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Reg. No. : | | | | | | | | | | | |

M 2491

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Fifth Semester

Chemical Engineering

MA 037 — SPECIAL FUNCTIONS, DIFFERENCE EQUATIONS AND Z-TRANSFORMS

(Common to Leather Technology and Textile Technology)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define ordinary and singular points of differential equation.

2. Find $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$

3. Give Bessel differential equation of order zero and order n.

4. Write modified Bessel function of order n.

5. State $P_0(x)$, $P_1(x)$ and $P_2(x)$.

6. State the orthogonality of legendre polynomials.

7. Write Hermite differential equation.

8. Give orthogonality of Laguerre polynomials.

9. Find the Z-transform of $\frac{1}{n}$.

10. If $z(f(n)) = F(z)$ then show that $f(0) = \underset{z \to \infty}{Lt} F(z)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\int_0^\infty x^n e^{-a^2x^2}$ in terms of Gamma function. (8)

(ii) Derive the relative between beta and Gamma function. (8)

Or

(b) Using power series method. Solve $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$. (16)

12. (a) (i) Show that $\sum_{n=-\infty}^{\infty} Jn(x)\, t^n = e^{\frac{x}{2}\left(t \frac{-1}{t}\right)}$. (10)

(ii) Express $J_6(x)$ in terms of $J_0(x)$ and $J_1(x)$. (6)

Or

(b) Show that one of the solutions of the differential equation

$4\frac{d^2y}{dx^2} + 9xy = 0$ is $y = \sqrt{x}\ J_{1/3}\left(x^{3/2}\right)$. (16)

13. (a) (i) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$. (8)

(ii) Show that $\int_{-1}^{+1} \frac{P_n(x)\ dx}{\sqrt{1-2xt+t^2}} = \frac{2t^n}{2n+1}$. (8)

Or

(b) (i) Show that $\int_{-1}^{1} x^m P_n(x)\, dx = 0$ if $m < n$ and $\int_{-1}^{+1} x^n P_n(x) dx = \frac{2^{n+1}(n!)^2}{(2_{n+1})!}$ (8)

(ii) Explain $x^3 - 8x^2 + 2x + 1$ in terms of Legendre Polynomials. (8)

14. (a) (i) Show that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$. (8)

(ii) Show that $\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x) dx = 0$ if $m \neq n$. (8)

Or

(b) (i) Prove that $L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}\left(x^n e^{-x}\right)$. (8)

(ii) Show that $\int_0^\infty e^{-x}(L_n(x))^2 dx = 1$. (8)

15. (a) (i) Solve $y_{n+3} - 12y_{n+2} + 48y_{n+1} - 64y_n = 5.4^n$ with $y_0 = y_1 = y_2 = 0$. (10)

(ii) Find the inverse z-transform of $\frac{z}{z^2 + 7z + 10}$, by residue method. (6)

Or

(b) (i) Find the z – transform, of

(1) $\frac{1}{n(n+1)}$ (4)

(2) $\cos\left(\frac{n\pi}{2}\right)$ (4)

(ii) Using z – transform, solve.

$y_{n+2} + 6y_{n+1} + 9y_n = 2^n$ with $y_0 = y_1 = 0$. (8)

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