), & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$. Hence prove that

$$\int_0^{\infty} \frac{Sin\, s - s\, Cos\, s}{s^3} Cos\left(\frac{s}{2}\right) ds = \frac{3\pi}{16}. \quad (8)$$

(ii) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence find $F_S[x e^{-a^2x^2}]$. (8)

(OR)

b (i) Evaluate $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$ by using Fourier transform. (8)

(ii) Find the Fourier sine transform of $f(x) = \begin{cases} x, & 0 < x < 1 \\ 2-x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$ (8)

25. a (i) Find the Z- transform of $\frac{1}{(n+1)(n+2)}$. (8)

(ii) Using convolution theorem evaluates $Z^{-1}\left[\frac{z^2}{(z-1)(z-3)}\right]$. (8)

(OR)

b (i) Solve the difference equation y(n+3) – 3y(n+1) +2y(n) =0 given that y(0) =4, y(1)=0 and y(2) = 8, by using Z-transform. (8)

(ii). State and prove Final value theorem of Z – transform (8)

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