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**Y 5235**

M.E. DEGREE EXAMINATION, MAY/JUNE 2008.

First Semester

Applied Electronics

MA 151 — APPLIED MATHEMATICS FOR ELECTRONICS ENGINEERS

(Common to M.E. – Communication Systems, M.E. – Digital Communication and Network Engineering, M.E. – Optical Communication and M.E. VLSI Design)

(Regulation 2002)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If  $u(x, t)$  is a function of two variables  $x$  and  $t$  and if  $U(x, s) = L[u(x, t); s]$  prove that

$$L(u_t; s) = sU(x, s) - u(x, 0)$$

2. A tightly stretched homogeneous string of length  $L$ , with its fixed ends at  $x = 0$  and  $x = L$  executes transverse vibrations. Motion is started with zero initial velocity by displacing the string into the form,  $f(x) = k(x^2 - x^3)$ . Find the deflection  $u(x, t)$  at any time  $t$  using D'Alembert's principle.
3. Show that the Bessel's function  $J_n(x)$  is an even or odd function according as  $n$  is even or odd integer.
4. Prove that  $\int x J_0(x) dx = x J_1(x)$ .
5. Express  $x^3 - 2x^2 - x - 3$  in terms of Legendre polynomials.

6. If the random variable  $X$  has the density function

$$f(x) = cx, \quad 0 \leq x \leq 2 \\ = 0, \quad \text{otherwise}$$

find the constant  $c$ .

7. If  $M_X(t)$  is the moment generating function of the random variable  $X$  and  $a$  and  $b$ , ( $b \neq 0$ ) are constants, find the moment generating function of  $\frac{X+a}{b}$ .
8. Find the probability that in a family of 4 children, there will be at least one boy.
9. Explain queue discipline.
10. Mention the commonly used measure of performance in a queueing situation.

PART B — (5 × 16 = 80 marks)

11. (a) Find the D'Alembert's solution of the one-dimensional wave equation  $u_{tt} = a^2 u_{xx}$  satisfying the initial condition  $u(x, 0) = f(x)$  and  $u_t(x, 0) = 0$ . (16)

Or

- (b) Using Laplace transformation method, solve the initial boundary value problem  $u_{tt} = u_{xx}$ ,  $0 < x < 1$ ,  $t > 0$  with the boundary condition  $u(0, t) = u(1, t) = 0$ ,  $t > 0$  and the initial condition  $u(x, 0) = \sin \pi x$ ,  $u_t(x, 0) = -\sin \pi x$ ,  $0 < x < 1$ . (16)
12. (a) Find the series solution of the Bessel's equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - n^2)y = 0. \quad (16)$$

Or

- (b) (i) State and prove Rodrigue's formula for Legendre polynomials.
- (ii) For  $n$  an integer, prove that  $J_n(x)$  is the coefficient of  $t^n$  in the expansion of  $e^{\frac{x}{2}\left(t - \frac{1}{t}\right)}$ . (8 + 8)

13. (a) Prove that

$$\int_{-1}^1 P_n(x)P_m(x) = 0, \quad m \neq n$$
$$= \frac{2}{2m+1}, \quad m = n$$
(16)

Or

(b) (i) If  $X$  is a normal random variable with mean zero and variance  $\sigma^2$ , find the probability density function of  $Y = e^X$ .

(ii) If  $X$  is a continuous random variable with probability density function  $f(x) = kx^2 e^{-x} \geq 0$ , find  $k$ , mean and variance. (8 + 8)

14. (a) (i) Find the moment generating function of the binomial distribution and hence find its mean and variance.

(ii) Find the regression coefficient of  $Y$  on  $X$  and  $X$  on  $Y$  from the data :  $\Sigma x = 50$ ,  $\bar{x} = 5$ ,  $\Sigma y = 60$ ,  $\bar{y} = 6$ ,  $\Sigma xy = 350$ ,  $\text{var}(x) = 4$  and  $\text{var}(y) = 9$ . (8 + 8)

Or

(b) If the joint pdf of a two dimensional random variable  $(X, Y)$  is given by  $f(x, y) = xy^2 + x^2/8$ ,  $0 \leq x \leq 2$ ,  $0 \leq y \leq 1$ , compute  $P(X > 1)$ ,  $P\left(Y < \frac{1}{2}\right)$ ,  $P(X < Y)$ ,  $P(X + Y \leq 1)$ . (16)

15. (a) For the  $[M/M/1]: (\infty/FIFO)$  queueing model, derive the expression for,  $L_s$ ,  $L_q$ ,  $L_w$ ,  $P(N > k)$ ,  $E(W_q)$  and  $E(W_s)$ . (16)

Or

(b) In a single server queueing system with poisson input and exponential service time, if the mean arrival rate is 3 calling units per hour, the expected service time is 0.25 h and the maximum possible number of calling units in the system is 2, find  $P_n (n \geq 0)$  average number of calling units in the system and in the queue and average waiting time in the system and in the queue. (16)