Register No: ……………………………

B.E. DEGREE EXAMINATIONS: APRIL/MAY 2011

Fourth Semester

U07MA401: NUMERICAL METHODS

(Common to Aeronautical Engineering, Civil Engineering, Mechatronics Engineering, Mechanical Engineering)

Time: Three Hours **Maximum Marks: 100**

Answer All Questions:-

PART A (10 x 1 = 10 Marks)

1. The order of convergence of Newton Raphson method is

a) 1 b) 2 c) 0 d) 3

2. The condition to apply Gauss – Jacobi method to solve a system of equations

a) diagonally dominant b) not diagonally dominant c) any systems d) none

3. A linear interpolating polynomial given the points $(0,0)$ and $(1,1)$.

a) y = -x b) $y = x^2$ c) y = x d) y = 0

4. The Newton's backward difference formula.

a) $y=y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \ldots\ldots\ldots$ b) $y=y_n + p\nabla y_n + \frac{p(p-1)}{2!}\nabla^2 y_n + \ldots\ldots\ldots$

c) $y = y_n - p\nabla y_n - \frac{p(p-1)}{2!}\nabla^2 y_n - \ldots\ldots\ldots$ d) $y=y_n - p\nabla y_n - \frac{p(p-1)}{2!}\nabla^2 y_n - \ldots\ldots\ldots$

5. The two point Gaussian quadrature formula to evaluate $\int_{-1}^{1} f(x)dx$

a) $f\left(-\frac{1}{\sqrt{3}}\right) - f\left(\frac{1}{\sqrt{3}}\right)$ b) $f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$ c) $f\left(\frac{2}{\sqrt{3}}\right) + f\left(-\frac{2}{\sqrt{3}}\right)$ d) $f\left(-\sqrt{\frac{3}{5}}\right) + f\left(\sqrt{\frac{3}{5}}\right)$

6. The n^{th} divided difference of a polynomial of the n^{th} degree are

a) n b) n! c) n^2 d) constant

7. The error in Simphson's 1/3 rule is

a) $o(h^4)$ b) $o(h^3)$ c) o (h) d) $o(h^2)$

8. The Euler's formula

a) $y_{n+1} = y_n + f(x_n, y_n)$ b) $y_{n+1} = y_n + hf(x_n, y_n)$ c) $y_{n+1} = y_n - f(x_n, y_n)$ d) $y_{n+1} = y_n - hf(x_n, y_n)$

9. $\nabla^2 u = 0$ is

a) one dimensional wave equation b) Laplace equation

c) one dimensional heat equation d) difference equation

10. Bender – Schmidt recurrence relation for one dimensional heat equation is

a) $u_{i,j+1} = \frac{1}{2}[u_{i+1,j} + u_{i-1,j}]$ b) $u_{i,j+1} = \frac{1}{2}[u_{i+1,j} - u_{i-1,j}]$

c) $u_{i,j-1} = ½ [u_{i+1,j} + u_{i-1,j}]$ d) $u_{i,j-1} = ½ [u_{i+1,j} - u_{i-1,j}]$

PART B (10 x 2 = 20 Marks)

11. State the formula of getting a root by False position method.
12. Compare Gauss – Elimination and Gauss – Sedial methods.
13. Obtain the divided difference table for the following data

x	-1	0	2	3
$f(x)$	-8	3	1	12

14. Find a polynomial which takes the following values

x	0	1	2
y	1	2	1

15. Write down the expression for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = x_n$ by Newton's backward difference formula.
16. In order to evaluate by Simpson's $1/3$ rule as well as by Simpson's $3/8$ rule, what is the restriction on the number of intervals?
17. Compute $y(0.1)$ by Taylor's series method to three places of decimals given that $\frac{dy}{dx} = x + y, y(0) = 1$.
18. State Adam's – Bash forth predictor corrector formula.
19. Obtain the finite difference scheme for the difference equation, $\frac{2d^2y}{dx^2} + y = 5$.
20. Write down the diagonal five point formula to solve the equation $U_{xx} + U_{yy} = 0$.

PART C (5 x 14 = 70 Marks)

21. a) (i) Solve the given system of equations by using Gauss – Seidal method

$$20x - y - 2z = 17$$
$$3x + 20y - z = -18. \quad (7)$$
$$2x - 3y + 20z = 25$$

(ii) Using Newton – Raphson method, solve $x \log_{10}^{x} = 12.34$ with $x_0 = 10$. (7)

(OR)

b) Determine the largest eigen value and the corresponding eigen vector of the matrix $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ by Power method.

22. a) (i) For the given values evaluate $f(9)$ using Lagrange's formula. (7)

x	5	7	11	13	17
$f(x)$	150	392	1452	2366	5202

(ii) Using Newton's forward interpolation formula find $f(2)$ (7)

x	0	5	10	15
$f(x)$	14	379	1444	3584

(OR)

b) The following values of x and y are given

x	1	2	3	4
y	1	2	5	11

Find the Cubic splines and evaluate $y(1.5)$

23. a) (i) The population of a certain town is shown in the following table

Year	1931	1941	1951	1961	1971
Population (in Thousands)	40.6	60.8	79.9	103.6	132.7

Find the rate of growth of the population in the year 1961. (7)

(ii) A river is 80 feet wide. The depth'd' in feet at a distance x feet from one bank is given by the following table

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of cross section of the river using Simpson's rule. (7)

(OR)

b) Evaluate the integral $\int_1^2 \int_1^2 \frac{dxdy}{x+y}$ using Trapezoidal rule with $h = k = 0.5$

24. a) The differential equation $\frac{dy}{dx} = y - x^2$ is satisfied by

$y(0) = 1, y(0.2) = 1.12186, y(0.4) = 1.4682, y(0.6) = 1.7379$. Compute the value of $y(0.8)$ by Milne's Predictor corrector formula.

(OR)

b) Using Runge – Kutta method of fourth order solve for y at $x = 1.2, 1.4$ from $\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x}$ with $x_0 = 1, y_0 = 0$.

25. a) Using Liebmann's iteration process solve $u_{xx} + u_{yy} = 0$ in $0 \le x \le 4, 0 \le y \le 4$. Given that $u(0, y) = 0, u(4, y) = 8 + 2y, u(x,0) = \frac{x^2}{2}$ and $u(x,4) = 2$ taking $h = k = 1$ .Obtain the result correct to one decimal.

(OR)

b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0) = 0, u(0,t) = 0$ and $u(1,t) = t$. Compute u for $t = \frac{1}{8}$ in two steps, using Crank – Nicolson formula

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