Reg. No. : | | | | | | | | | | | |

T 3326

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2008.

Fourth Semester

Biotechnology

MA 1255 — PROBABILITY AND STATISTICS

(Regulation 2004)

Time : Three hours

Maximum : 100 marks

Use of Statistical and Control Chart tables is permitted.

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If A, B, C are any three events such that $P(A)=P(B)=P(C)=\frac{1}{4}$; $P(A \cap B)=P(B \cap C)=0$; $P(C \cap A)=\frac{1}{8}$; find the probability that at least one of the events A, B and C occurs.

2. Find the mean of the Geometric distribution.

3. One ball is drawn at random from a box containing 2 white, 3 red and 4 black balls. If X denotes the no. of white balls drawn and Y denotes the no. of red balls drawn, find the joint probability mass function of (X,Y).

4. If $Y=-2X+3$, find the $\text{cov}(X,Y)$.

5. Classify random processes.

6. Define strict sense stationary process.

7. What are the two types of maintenance?

8. The density function of the time to failure (in years) of a component is given by $f(t) = \frac{200}{(t+10)^3}$, $t \geq 0$. Find the reliability function.

9. Compare Randomized block design and Latin square design.

10. When do you say that a process is out of control?

PART B — (5 × 16 = 80 marks)

11. (a) (i) There are four candidates for an officer position. The respective probabilities that they will be selected are 0.3, 0.2, 0.4 and 0.1. The probabilities for a project's approval are 0.35, 0.85, 0.45 and 0.15 depending on which of the 4 candidates is selected. What is the probability of the project being approved? (8)

(ii) Find the moment generating function of Binomial distribution and hence find its mean and variance. (8)

Or

(b) (i) A country filling station is supplied with petrol once a week. If its weekly volume X of sales in thousands of litres is distributed by

$$f(x) = \begin{cases} 5(1-x)^4, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

What must be the capacity of the tank in order that the probability that its supply will be exhausted in a given week shall be 0.01? (8)

(ii) Given the random variable X has the density function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the density function of $Y = 8X^3$. (8)

12. (a) (i) For the joint pdf

$$f(x, y) = \begin{cases} k(1+xy), & |x| < 1, |y| < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find k and the marginal pdfs. Are X and Y independent? (8)

(ii) If $X_1, X_2, \ldots X_n$ are Poisson variates with parameter $\lambda = 2$, use the Central limit theorem to estimate $P(120 \leq S_n \leq 160)$ where $S_n = X_1 + X_2 + \ldots + X_n$ and $n = 75$. (8)

Or

(b) (i) If the joint pdf of (X, Y) is

$$f(x, y) = x + y; \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1 \end{cases}.$$

find the joint pdf of XY. (8)

(ii) Show that the correlation coefficient of two random variables X and Y is independent of change of origin and scale of random variables. (8)

13. (a) (i) Show that the random process $X(t) = A\cos(wt + \theta)$ is wide sense stationary, if A and w are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (8)

(ii) Suppose that customers arrive at a counter independently from 2 different sources. Arrivals occur in accordance with a Poisson process with mean rate of 6 per hour from the first source and 4 per hour from the second source. Find the mean interval between any two successive arrivals. (8)

Or

(b) (i) Show that addition of two Poisson processes is again a Poisson process. (6)

(ii) A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However if he sells either in B or C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady states? (10)

14. (a) The time to failure in operating hours of a critical solid state power unit has the hazard rate function $\lambda(t) = 0.003\left(\dfrac{t}{500}\right)^{0.5}$, $t \ge 0$.

(i) What is the reliability if the power unit must operate continuously for 50 hours? (5)

(ii) Determine the design life if a reliability of 0.90 is desired. (4)

(iii) Compute the MTTF. (3)

(iv) Given that the unit has operated for 50 hours. What is the probability that it will survive a second 50 hours of operation? (4)

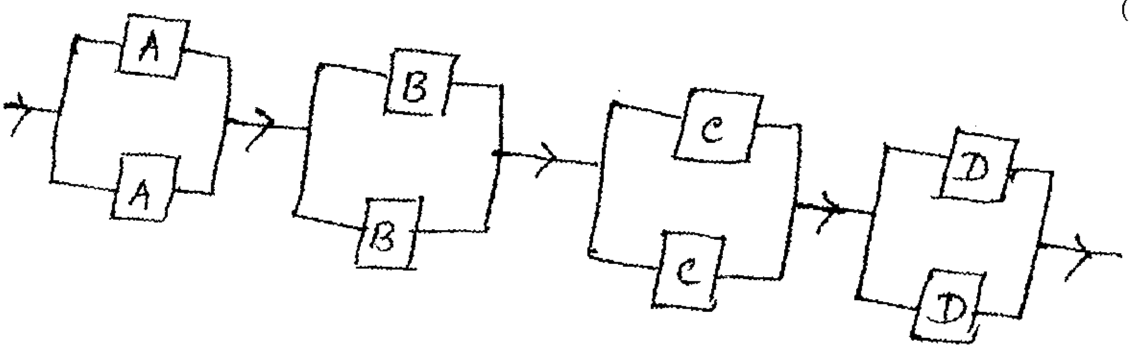
Or

(b) (i) If a device has a failure rate of $\lambda(t) = (0.015 + 0.02t)/\text{year}$, where t is in years,

(1) Calculate the reliability for a 5 year design life assuming that no maintenance is performed.

(2) Calculate the reliability for a 5 year design life assuming that annual preventive maintenance restores the device to an as good as new condition. (8)

(ii) Compute the reliability of the system for the connection given in figure, if the reliabilities of A, B, C, D are 0.90, 0.95, 0.96 and 0.98 respectively. (8)



15. (a) The following data resulted from an experiment to compare three burners B_1, B_2 and B_3. A latin square design was used as the tests were made on 3 engines and were spread over 3 days.

	Engine 1	Engine 2	Engine 3
Day 1	$B_1 - 16$	$B_2 - 17$	$B_3 - 20$
Day 2	$B_2 - 16$	$B_3 - 21$	$B_1 - 15$
Day 3	$B_3 - 15$	$B_1 - 12$	$B_2 - 13$

Test the hypothesis that there is no difference between the burners. (16)

Or

(b) Given below are the values of sample mean $\overline{X}$ and sample range R for 10 samples, each of size 5. Draw the appropriate mean and range charts and comment on the state of control of the process. (16)

Sample No.	1	2	3	4	5	6	7	8	9	10
Mean	43	49	37	44	45	37	51	46	43	47
Range	5	6	5	7	7	4	8	6	4	6