Register Number:………………..

B.E/B.TECH., DEGREE EXAMINATIONS NOV/DEC 2012

Second Semester

MAT102: ENGINEERING MATHEMATICS II

(Common to AE/AUE/CE/ECE/EEE/EIE/MECH/MECHATRONICS)

Time: Three Hours **Maximum Marks:100**

Answer ALL Questions:-

PART A (10x1=10 Marks)

1. Change the order of integration in $\int_0^2 \int_0^x f(x, y)\,dy\,dx$ is __________

a) $\int_0^x \int_0^2 f(x, y)\,dy\,dx$ b) $\int_0^y \int_0^2 f(x, y)\,dx\,dy$

c) $\int_0^2 \int_y^2 f(x, y)\,dx\,dy$ d) $\int_0^2 \int_y^2 f(x, y)\,dy\,dx$

2. $\iint dx\,dy$ over the region bounded by $x = 0$, $x = 2$, $y = 0$ and $y = 2$ is

a)1 b) 2 c) 3 d) 4

3. The Value of Div (curl $\vec{f}$) is _________

a) Curl (div $\vec{f}$) b) Zero c) 1 d) curl (curl $\vec{f}$)

4. div (grad ϕ) = _________

a) $\vec{0}$ b) div ϕ c) grad ϕ d) $\nabla^2 \phi$

5. if f (z)=e^{2z} then the real part of f(z) is

a) $e^y \sin x$ b) $e^x \cos y$ c) $e^{2x} \cos 2y$ d) $e^y \cos y$

6. Cauchy – Riemann equation in polar co – ordinates is

a) $u_r = \frac{1}{r} v_\theta$ and $v_r = \frac{-1}{r} u_\theta$ b) $\frac{1}{r} u_r = v_\theta$ and $\frac{1}{r} v_r = -u_\theta$

c) $u_r = \frac{-1}{r} v_\theta$ and $v_r = \frac{1}{r} u_\theta$ d) $\frac{1}{r} u_r = -v_\theta$ and $\frac{1}{r} v_r = u_\theta$

7. The value of $\int_c \frac{3z^2+7z+1}{z+1} dz$ if c is $|z| = \frac{1}{2}$ is

a) 1 b) 0 c) -1 d) 2

8. f (z) = $\frac{1}{z^2+1}$ is analytic everywhere except

a) z =1 b) z = ± 1 c) z = $\pm$ i d) z = i

9. L $[tf(t)]$ =

a) $\frac{Lf(t)}{s}$ b) $\frac{-d}{ds}$ Lf (t) c) $\frac{d}{ds}$ Lf (t) d) $\frac{-Lf(t)}{s}$

10. $L^{-1}\left(\frac{1}{s^2+a^2}\right)$ =

a) $\frac{\cos at}{a}$ b) $\sin at$ c) $\frac{\sin at}{a}$ d) $\cos at$

PART B (10 x 2 = 20 Marks)

11. Evaluate $\int_1^a \int_1^b \frac{dydx}{xy}$

12. Evaluate $\int_0^{\pi} \int_0^5 r^4 \sin\theta dr d\theta$

13. Find the unit vector normal to the surface of the sphere $x^2+y^2+z^2=1$.

14. If S is any closed surface enclosing a volume v and $\vec{F} = x\vec{i} + 2y\vec{j} + 3z\vec{k}$, find $\int_c \bar{F}.\hat{n}\, ds$

15. Test whether the function $x^4 - 6x^2y^2 + y^4$ is harmonic or not.

16. Find the invariant points of the transformation w = $\frac{2z+4i}{iz+1}$

17. State Cauchy's integral formula.

18. Evaluate $\int_c \frac{dz}{z^2 e^z}$ where C is $|z| = 1$

19. State the conditions for the existence of Laplace transform.

20. Evaluate $L^{-1}\left(\frac{s-1}{s^2-2s+5}\right)$

PART C (5 x 14 = 70 Marks)

21. a) (i) Evaluate $\iiint_R (x - y + z)dxdydz$ where R is given by $1 \le x \le 2$,

$2 \le y \le 3, 1 \le z \le 3$. (7)

(ii) Find the area between the parabolas $y^2 = 9x$ and $x^2 = 9y$ (7)

(OR)

b) (i) Change the order of integration in $\int_o^a \int_y^a \frac{xdydx}{x^2 + y^2}$ and hence evaluate it. (7)

(ii) Change in to Polar Co – ordinates and then evaluate (7)

$$\int_o^2 \int_0^{\sqrt{2x-x^2}} \frac{xdy}{\sqrt{x^2 + y^2}} \, dx$$

22. a) (i) Find the Value of the constant a, b, c so that the vector

$\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. (4)

(ii) Verify Green's theorem for $\int_c (x^2 - y^2)dx + 2xydy$ Where 'C' is the boundary of the rectangle in the XOY – plane bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$ (10)

(OR)

b) Verify divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\bar{J} + (z^2 - xy)\bar{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.

23. a) (i) Find the analytic function f(z)=u+ iv given that $u - v = e^x(\cos y - \sin y)$ (7)

(ii) Find the bilinear transformation which maps Z=1, i, -1 respectively on to w = i, 0, - i respectively. (7)

(OR)

b) (i) If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (8)

(ii) Find the image of the circle $|z - 1| = 1$ in the complex plane under the mapping $w = \frac{1}{z}$ (6)

24. a) (i) Using Cauchy's integral formula Evaluate $\int_c \frac{z+1}{z^2+2z+4} dz$ where C is the circle $|z+1+i| = 2$ (7)

(ii) Obtain Taylor's series to represent the function $\frac{z^2-1}{(z+2)(z+3)}$ in the region $|Z| < 2$. (7)

(OR)

b) (i) Evaluate $\int_c \frac{e^z}{(z+1)^2} dz$ around the circle $|z-1| = 3$ using residue theorem. (6)

(ii) Evaluate $\int_o^{2\pi} \frac{d\theta}{13+5\sin\theta}$ (8)

25. a) (i) Find $L\left[t^2 e^{3t} \sinh t\right]$ (6)

(ii) Find the Laplace transform of the function $f(t) = \begin{cases} t & 0 < t < b \\ 2b - t, & b < t < 2b \end{cases}$ with $f(t+2b) = f(t)$. (8)

(OR)

b) (i) Find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$ using convolution theorem. (7)

(ii) Using Laplace transform solve $y'' + y = 2e^t$, $y(0) = 1, y'(0) = 2.$ (7)

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