Register Number:………………..

B.E/B.TECH., DEGREE EXAMINATIONS NOV/DEC 2012

Second Semester

MAT103: ENGINEERING MATHEMATICS II

(Common to CSE/IT/TXT/FT/BT)

Time: Three Hours **Maximum Marks:100**

Answer ALL Questions:-

PART A (10x1=10 Marks)

1. The value of $\int_0^1 \int_0^{1-x} y\, dy\, dx$ is --------.

A) 1/5 B) 1/7 C) 1/6 D) 1/9

2. Area in polar coordinates = ---------------

A) $\iint xydxdy$ B) $\iint dxdy$ C) $\iint \frac{1}{r}\; dr\, d\theta$ D) $\iint rdrd\theta$

3. The gradient of Φ where Φ is $3x^2y-y^3z^2$ at the point (1,-2,-1) is ---------.

A) $-12\vec{i} - 9\vec{j} - 16\vec{k}$ B) $-12\vec{i} - 19\vec{j} - 16\vec{k}$

C) $-12\vec{i} - 9\vec{j} - 36\vec{k}$ D) $-52\vec{i} - 9\vec{j} - 16\vec{k}$

4. The value of a such that the vector $\vec{F} = (3x-2y+z)\vec{i} + (4x+ay-z)\vec{j} + (x-y+2z)\vec{k}$ solenoidal is --------------

A) 5 B) -5 C) 10 D) -10

5. If u and v are harmonic, then u+iv is ------------

A) not analytic B) analytic C) orthogonal D) invariant

6. If f(z) = u+iv is an analytic function of z = x+iy ,then u and v satisfy ------------ equation.

A) Poisson B) Laplace C) Elliptic D) Hyperbolic

7. The cross ratio of the points z_1, z_2, z_3, z_4 is ------------------

A) $\frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_4)(z_3-z_2)}$ B) $\frac{(z_1+z_2)(z_3+z_4)}{(z_1+z_4)(z_3+z_2)}$ C) $\frac{(z_1+z_2)(z_3+z_4)}{(z_1-z_4)(z_3-z_2)}$ D) $\frac{(z_1-z_2)(z_3-z_4)}{(z_1+z_4)(z_3+z_2)}$

8. The invariant points of $w = \frac{2z+6}{z+7}$ is -------------

A) (6,-1) B) (5, 2) C) (1,-6) D) (-5,-2)

9. The poles of cotz are --------------

A) z = 0,1,2,3…… B) z = -1,-2,-3 …. C) z = $\pi, 2\pi, 3\pi$ …. D) z = 0,$\pm\pi, \pm 2\pi$, $\pm 3\pi$ … ..

10.If c is the circle with centre at origin and radius 2 ,then $\int_c \frac{z}{z-4}\, dz$ is -------

A) 1 B) -1 C) 0 D) 6

PART B (10x2 =20 Marks)

11. Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} xydydx$

12. Transform into polar coordinates and hence evaluate $\iint xy(x^2+y^2)^{\frac{3}{2}}\, dxdy$ over the positive quadrant of the circle $x^2 + y^2 = k^2$.

13. Find the constants a, b, c so that the vector $\vec{F} = (x+2y+az)\vec{i} + (bx+3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational.

14. Find the unit vector normal to the surface $x^2 + y^2 - z^2 = 1$ at (1, 1, 1).

15. Show that the function f(z) = $\bar{z}$ is no where differential.

16. Prove that an analytic function with constant real part is constant.

17. Find the image of the circle |z| =C by the transformation w =5z.

18. Define fixed point.

19. Find the singularities of f(z) =$\frac{cot\pi z}{(z-a)^3}$.

20. Evaluate $\int_c \frac{z^2+5}{z-3}$ dz where c is the circle |z| =4.

PART C (5x 14 = 70 Marks)

21. a) (i) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xydydx$ and hence evaluate it.

(7)

(ii) Find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (7)

(OR)

b) (i) Find the area of the region outside the inner circle r = 2cosθ and inside the outer circle r = 4cosθ by double integration. (7)

(ii) Evaluate the integral $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ (7)

22. a) Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{\imath} + (yz + 4)\vec{\jmath} - xz\vec{k}$ where s is the surface of the cube x=0,x=2,y=0,y=2,z=0 and z=2 above xy-plane.

(OR)

b) Verify Gauss divergence theorem for the function $\vec{F} = 4x\vec{\imath} - 2y^2\vec{\jmath} + z^2\vec{k}$ taken over the surface region bonded by the planes x=0, x=1, y=0, y=2,z=0 and z=3.

23. a) (i) Prove that $u = e^{x^2-y^2}$, $\cos y$ is harmonic and find its conjugate harmonic. (7)

(ii) If f(z) is an analytic function of z , prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$ (7)

(OR)

b) (i) If u = $\frac{sin2x}{cosh2y+cos2x}$, find the corresponding analytic function f(z). (7)

(ii) If f(z) = u(x, y) +i v(x, y) is an analytic function, then prove that the two curves u(x,y) =C_1 and v(x,y)=C_2 where C_1 and C_2 are constants are cut orthogonally. (7)

24. a) (i) Show that the transformation $w = \frac{1}{z}$ transforms circles and straight line in the z-plane into circles or straight lines in the w-plane. (8)

(ii) Find the critical points for the transformation $\omega^2 = (z-\alpha)(z-\beta)$ (6)

(OR)

b) (i) Find the bilinear transformation which maps the points z= -i,0,i into the points w= -1,i,1 respectively. (6)

(ii) Discuss the transformation w = z^2 (8)

25. a) (i) Using Cauchy's integral formula, evaluate $\int_c \frac{sin\pi z^2+cos\pi z^2}{(z-2)(z-3)}\ dz$ where c is the circle $|z| = 4$. (7)

(ii) Find the Laurent's series expansion of $f(z) = \frac{7z-2}{z(z-2)(z+1)}$ in the region $1< |z+1|<3$. (7)

(OR)

b) (i) Using contour integration , solve $\int_0^\infty \frac{cosmx}{x^2+a^2}\ dx$ where $m\geq 0$. (7)

(ii) Evaluate $\int_c \frac{e^z}{(z^2+\pi^2)^2}\ dz$, where C is the circle $|z|=4$ by using Cauchy's Residue theorem. (7)

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