Reg. No. : | | | | | | | | | | | |

Y 5230

M.E. DEGREE EXAMINATION, MAY/JUNE 2008.

First Semester

Structural Engineering

MA 145 — APPLIED MATHEMATICS

(Regulation 2002)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $u(x,t)$ is a function of two variables x,t. Find $L\left\{\frac{\partial u}{\partial t}\right\}$.

2. State conditions under which Laplace transform exists.

3. Define Dirichlet problem for a rectangle.

4. Find Fourier transform of $e^{-|x|}$.

5. State Euler poisson equation.

6. Write the ostrogradsky equation for the functional $I\{Z(x,y)\} = \iint_D \left(z_x^2 + z_y^2\right) dx\,dy$.

7. **Given :**

$$F(x) = \begin{cases} 0 & -\infty \le x < 0 \\ \dfrac{x^2}{4} & 0 \le x < 1 \\ \dfrac{2x-1}{4} & 1 \le x < 2 \\ -\dfrac{x^2}{4} + \dfrac{3x}{2} - \dfrac{5}{4} & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$$

Find $f(x)$.

8. Define conditional density function of Y given X and conditional density function of X given Y.

9. Define Multiple and Partial correlation.

10. Define Parameter and Statistics.

PART B — (5 × 16 = 80 marks)

11. (a) Using Laplace transform method solve $u_{tt} = u_{xx}$ $0<x<1$ $t>0$

Boundary conditions : $u(0,t)=0=u(1,t) t>0$

$$u(x,0)=\sin \pi x$$

Initial conditions : $u_t(x,0)=-\sin \pi x$ $0<x<1$. (16)

Or

(b) Solve $ku_{xx}=u_t$ $0<x<\infty$; $t>0$

Boundary condition : $u(0,t)=u_0$ $t>0$

Initial condition : $u(x,0)=0$; $0<x<\infty$; u and $u_x \to 0$ as $x \to \infty$. (16)

12. (a) Prove :

(i) If the Dirichlet's problem for a bounded region has a solution, then it is unique. (8)

(ii) If the Newmann problem for a bounded region has a solution, then it is either unique or it differs from one another by a constant only. (8)

Or

(b) Solve :

$u_{xx} + u_{yy} = 0;\ 0 \le x \le l,\ 0 \le y \le \infty;$

$u(0,y) = e^{-2y};\ u(l,y) = 0;\ y < 0;\ u_y(x,0) = 0\ \ 0 \le x \le l.$

Find $u(x,y)$. (16)

13. (a) (i) Find the extremals of the functional $\int_{x_0}^{x_1} (2yz - 2y^2 + y'^2 - z'^2)\,dx$. (8)

(ii) Find the extremals of the functional $\int_{x_0}^{x_1} (x^2 y'^2 + 2y^2 + 2xy)\,dx$. (8)

Or

(b) Using Ritz method find an approximate solution to the problem of the minimum of the functional $v\{y(x)\} = \int_0^1 (y'^2 - y^2 + 2xy)\,dx;\ \ y(0) = y(1) = 0$, Seeking the solution in the form $y(x) = e^{x(1-x)}$. Also compare with exact solution at $x = 0, .25, .5, .75$ and 1.0. (16)

14. (a) (i) The probability density function

$$f(x) = \begin{cases} kx & 0 \le x \le 5 \\ k(10-x) & 5 \le x \le 10 \\ 0 & \text{elsewhere} \end{cases}$$

Find k, also find $P(2.5 \le x \le 7.5)$. (8)

(ii) A random variable X has probability mas function $p(x) = \dfrac{1}{2^x}$ $x = 1, 2, 3, \ldots$. Find moment generating function and hence mean and variance. (8)

Or

(b) (i) The joint probability of X and Y is

$$f(x,y) = xye^{-\left(\frac{x^2+y^2}{2}\right)} \quad x \geq 0, y \geq 0$$
$$= 0 \quad \text{otherwise}$$

Find marginal probability density functions $g(x)$ and $h(y)$. Are X and Y independent. (8)

(ii) The joint density function of random variable X and Y is

$$f(x,y) = 2-x-y \quad 0 \leq x \leq 1, 0 \leq y \leq 1$$
$$= 0 \quad \text{otherwise}$$

Find $Cov(X,Y)$. (8)

15. (a) (i) In a trivariate distribution $r_{12} = 0.7$, $r_{23} = r_{31} = 0.5$. Find $r_{23.1}$, $R_{1.23}$. (8)

(ii) $\{x_1, x_2, x_3\}$ is a random sample from a normal population with mean μ and variance σ^2. Show that the estimator's $\hat{\vartheta}_1$ and $\hat{\theta}_2$ for μ defined below are unbiased. Also compare their efficiencies. $\hat{\theta}_1 = \frac{x_1 + x_2 + x_3}{3}$, $\hat{\theta}_2 = \frac{x_1 + 2x_2 + 3x_3}{6}$. (8)

Or

(b) (i) Find regression equation of X_1 on X_2 and X_3 given (8)

$\overline{X}_1 = 28.02$	$\overline{X}_2 = 4.91$	$\overline{X}_3 = 594$
$\sigma_{X_1} = 4.42$	$\sigma_{X_2} = 1.1$	$\sigma_{X_3} = 85$
$r_{12} = 0.8$	$r_{23} = -.56$	$r_{31} = -.4$

(ii) Find the MLE for the parameter λ of a Poisson distribution on the basis of a sample of size 'n'. Also find its variance. (8)