Reg. No. : | | | | | | | | | | | |

L 1705

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

First Semester

Civil Engineering

MA 131 – MATHEMATICS – I

(Common to all branches of B.E/B.Tech except Marine Engineering)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that two similar matrices have the same characteristic roots.

2. Define orthogonal matrix.

3. Find the direction cosines of the line joining the points $(2,3,-6)$ and $(3,-4,5)$.

4. Find the equation of the sphere having the circle $x^2+y^2+z^2=9$; $x-2y+2z=5$ as a great circle.

5. Find the radius of the curvature of the curve $xy=c^2$ at (c,c).

6. Find the envelope of $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$, where θ is the parameter.

7. Find Taylor's series expansion of $e^x \sin y$ near the point $\left(-1,\frac{\pi}{4}\right)$ up to the first degree terms.

8. If $u=f(x-y,y-z,z-x)$ find $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

9. Solve : $p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$, where $p = \frac{dy}{dx}$.

10. Solve : $(D^2 + 2D + 1)y = e^{-x}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and the eigenvectors of

$$\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$$ (8)

(ii) Using Cayley – Hamilton theorem, Find A^{-1} if

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{pmatrix}$$

Also verify the theorem. (8)

Or

(b) Reduce the quadratic form $10x_1^2 + 2x_2^2 + 5x_3^2 + 6x_2x_3 - 10x_3x_1 - 4x_1x_2$ to a canonical form by orthogonal reduction. Find a set of values of x_1, x_2, x_3 which will make the form vanish. (16)

12. (a) (i) Find the angle between the straight lines whose direction cosines are given by the relations $31 + m + 5n = 0$ and $6mn - 2nl + 5lm = 0$. (8)

(ii) Prove that a point, at which the sum of the squares of whose distances from the planes $x+y+z=0$, $x-z=0$, $x-2y+z=0$ is 9, lies on the sphere $x^2+y^2+z^2=9$. (8)

Or

(b) (i) Find the equation of the sphere through the circle $x^2+y^2+z^2=9$, $2x+3y+4z=5$ and the point $(1,2,3)$. (8)

(ii) Find the length and the equations of the shortest distance between the lines $x-10 = \frac{y-9}{3} = \frac{z+2}{-2}$ and $\frac{x+1}{2} = \frac{y-12}{4} = z-5$. (8)

13. (a) (i) For the curve $x=a(\cos\theta+\theta\sin\theta)$, $y=a(\sin\theta-\theta\cos\theta)$, prove that the radius of curvature is $a\theta$. (8)

(ii) Find the radius of curvature at any point $(a\cos^3\theta, a\sin^3\theta)$ on the curve $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$. (8)

Or

(b) (i) Find the equation of the evolute of the parabola $y^2=4ax$. (8)

(ii) Considering the evolute of a curve as the envelope of its normals, find the envelope of $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (8)

14. (a) (i) Given the transformation $u=e^x\cos y$, $v=e^x\sin y$ and that ϕ is a function of u and v also of x and y, prove that $\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=(u^2+v^2)\left(\frac{\partial^2\phi}{\partial u^2}+\frac{\partial^2\phi}{\partial v^2}\right)$. (8)

(ii) Show that $\frac{d}{da}\int_0^{a^2}\tan^{-1}\left(\frac{x}{a}\right)dx = 2a\tan^{-1}(a)-\frac{1}{2}\log(a^2+1)$. (8)

Or

(b) (i) Expand $e^x\log(1+y)$ in powers of x and y up to terms of third degree. (8)

(ii) Find the shortest and the longest distances from the point (1,2,–1) to the sphere $x^2+y^2+z^2=24$. (8)

15. (a) (i) Solve : $(7x-3y-7)\,dx+(3x-7y-3)dy=0$. (8)

(ii) Solve : $(D^2+4)y=\sinh 2x+\pi$, $(D^2-6D+13)y=5e^{2x}$. (8)

Or

(b) (i) Solve : $(x^2D^2-3xD+4)y=x^2\cos(\log x)$. (8)

(ii) Solve : $y''+a^2y=\tan ax$ by the method of variation of parameters. (8)