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**M.C.A DEGREE EXAMINATIONS: JUNE 2012**

Second Semester

**MASTER OF COMPUTER APPLICATIONS**

MAT509: Mathematical Foundations of Computer Science

**Time: Three Hours** **Maximum Marks: 100**

**Answer all the Questions:**

**PART A (10 x 2 = 20 Marks)**

1. State the condition in terms of ranks for a system of equations AX = B to possess a solution.

2. Find the sum of the squares of the eigen values of A = $\begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

3. Prove that $A - B = B^c - A^c$.

4. Write down the matrix of the relation R = {(1,1) (1,3) (2,2) (3,3)}

5. Construct the truth table for $P \rightarrow (P \vee Q)$.

6. Symbolize the following expression “ X is the father of the mother of Y”.

7. Define Universal quantifiers with example.

8. Give a grammar for palindromes over {a, b}.

9. Define Finite state automata.

10. Distinguish between DFA and NFA

**PART B (5 x 16 = 80 Marks)**

11.a) (i) Test for consistency and solve if consistent.

$x + 2y + z = 3;\ 2x + 3y + 2z = 5;\ 3x - 5y + 5z = 2;\ 3x + 9y - z = 4.$

(ii) Verify Cayley-Hamilton theorem for the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{pmatrix}$ and hence find $A^{-1}$ and $A^4$.

**(OR)**

b) (i) Find the eigen values and eigen vectors of the matrix $\begin{pmatrix} 3 & -4 & 4 \\ 1 & -2 & 4 \\ 1 & -1 & 3 \end{pmatrix}$.

(ii) Prove that the eigen values of a real symmetric matrix are real.

12.a) (i) In a survey of 60 people, it was found that 25 read Newsweek magazine, 26 read Time, 26 read Fortune. 9 read both Newsweek and Fortune, 11 read both Newsweek and Time, 8 read both Time and Fortune, 3 read all three magazines.

1. Find the number of people who,read atleast one of the three magazine.
2. Find the number of people who read exactly one magazine.

(ii) Let A be a set of non zero integers and let R be the relation on A x A defined by (a,b)R(c,d) whenever ad = bc. Prove that R is an equivalence relation.

**(OR)**

b) (i) If A,B,C are any three sets then prove that $A-(B\cap C)=(A-B)\cup(A-C)$.

(ii) Check whether the following functions are bijective or not.

1. $f(x)=-2x$ 2. $g(x)=x^2-1$ 3. $\begin{aligned} h(x)&=x \quad if\ x<0 \\ &=x^2\ \ if\ x\geq 0 \end{aligned}$.

13.a) (i) Obtain the PDNF and PCNF of $P\rightarrow\left[(P\rightarrow Q)\wedge\neg(\neg Q\vee\neg P)\right]$.

(ii) Prove that the following argument is valid.” In a Triangle XYZ, there is no pair of angles of equal measure. If a triangle has two sides of equal length then it is isosceles. If a triangle is isosceles then it has two angles of equal measure. Hence triangle XYZ has no two sides of equal length.

**(OR)**

b) (i) Prove that $P\rightarrow Q,\neg Q\vee R,\neg(R\wedge\neg S),P\Rightarrow S$ by direct method.

(ii) Prove that $(x)(P(x)\vee Q(x))\Rightarrow(x)P(x)\vee(\exists x)Q(x)$ .

14.a) (i) Construct a grammar for the language $L=\left\{a^nb^n : n\geq 1\right\}$

(ii) State Pumping lemma for regular languages. .Also prove that the set $L=\left\{a^nba^n : n\geq 1\right\}$ is not regular.

**(OR)**

b) (i) What are the various types of a phrase structure grammar with examples to each

(ii) Derive the strings 1) ababbbba 2) bbbcbba

given G1: $\left\{S\rightarrow AbS, A\rightarrow aS, S\rightarrow ba\ and\ A\rightarrow b\right\}$ and

G2: $\left\{S\rightarrow bcS, S\rightarrow bbS, S\rightarrow cb\ and\ S\rightarrow a\right\}$

15.a) (i) Construct DFA equivalent to the NFA $M = \{(p,q,r,s),(0,1),\delta, p,\{s\}\}$ where $\delta$ is given by

| State \ Input | 0 | 1 |
|---|---|---|
| p | p,q | p |
| q | r | r |
| r | s | $\phi$ |
| s | s | s |

(ii) Construct FSA that accepts the set of Natural numbers x which are divisible by three.

**(OR)**

b) (i) Construct NFA recognizing L(G) where G is the grammar

$S \rightarrow aS / bA / b : A \rightarrow aA / bS / a$ .

(ii) Construct DFA accepting the set of all strings ending in 00 over the alphabet{0,1}.

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