Register No: ……………………

B.E./B.TECH DEGREE EXAMINATIONS: APRIL/MAY 2011

Second Semester

U07MA201: MATHEMATICS – II

(Common to B.E - Civil, Aeronautical, Mechanical, Electronics & Instrumentation, Electronics & Communication, Computer Science & Engineering ,& B.Tech - Information Technology, Textile Technology, Fashion Technology)

Time: Three Hours **Maximum Marks: 100**

Answer All the Questions:-

PART A (10 x 1 = 10 Marks)

1. The value of $\int_0^a \int_0^{\sqrt{ay}} xy\, dx\, dy$ is

   a) $\frac{a^6}{6}$ b) $\frac{a^4}{6}$ c) $\frac{a^3}{3}$ d) $\frac{a^2}{2}$

2. $\int_0^a \int_0^b dx dy$ represents

   a) Area of triangle b) Area of circle c) Area of rectangle d) Area of square

3. If the vector $\vec{F} = (x-3y)\vec{i} + (y-2z)\vec{j} + (x+kz)\vec{k}$ is solenoidal, then the value of 'k' is

   a) -2 b) 4 c) 2 d) 1

4. A vector $\vec{F}$ is conservative if it is a

   a) solenoidal vector b) irrotational vector c) scalar potential d) unit vector

5. An analytic function with constant modulus is

   a) a function of x alone. b) a function of y alone.

   c) a null function d) a constant function

6. The invariant points of the transformation $w = -\left(\frac{2z+4i}{iz+1}\right)$ are

   a) -4i, -i b) 4i, i c) 4i, -i d) -4i, i

7. The value of $\int_c \frac{z}{z^2-1} dz$ where C is $|z| = 1/2$ is

   a) 6π i b) 18π i c) 14π i d) 0

8. If the Laurent's expansion of f (z) contains only a finite number of negative powers of (z - a), then z = a is called a

   a) pole b) residue

   c) removable singularity d) essential singularity

9. $L[t\sin 2t]$ is

a) $\frac{4s^2}{(s^2+2)^2}$ b) $\frac{4s}{(s^2+4)^2}$ c) $\frac{4}{(s^2+4)^2}$ d) $\frac{-4}{(s^2+4)^2}$

10. The value of $\int_0^\infty e^{-3t}\cos 2t\,dt$ is

a) $\frac{2}{13}$ b) $\frac{3}{13}$ c) $\frac{6}{13}$ d) 0

PART B (10 x 2 = 20 Marks)

11. Evaluate $\int_0^1\int_1^2 x(x+y)\,dy\,dx$.

12. Change the order of integration $\int_0^1\int_0^x f(x,y)\,dy\,dx$.

13. Find $\nabla(r)$.

14. Prove that $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is solenoidal.

15. Show that the function $U = \frac{1}{2}\log(x^2+y^2)$ is harmonic.

16. Find the points at which the mapping $\omega^2 = (z-\alpha)(z-\beta)$ is not conformal.

17. Evaluate $\int_c \frac{z\,dz}{z-2}$ where $C : |z| = 1$.

18. Expand $\log(1+z)$ as a Taylor series about $z = 0$.

19. State convolution theorem on Laplace transform

20. Find $L^{-1}\left[\frac{s}{(s+2)^2}\right]$.

PART C (5 x 14 = 70 Marks)

21. a) Change the order of integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dx$.

(OR)

b) (i) Using double integral find the area bounded by $y = x$ and $y = x^2$. (7)

(ii) Find the volume of the sphere $x^2+y^2+z^2 = a^2$. (7)

22. a) Verify Green's theorem in XY plane for $\int_c (3x-8y^2)\,dx + (4y-6xy)\,dy$ where C is the boundary of the region given by $x = 0, y = 0$ and $x + y = 1$.

(OR)

b) (i) Find the directional derivative of $\varphi = x^2 y z + 4xz^2$ at $(1,-2,-1)$ in the direction of $2\vec{i} - \vec{j} - 2\vec{k}$. (7)

(ii) Find $\iint \vec{r}\, ds$ where S is the surface of the tetrahedron whose verities are $(0,0,0), (1,0,0), (0,1,0), (0,0,1)$. (7)

23. a) (i) If $f(z) = u + iv$ is a regular function then prove that $\nabla^2 |f(z)|^2 = 4|f^1(z)|^2$. (7)

(ii) Find the image of $|z - 2i| = 2$ under the transformation $\omega = \frac{1}{z}$. (7)

(OR)

b) (i) Find the bilinear transformation that maps the points $\infty, i, 0$ on to $0, 1, \infty$ respectively. (7)

(ii) Find an analytic function whose real part is $e^x (x\cos y - y\sin y)$. (7)

24. a) Evaluate $\int_0^{2\pi} \frac{d\theta}{5 + 4\cos\theta}$ by using contour integration

(OR)

b) (i) Find the Laurent's series expansion of $\frac{z-1}{(z+2)(z+3)}$ valid in the region $2 < |z| < 3$. (7)

(ii)Using Cauchy's integral formula evaluate $\int_c \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where c is $|z| = 3$. (7)

25. a) Find the Laplace transform of $f(t) = \begin{cases} t & ,0 < t < a \\ 2a - t, & a < t < 2a \end{cases}$ with $f(t + 2a) = f(t)$.

(OR)

b) (i) Using Convolution theorem find $L^{-1}\left[\frac{s}{(s^2 + a^2)^2} \right]$. (7)

(ii) Solve $\frac{d^2 y}{dx^2} - \frac{dy}{dx} + 2y = 0$ given that $y = \frac{dy}{dx} = 1$ at $x = 0$ using Laplace transform method. (7)

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