Register No: …………………………………

MCA DEGREE EXAMINATIONS APRIL/MAY 2011

Second Semester

MASTER OF COMPUTER APPLICATIONS

MAT509: Mathematical Foundations of Computer Science

Time: Three Hours **Maximum Marks: 100**

Answer all Questions

PART A (10 x 2 = 20 Marks)

1. Define a rank of a matrix.
2. If the eigen values of a matrix A are 1, 2, 3 then what are the eigen values of adjA?
3. Prove analytically that $(A\cap B) \cup (A\cap B^c) = A$.
4. State the nature of the relation defined on the set of integers by aRb iff ab>0.
5. Write the statement "To be poor is to be happy" in symbolic form.
6. Translate the statement "The sum of two positive integers is positive" into a logical expression.
7. Give a grammar for Palindromes over 0 and 1.
8. State the pumping lemma for regular languages.
9. Design a DFA that accepts the strings having even number of a's over {a,b}.
10. Define non-deterministic finite state automata.

PART B (5 x 16 = 80 Marks)

11. a) (i) Show that the system 3x +y +z =2; x -3y +2z =1; 7x -y +4z =5 has one- parameter family of solutions. (8)

(ii) Verify Cayley-Hamilton theorem, for

$$A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$$ and hence find A^{-1}.

(OR)

b) Find the eigen values and eigen vectors of (8)

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

12. a) (i) If A, B, C and D are sets prove algebraically that $(A \cap B) \times (C \cap D) = (A \times C) \cap (B \times D)$. (8)

(ii) If R is the relation on the set of integers such that $(a, b) \in R$ if and only if $3a+4b = 7n$ for some integer n, prove that R is an equivalence relation. (8)

(OR)

b) (i) Determine the number of positive integers 'n' where $1 \leq n \leq 200$ and 'n' is not divisible by 2, 3, 5 and divisible by 7. (8)

(ii) If $f: Z \rightarrow N$ defined by

$$f(x) = 2x-1, \text{ if } x>0$$
$$= -2x, \text{ if } x \leq 0$$ (8)

Find f^{-1} if it exists.

13. a) (i) Obtain the pdnf and pcnf of $(P \rightarrow (Q \wedge R)) \rightarrow (\sim P \wedge (\sim Q \wedge R))$ (8)

(ii) Check the validity of the following arguments:

Lions are dangerous animals. There are Lions. Therefore there are dangerous animals. (8)

(OR)

b) (i) Show that the hypothesis "If you send me an e-mail message, then I will finish writing the program". "If you do not send me an e-mail message, then I will go to sleep early", and " If I go to sleep early, then I will wake up feeling refreshed" leads to the conclusion that " If I do not finish writing the program, then I will wake up feeling refreshed". (8)

(ii) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ (8)

14. a) (i) Explain various types of grammar with example. (8)

(ii) Construct a context-free grammar for the language $L=\{a^n b^m a^m b^n: m, n \geq 1\}$. (8)

(OR)

b) (i) Construct a grammar for the language $L = \{a^n b^n : n \geq 1\}$ (8)

(ii) Show that $L = \{0^i 1^i\}$ is not regular (8)

15. a) (i) Construct a automation M which will accept those words over {a, b}where the number of b's is not divisible by three. (8)

(ii) Construct a deterministic finite automation equivalent to the grammar

$S \rightarrow aS/bS/aA,\ A \rightarrow bB,\ B \rightarrow aC,\ C \rightarrow \lambda.$ (8)

(OR)

b) (i) Prove that for every NFA there is an equivalent DFA (8)

(ii) Construct a FSA that accept the set of natural numbers x which are divisible by three (8)

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