Reg. No. : ☐☐☐☐☐☐☐☐☐☐☐☐

L 1460

B.E/B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Second Semester

Civil Engineering

MA 132 — MATHEMATICS — II

(Common to all branches except Information Technology)

Time : Three hours

Maximum : 100 marks

Use of statistical tables permitted

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Change into polar coordinates, the integral $\int\limits_{-a}^{a}\int\limits_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\, dx$

2. Determine the value of $\Gamma\left(\dfrac{11}{2}\right)$.

3. Determine whether $\vec{F} = \left(2xy + Z^3\right)\hat{i} + x^2\hat{j}\; 3xz^2\,\hat{k}$ is a conservative force field.

4. State Stokes theorem.

5. Find the real and imaginary parts of $f(z) = 2z^3 - 3z$.

6. Find the points in the z –plane at which the mapping $w = z + z^{-1}, (z \neq 0)$ fails to be conformal.

7. Find the radius of convergence of the series $\sum\limits_{n=0}^{\infty}\dfrac{(z-a)^n}{n!}$ where a is any number.

8. Determine the residue at the pole $z = -2$ of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$.

9. Write down the formula for μ_3 in terms of moment about any point.

10. Prove that correlation coefficient is the geometric mean of between the regression coefficients.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Evaluate $\iint r^2 dr\, d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$. (8)

    (ii) Evaluate $\int_0^\infty \int_0^x x e^{\frac{-x^2}{y}} dy\, dx$ by changing the order of integration. (8)

    Or

    (b) (i) Find by triple integration, the volume in the positive octant bounded by the coordinate planes and the plane $x + 2y + 3z = 4$. (8)

    (ii) Prove that $y\beta(x+1, y) = x\beta(x, y+1)$. (3)

    (iii) Evaluate $\int_0^1 \dfrac{x^2 dx}{\sqrt{1-x^4}}$ in terms of gamma function. (5)

12. (a) (i) Find the work done when a force $\vec{F} = (y+3)\hat{i} + xz\hat{j} + (yz-x)\hat{k}$ moves a particle from $(0,0,0)$ to the point $(2,1,1)$ along the curve $x = 2t^2, y = t, z = t^3$ (6)

    (ii) State the divergence theorem. Verify the theorem for $\vec{F} = 2x^2 y\hat{i} - y^2\hat{j} + 4xz^2\hat{k}$ taken over the region in the first octant bounded by $y^2 + z^2 = 9$ $x = 0$ and $x = 2$. (10)

    Or

    (b) (i) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that grad $\left(\dfrac{1}{r}\right) = \dfrac{-\vec{r}}{r^3}$. (4)

    (ii) What is the greatest rate of increase of $u = x^2 + yz^2$ at the point $(1, -1, 3)$? (4)

    (iii) State Green's theorem. Verify the theorem for $\oint_C (xy^2 - 2xy)dx + (x^2y + 3)dy$ around the boundary of C of the region enclosed by $y^2 = 8x$ and $x = 2$. (8)

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13. (a) (i) If $f(z) = u + iv$ is an analytic function, find $f(z)$ if
$v = \log(x^2 + y^2) + x - 2y$ (8)

(ii) Find the image of the strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation
$w = \frac{1}{z}$. Also show the regions graphically. (8)

Or

(b) (i) Explain the Milne Thomson's method to determine the analytic
function $f(z)$ when its real part or imaginary part is given. Apply
the method to find $f(z)$ when the real part of $f(z)$ is
$e^{2x}(x \cos 2y - y \sin 2y)$. (10)

(ii) Find the bilinear transformation which maps the points $z = 1,\ i, -1$
into points $w = i, 0, -i$ (6)

14. (a) (i) Evaluate $\int_{0}^{2+i} (Z^{-2}) dz$, along the real axis to 2 and then vertically to
$2 + i$. (6)

(ii) Evaluate $\int_{0}^{2\pi} \frac{d\theta}{2 + \cos\theta}$. (10)

Or

(b) (i) Expand $\frac{1}{z^2 - 4z + 3}, 1 < |z| < 3$ as a Laurent series. (8)

(ii) Evaluate $\oint_{C} \frac{2z - 1}{z(z+1)(z-3)} dz$ where C is the circle $|z| = 2$. (8)

15. (a) (i) Show that the correlation coefficient between X and Y remains
unaltered by a linear change of variables. (8)

(ii) Given the following pairs of measurements for two variables.

X : 5 8 3 9 10 12

Y : 9 12 5 15 18 20

Use the regresion line of y on x to estimate the value of y when
$x = 1$. (8)

Or

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(b) (i) A survey of 320 families with 5 children each revealed the following distribution.

No. of boys :   5   4   3   2   1   0

No. of families : 14 56 110 88 40 12

Is this result consistent with the hypothesis that male and female births are equally likely. Test at 0.05 level of significance.   (8)

(ii) Two independent samples of eight and seven items respectively had the following values of the variables

Sample I   9   11 13 11 15   9 12 14

Sample II 10 12 10 14   9   8 10

Do the two estimates of population mean differ significantly at 5% level of significance.   (8)

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